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Experimental investigation of an oscillator with discontinuous support considering different system aspects

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Abstract

Non-smooth characteristics are, in general, the source of difficulties for the modeling and simulation of natural systems. These characteristics are usually related to either the friction phenomenon or the discontinuous behavior as intermittent contacts. This article develops an experimental investigation concerning non-smooth systems with discontinuous support. An experimental apparatus is developed in order to analyze the nonlinear dynamics of a single-degree of freedom system with discontinuous support. The apparatus is composed by an oscillator constructed by a car, free to move over a rail, connected to an excitation system. The discontinuous support is constructed considering mass–spring systems separated by a gap to the car position. This apparatus is instrumented to obtain all the system state variables. System dynamical behavior shows a rich response, presenting dynamical jumps, bifurcations and chaos. Different configurations of the experimental set up are treated in order to evaluate the influence of the internal impact within the car and also support characteristics in the system dynamics.

1. Introduction

Non-smooth nonlinearity is usually related to either the friction phenomenon or the discontinuous characteristics of dynamical systems, appearing in many kinds of engineering systems and also in everyday life. Examples may be mentioned by the stick–slip oscillations of a violin string or grating brakes [18]. Some related phenomena as chatter and squeal causes serious problems in many industrial applications and, in general, these forms of vibrations are undesirable because of their detrimental effects on the operation and performance of mechanical systems [1]. Therefore, non-smooth systems have being analyzed in order to understand various engineering problems: Oil drilling [17,4,7], rotor dynamics [5] and manufacturing [13] are some interesting examples.

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The mathematical modeling and numerical simulations of non-smooth systems present many difficulties, which turns their description unusual complex. Moreover, the dynamical behavior of these systems is complex, presenting a rich response. Literature presents reports dealing with different aspects of non-smooth systems [3,9].

Since non-smooth systems present an unusual complex behavior and their description involves many mathematical and numerical difficulties [14,6,3,9], experimental studies are of great importance. Literature presents references focusing experimental approaches to verify the proposed numerical methods, and also discussing just the experimental point of view of non-smooth systems. Wiercigroch et al. [16] and Wiercigroch and Sin [15] present an experimental analysis of a base excited symmetrically bilinear oscillator. Virgin and co-workers also develops interesting experimental analyses related to non-smooth systems [12,11,2,10,8].

This article deals with the nonlinear analysis of dynamical systems with discontinuities. This issue is previously addressed in Divenyi et al. [3] and Savi et al. [9] where the mathematical modeling, numerical simulation and experimental verification of a one-degree of freedom oscillator with discontinuous support are of concern. Now, this problem is revisited considering an experimental investigation of some variations of this system. The experimental apparatus is composed by an oscillator constructed by a car, free to move over a rail, connected to an excitation system. The discontinuous support is constructed considering a mass–spring system separated by a gap related to the car position. This apparatus is instrumented to obtain all system state variables. Different aspects related to the system dynamics are carried out considering the influence of internal impacts within the car and also support characteristics. Despite the deceiving simplicity of this system, its nonlinear dynamics is very rich.

2. Experimental apparatus

The dynamical response of discontinuous systems is analyzed from a single-degree of freedom oscillator with discontinuous support, shown in Fig. 1. The oscillator is composed by a mass m and part of this mass, m_i, is free to move through a guide within the car, impacting at its ends. The mass is connected to linear springs with stiffness k. The dissipation process can be modeled by linear viscous damping with coefficient c. The support is a linear oscillator composed by a mass m_s, a spring with stiffness k_s and, again, a dissipation process is represented by a linear damping with coefficient c_s. The mass displacement is denoted by x, relative to the equilibrium position and the original distance between the mass and the support is defined by a gap g. Since it is possible to consider different properties and also different gaps to each support, it is employed the superscripts L and R to respectively identify the left and right hand side support. This system has two possible modes, represented by a situation where the mass presents contact with the support and another situation when there is no contact. Moreover, it is assumed that the system is subjected to a harmonic excitation $F(t) = \rho \sin(\omega t)$. There are many simplifications that can be done to this general system, depending on

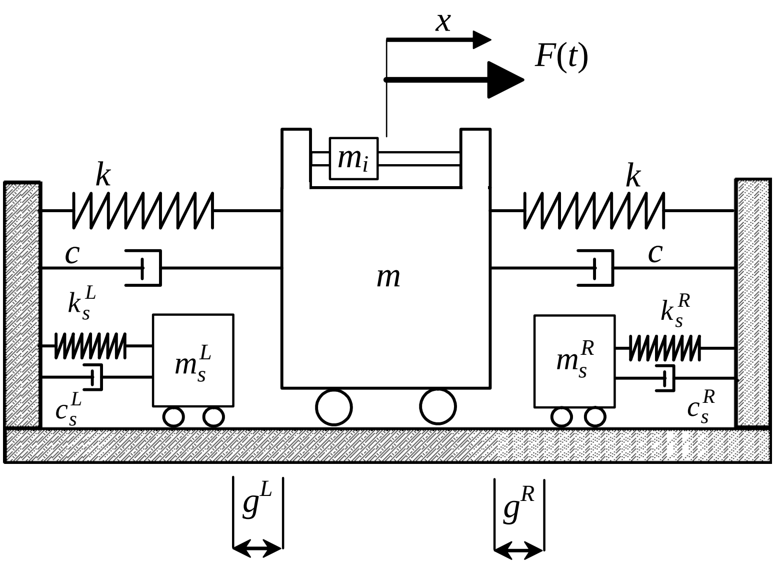

Fig. 1. Non-smooth system with discontinuous support.

the system characteristics. When the support relaxation time is much smaller than the time between two contact events, the support dynamics can be neglected and therefore, it may be assumed massless. Another simplification arises when the impact occurs just in one side. Concerning the internal impact, the impact mass may be fixed to the car and, therefore, the system may not present this kind of behavior.

