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CHAOS AND HYPERCHAOS IN SHAPE MEMORY SYSTEMS

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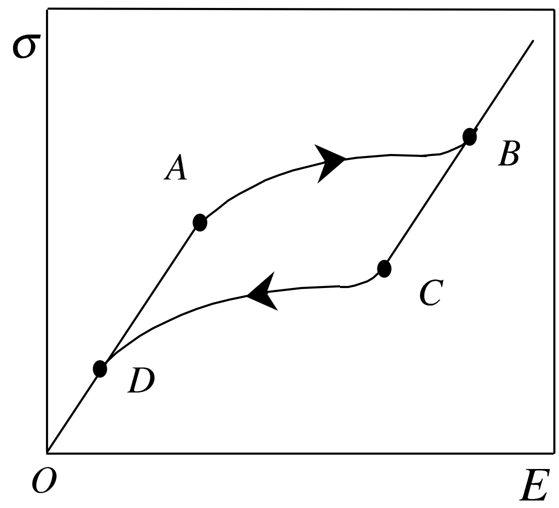
Shape memory and pseudoelastic effects are thermomechanical phenomena associated with martensitic phase transformations, presented by shape memory alloys. The dynamical analysis of intelligent systems that use shape memory actuators involves a multi-degree of freedom system. This contribution concerns with the chaotic response of shape memory systems. Two different systems are considered: a single and a two-degree of freedom oscillator. Equations of motion are formulated assuming a polynomial constitutive model to describe the restitution force of oscillators. Since equations of motion of the two-degree of freedom oscillator are associated with a five-dimensional system, the analysis is performed considering two oscillators, both with single-degree of freedom, connected by a spring-dashpot system. With this assumption, it is possible to analyze the transmissibility of motion between two oscillators. Results show some relation between the transmissibility of order, chaos and hyperchaos with temperature.

Keywords: Chaos; spatiotemporal chaos; hyperchaos; shape memory alloys.

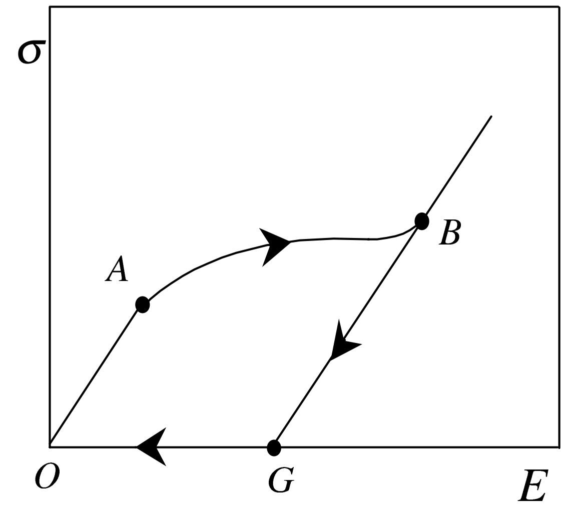
1. Introduction

Shape memory alloys (SMAs) are a family of metals with the ability of changing shape depending on their temperature. SMAs undergo thermoelastic phase transformations, which may be induced either by temperature or stress. Shape memory and pseudoelasticity are effects presented by these alloys which are associated with these phase transformations.

When a specimen of SMA is stressed at a constant temperature, inelastic deformation is observed above a critical stress. This inelastic process, however, fully recovers during the subsequent unloading. The stress–stain curve, which is the macroscopic manifestation of the deformation mechanism of the martensite, forms a hysteresis loop. At a lower temperature, some amount of strain remains after complete unloading. This residual strain may be recovered by heating the specimen. The first case, is the pseudoelastic effect [Fig. 1(a)], while the last is the shape memory effect (SME) or one-way SME [Fig. 1(b)] [Tanaka, 1990]. These effects are inter-related in the sense that, if the hysteresis cycle in the pseudoelastic case is not completed when applied stress is removed, then reversion of the residual martensite must be induced upon heating, by employing the SME [Sun & Hwang, 1993]. In the process of returning to their remembered



(a)



(b)

Fig. 1. (a) Pseudoelastic effect; (b) Shape memory effect.

shape, alloys can generate large forces which may be useful for actuation [Rogers, 1995].

After subjecting the specimen to a "training routine", such as a series of SME cycles or a series of stress induced martensite cycles, it is possible to obtain changes in shape in both directions as a function of temperature (heating and cooling). Therefore, both high and low temperature shapes may be remembered. This phenomenon is the two-way SME [Zhang *et al.*, 1991; Perkins, 1984].

The SME and other related thermoelastic processes associated with martensitic phase transformations have been known since at least 1938. But it has been the investigations of Buehler *et al.* [1963] on phase changes in Ni–Ti alloys that instigated the technological interest in the SMAs. The alloys Ni–Ti, Cu–Zn, Cu–Zn–Al, Mg–Cu, Fe–Mn–Si, Cr–Ni are some of the SMAs. Their properties are very sensitive to composition and processing variables. Ni–Ti (Nitinol) is the most popular SMA as a consequence of a combination of shape memory response with good engineering properties. Strains that elongate up to 8% can be reversed by heating the alloy, typically with electric current [Tuominen & Biermann, 1988].

Due to such remarkable properties, SMAs have found a number of applications in engineering sciences. They are ideally suited for use as fastener, seals, connectors and claps [Borden, 1991]. Self-actuating fastener, thermally actuators switches, a number of bioengineering devices are some examples of these applications [Schetky, 1979]. The use of SMAs can help to solve important problems concerning space saving in aerospace: self-erectable structures, stabilizing mechanisms, solar batteries, nonexplosive release devices are some possibilities [Pacheco & Savi, 2000; Chernyavsky *et al.*, 1993; Busch *et al.*, 1992; Schetky, 1979]. Micromanipulators and robotics actuators are using SMAs to mimic the smooth motions of human muscles [Rogers, 1995]. Also, SMAs are being used as actuators for vibration control of flexible structures. SMAs wires embedded in composite materials have been used to modify vibrational characteristics [Rogers, 1995; Rogers *et al.*, 1991]. The main drawback of SMAs is their slow rate of change.

Since the phenomena associated with martensitic transformation are intrinsically nonlinear, its dynamical response may present some characteristics not observed in linear systems. As an example one could mention chaotic motion which study considers proper mathematical and geometrical aspects [Alligood *et al.*, 1997; Hilborn, 1994; Mullin, 1993; Ott, 1993; Moon, 1992; Kapitaniak, 1991; Wiggins, 1990; Thompson & Stewart, 1986; Guckenheimer & Holmes, 1983]. Savi and Braga [1993a, 1993b] discussed the chaotic response of shape memory oscillators where the restitution force is provided by SMA helical