An experimental apparatus of this system is developed considering an oscillator composed by a car (4), free to move over a rail (2), connected to an excitation system composed by springs (3), strings and a DC motor (1) (PASCO ME-8750 with 0–12 eV, 0–0.3 A). Dissipation characteristics may be adjusted by a magnetic damping device (6). The car has an internal mass which can move through a guide (7). The discontinuous support is constructed, at the left hand side, by considering a spring separated by a gap related to the car position (5). On the other hand, at the right hand side, there is a mass connected to the support spring and fixed by a string in order to always present compressive behavior (8). Actually, different supports may be used depending on the analysis. The movement is measured with the aid of a rotary sensor (9), PASCO CI-6538, which has a precision of $\pm 0.25°$, maximum velocity of 30 rev/s and maximum sampling frequency of 1000 Hz. The apparatus is shown in Fig. 2, together with detailed pictures of some parts.

Parameters identification may be done by different procedures. Savi et al. [9] develop numerical and experimental investigations dealing with an oscillator without internal impact, where the discontinuity is represented by a single support, massless. This reference employs a weight scale to measure the system mass while the spring stiffness is evaluated through the slope of a force–displacement curve, plotted with the aid of two sensors: the rotary sensor shown in Fig. 2 and a force transducer. Concerning the dissipation characteristics, it is assumed a linear viscous damping in the considered range and the determination of the oscillator dissipation parameter is done analyzing the system frequency response. The support dissipation parameter is identified assuming the logarithmic decrement procedure, which is defined verifying the ratio between two consecutive displacement amplitudes. Moreover, forcing characteristics, namely the amplitude and frequency forcing, are respectively related to the motor length of rotating crank and motor voltage.

In this article, the system dynamical response is analyzed evaluating the influence of different characteristics related to either the car characteristics or the support and, as a matter of fact, parameters identification is not the main issue. With respect to the car characteristics, it is considered a further impact system within the car. Concerning the support

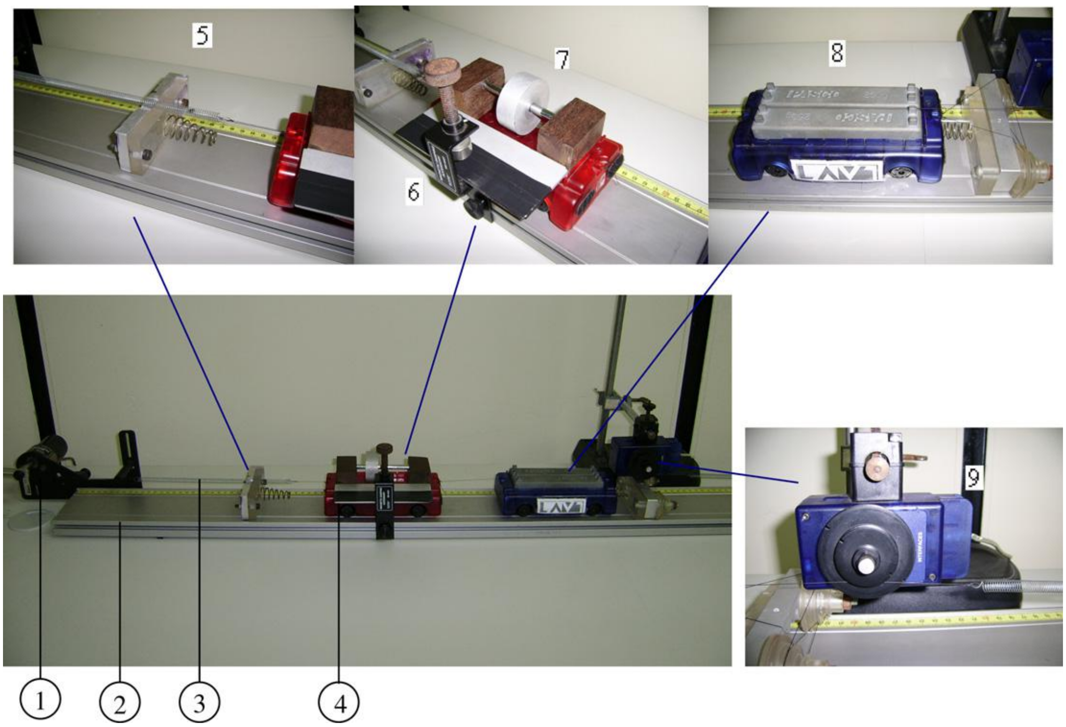

Fig. 2. Experimental apparatus.

characteristics, the inclusion of two supports and also different properties as stiffness and damping are considered. Moreover, it is investigated the inclusion of inertia aspects, connecting a mass to the support.

3. Influence of the car characteristics

The influence of the car characteristics is now investigated by considering a single support. Under this condition, there is only a single gap, denoted by $g = g^R$. The goal is to consider an internal impact mass, which may be free to move through a guide within the car, impacting at its ends. Since this impact mass may be fixed to the car, it is possible to establish a comparison between the dynamics with or without internal impacts. The car has a total mass $m = 0.471$ kg and different configurations are analyzed by changing the initial position of the impact mass and also the guide lubrication.

A comparison of the system dynamics considering the car with and without the internal impacts is presented in Fig. 3 assuming $g = 22.65$ mm, $\rho = 0.76$ N and $\omega = 1.51$ Hz. The situation without impact is generated restricting the internal mass movement. Results with impacts present smaller amplitudes than those without impacts where the car is in the imminence to jump out of the rail. Internal impacts tend to dissipate system energy and, therefore, small amplitude responses are expected.

At this point, it is analyzed the impact mass initial position influence. Two different situations are considered (Fig. 4): In the first one, the impact mass starts at the end near the support and therefore, it is associated with a smaller first impact. The second configuration, on the other hand, is associated with the impact mass starting the movement in the opposite end of the support being related to a greater first impact.

A comparison between the responses of both configurations for $g = 62.1$ mm, $\rho = 0.76$ N, $\omega = 1.03$ Hz is presented in Fig. 5. When the system has a smaller first internal impact, the system response is related to a period-2 orbit. By considering the other situation, the first internal impact changes the orbit and the system response is related to a period-1 orbit. The right side of Fig. 5 presents the transient response showing the influence of the first impact that causes the orbit change responsible for the change in the system response.

Now, the mass guide is lubricated causing differences in the system response. A comparison between the system with and without a special lubrication assuming $g = 50.4$ mm, $\rho = 0.76$ N, $\omega = 1.41$ Hz is presented in Fig. 6. Notice that internal impacts cause perturbations in the system response, and the original orbit presents oscillations during a cycle. These oscillations are probably due to the force increase during internal impacts. Other situations may also cause the increase of the number of impacts for the same energy level.

4. Influence of the support characteristics

In order to analyze the influence of the support characteristics in the system dynamics, the experimental set up is altered. The analysis begins by assuming a single support ($g = g^R$) and the first consideration is just the change of the support elastic element to a spring with higher stiffness and also to a rigid rubber. The impact mass is now restrict

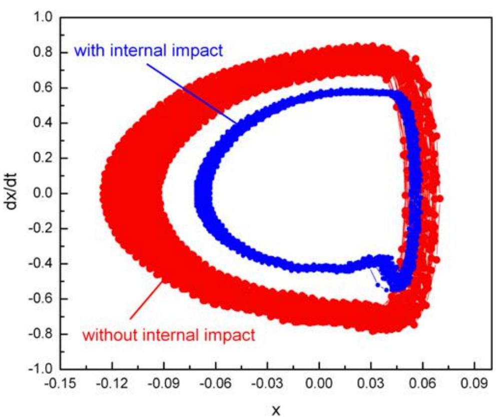

Fig. 3. Effect of internal impact. $g = 22.65$ mm, $\rho = 0.76$ N, $\omega = 1.51$ Hz.

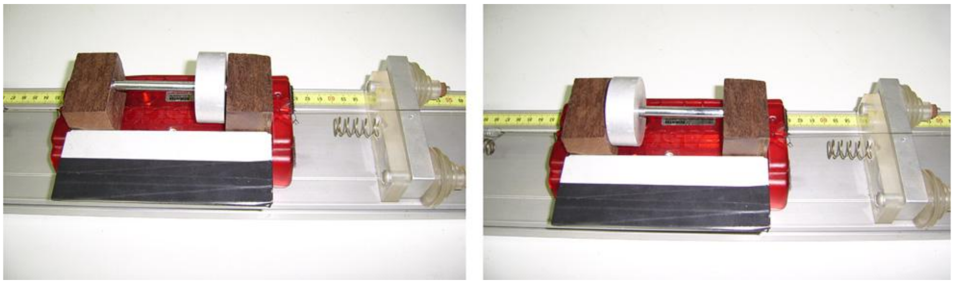

Fig. 4. Different configurations related to the impact mass initial condition.

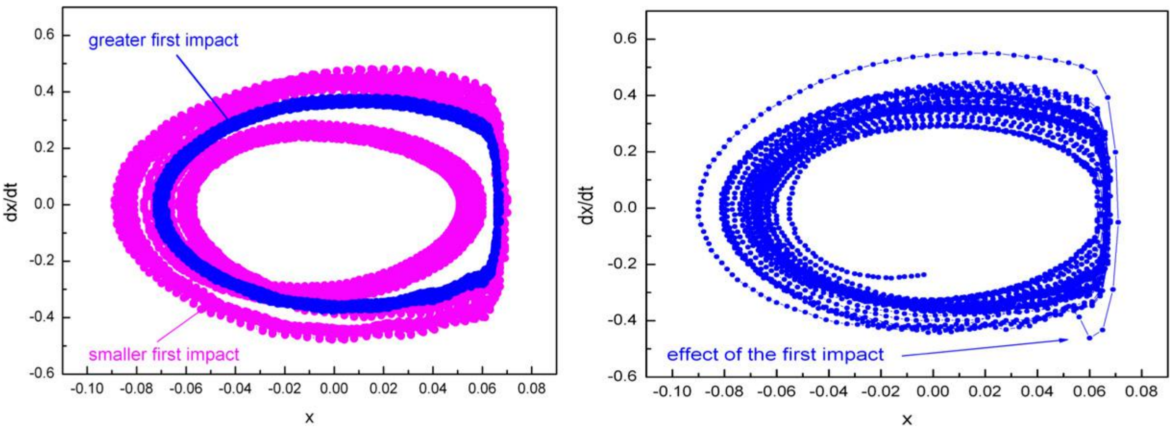

Fig. 5. Effect of internal impact: comparison between different initial positions of the internal mass. $g = 62.1$ mm, $\rho = 0.76$ N, $\omega = 1.03$ Hz.

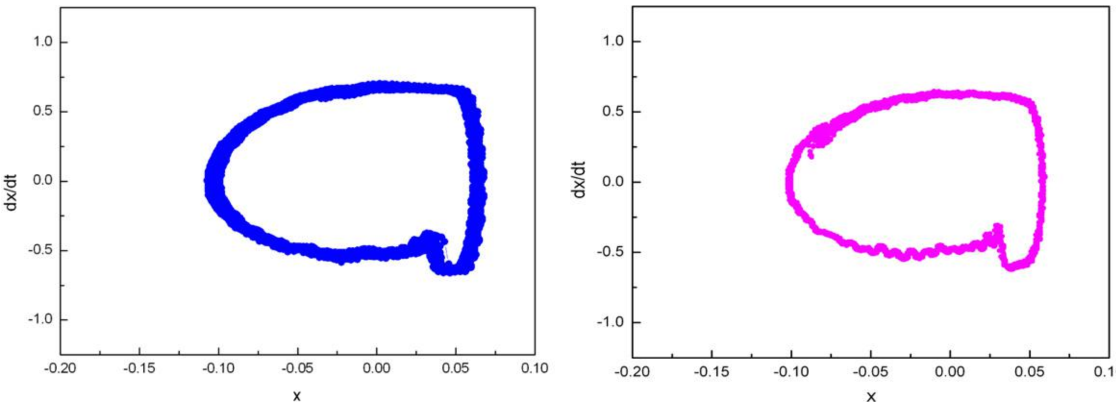

Fig. 6. Effect of internal impact: lubricated guide. $g = 50.4$ mm, $\rho = 0.76$ N, $\omega = 1.41$ Hz.

to move through the guide and, therefore, there are no internal impacts. Fig. 7 shows different supports employed in the analysis representing three situations: low stiffness spring, high stiffness spring and rubber.

Fig. 8 shows the response for each support considering an oscillator with a mass $m = 0.838$ kg, excitation parameters $\rho = 0.75$ N, $\omega = 0.80$ Hz and three different gaps: 2.5, 11.0 and 16.3 mm. The increase of the support stiffness causes a reduction on the phase space since the mass is not capable to reach certain positions in the contact region. For the small gap, $g = 2.5$ mm, it is noticeable that the low stiffness spring impacts just one time in a cycle. The other two supports,

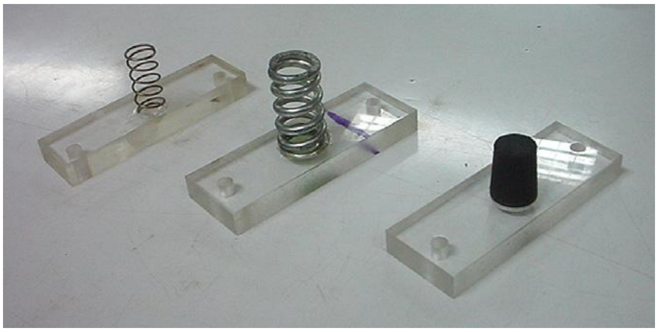

Fig. 7. Different supports.

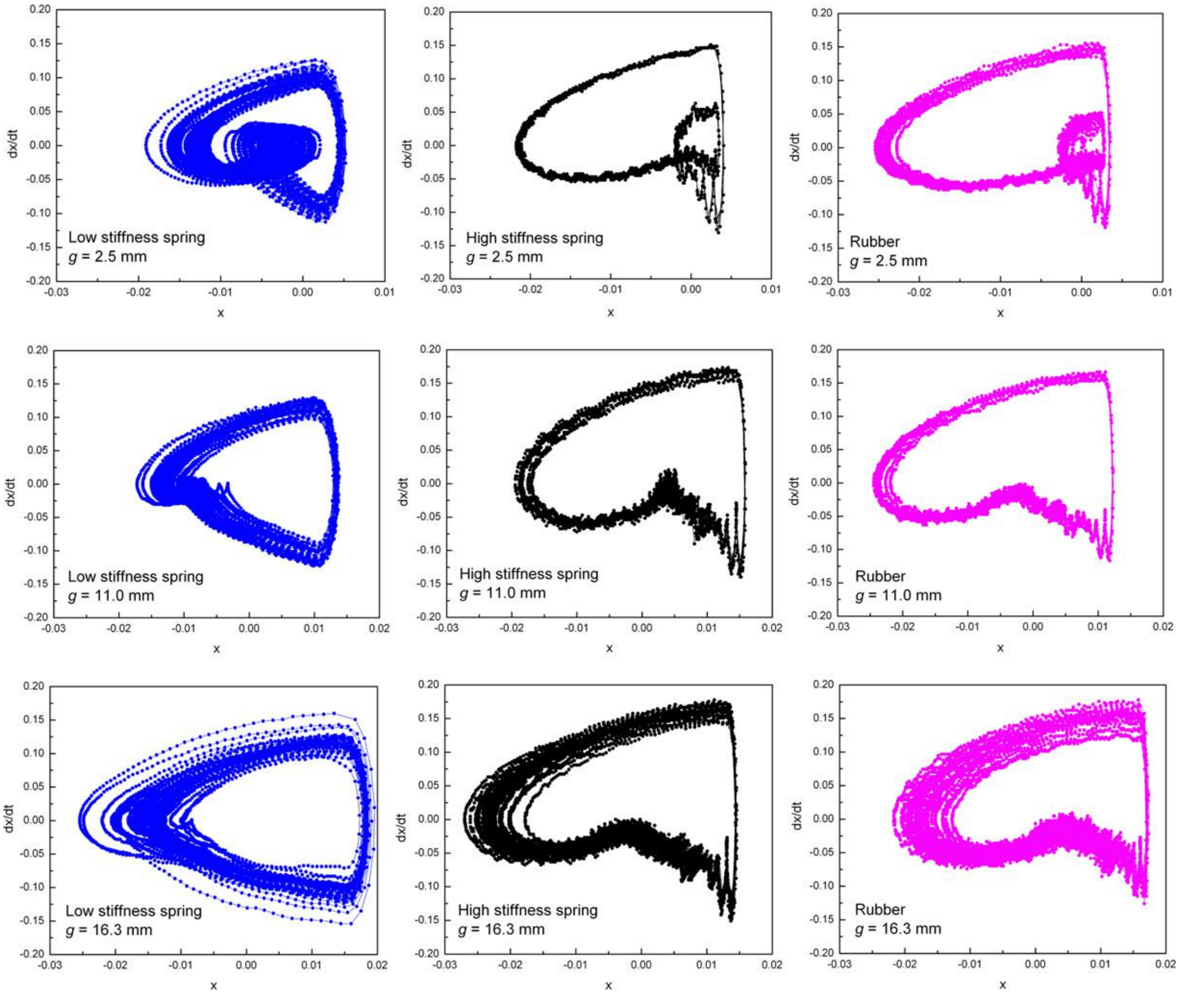

Fig. 8. Influence of different support characteristics.

high stiffness spring and rubber, on the other hand, impacts twice. Besides, low stiffness spring presents smaller amplitudes than those obtained by the other supports. By changing the gap, the system presents similar qualitative results for all supports.

The support inertia is now focused on. This investigation is done considering a system with a single support where a new car is connected to the support (Fig. 2). It should be pointed out that the car is fixed by a string and, therefore, its response is always associated with spring compressive behavior. Different forcing frequencies and support masses are analyzed by considering an oscillator with mass $m = 0.471$ kg. The first noticeable effect associated with the support mass increase is the velocity decrease when the car looses the contact. This behavior may be understood just thinking in terms of momentum conservation. By observing the state space orbit, it is also perceptible the orbit inclination. The forthcoming results are obtained assuming $g = 13.35$ mm and $\rho = 0.75$ N. A comparison between the system with and without support inertia for $\omega = 0.75$ Hz is shown in Fig. 9. The same comparison is presented in Figs. 10 and 11 for $\omega = 0.97$ Hz and $\omega = 1.29$ Hz, respectively.

The next results show the influence of the support mass increase, considering three m_s values: 0.261 kg, 0.511 kg, 0.761 kg. This analysis is done assuming $g = 5.55$ mm, $\rho = 0.75$ N and $\omega = 0.88$ Hz. Fig. 12 shows that together with the inclination of the orbit, similar to those of the previous results, there is a decrease of the second oscillation amplitude (internal loop). Notice again the velocity decrease (related to the contact loss instant) as the support mass is increased.

At this point, it is investigated the combination of the support inertia with the internal impact. It is assumed an oscillator with $m = 0.471$ kg and a support inertia of $m_s = 0.761$ kg. Moreover, $g = 49.7$ mm, $\rho = 0.65$ N and $\omega = 0.94$ Hz. Fig. 13 shows the difference between the system dynamics with and without internal impact. The left side picture presents the dynamics related to the non-impact system while the right side picture presents the system where the impact mass is free to move through the guide. Notice that the internal impact tends to increase the response complexity.

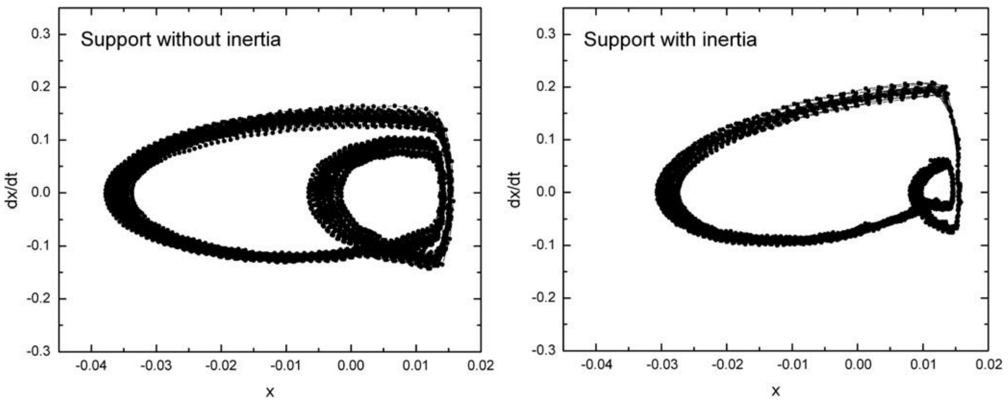

Fig. 9. Effect of support inertia for $\omega = 0.75$ Hz.

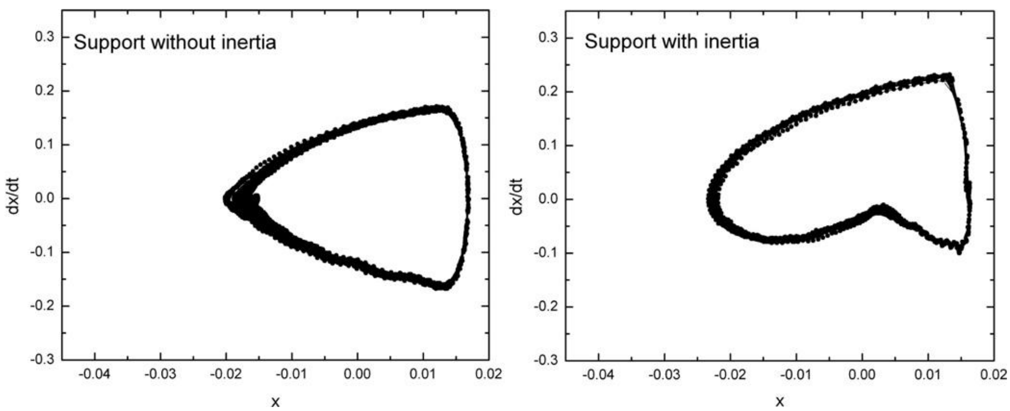

Fig. 10. Effect of support inertia for $\omega = 0.97$ Hz.

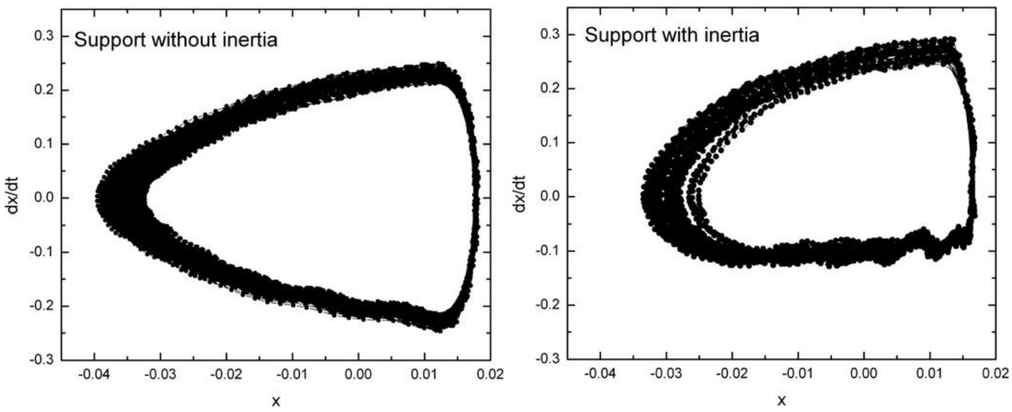

Fig. 11. Effect of support inertia for $\omega = 1.29$ Hz.

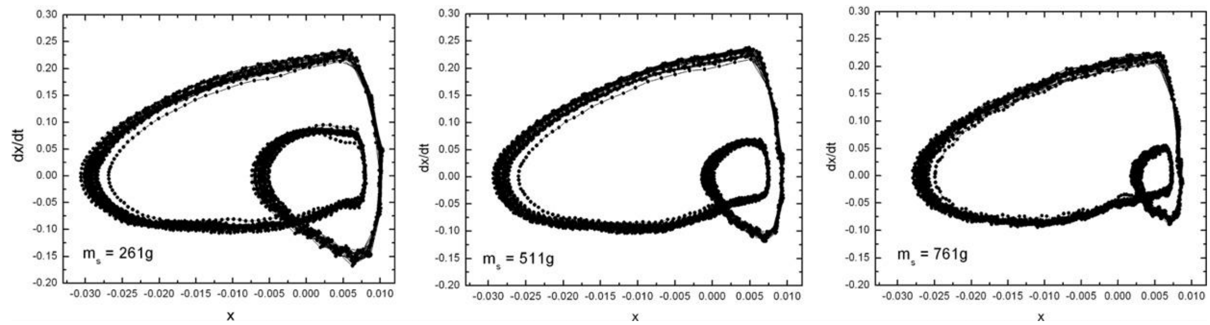

Fig. 12. Comparison of the system dynamics for different values of the support inertia.

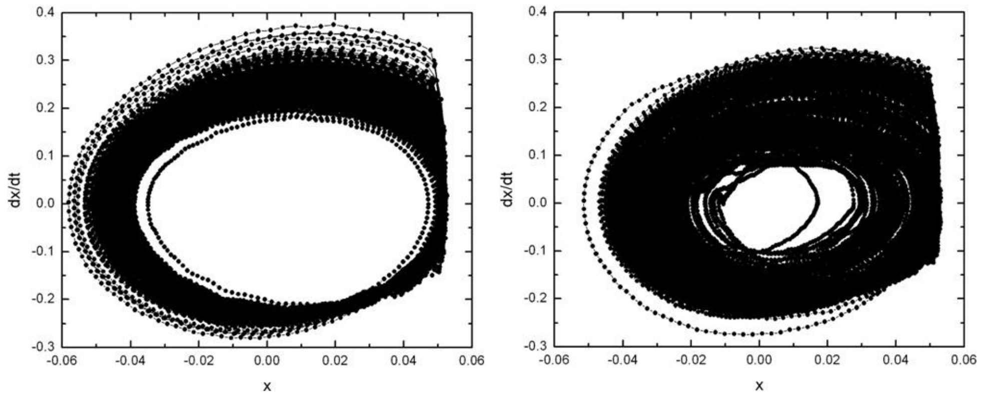

Fig. 13. Comparison of the system dynamics with support inertia and internal impacts.

5. Influence of two supports

At this point, the experimental set up is altered in order to consider two supports. At first, the influence of different support stiffness combinations is of concern assuming $\rho = 0.79$ N, $\omega = 1.03$ Hz, $g^L = -9.5$ mm, $g^R = 9.5$ mm and $m = 0.838$ kg. The analysis considers the same left support, using three different right supports, basically associated with low, intermediate and high stiffness springs. Results considering these three configurations are presented in Fig. 14 showing that, as expected, the increase in support stiffness reduces the phase space since the mass is not capable to reach

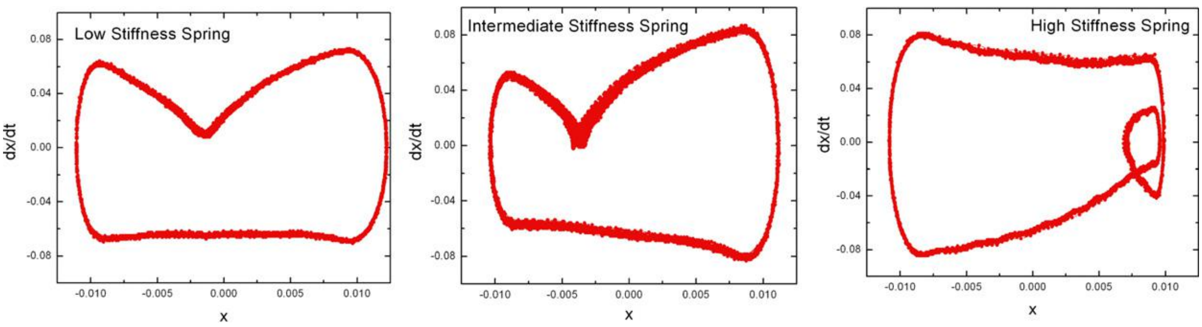

Fig. 14. Comparison of the system dynamics with two supports and different stiffness.

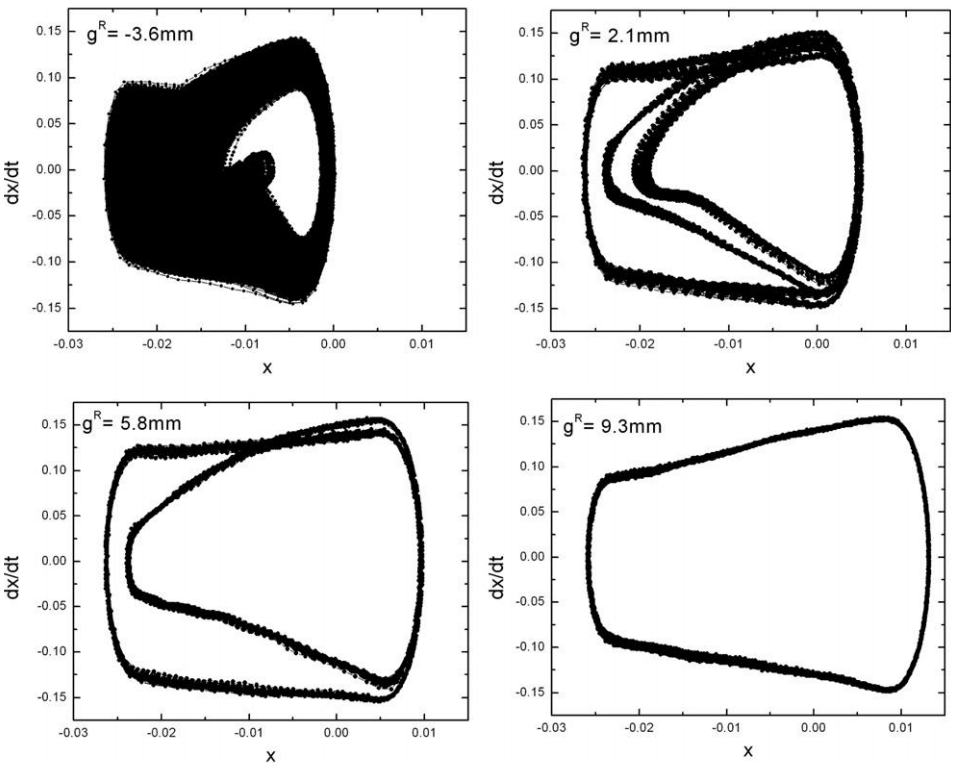

Fig. 15. Comparison of the system dynamics with two supports and different gaps.

certain positions in the contact region. Besides, this can change the dynamical response causing period-2 response presented by the high stiffness spring responses.

The gap influence is now evaluated considering $\rho = 0.79$ N and $\omega = 1.53$ Hz. The left side support is fixed with a gap $g^L = -23.95$ mm, while the right side support gap is varied assuming different values: -3.6 mm, 2.1 mm, 5.8 mm, 9.3 mm. Results obtained from these configurations are presented in Fig. 15. Notice that, as the gap increases, the complexity of the system response decreases. For the first gap, the system presents a chaotic-like response and after that, the gap decrease causes a period-4, period-2 and finally, a period-1 response. These results show a qualitative change on system dynamics, being related to bifurcations among these results.

The influence of internal impacts on a system with two supports is now focused on. Therefore, it is considered situations with and without internal impact and different gaps, assuming $\rho = 0.79$ N and $\omega = 1.44$ Hz. The right side support is fixed with a gap $g^R = 34.2$ mm, while the left side support gap is varied assuming different values: -12.5 mm, -34.2 mm and -72.2 mm. Results obtained without internal impact are presented in Fig. 16, while results with impacts

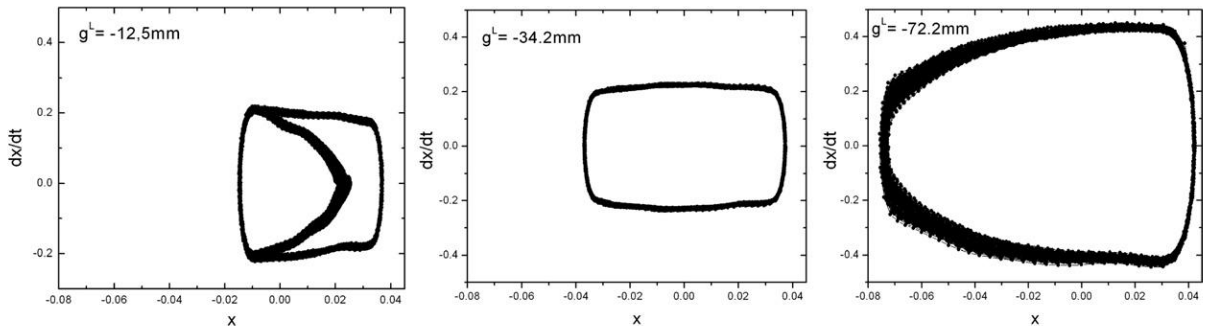

Fig. 16. System dynamics without internal impact considering two supports and different gaps.

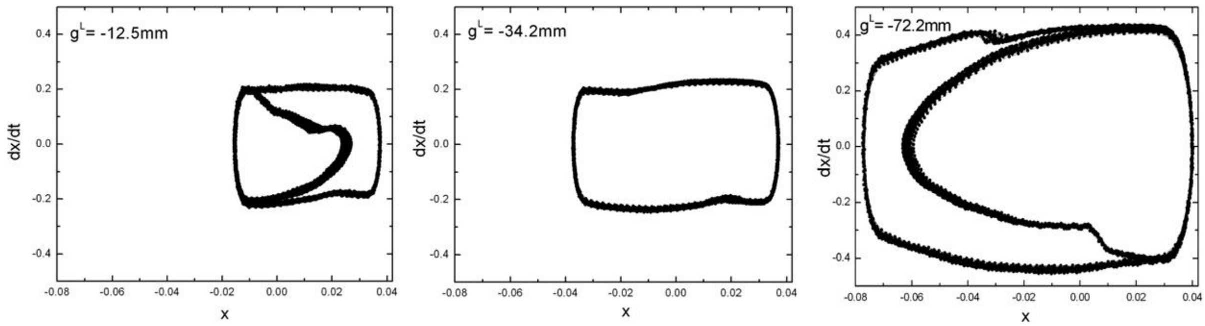

Fig. 17. System dynamics with internal impact considering two supports and different gaps.

are presented in Fig. 17. Notice that the impact influence on system dynamics depends on the gap. By considering large values of gap the system may present a different qualitative behavior.

6. Conclusions

The experimental analysis of a non-smooth system with discontinuous support is presented in this contribution. An experimental apparatus is constructed in order to evaluate the nonlinear dynamics of this system, verifying the influence of the car characteristics and also the support in the system dynamics. In general, despite the deceiving simplicity of this system, its nonlinear dynamics is very rich, presenting bifurcations and chaos. With respect to the car characteristics, an internal impact is introduced considering a mass free to move through a guide. Concerning the support, different kinds are considered changing the stiffness and also the influence of support inertia. Besides, the system response with two supports is of concern, analyzing the influence of stiffness and gaps. These aspects can change the system dynamics in a qualitative point of view and also may be desirable in order to dissipate energy.

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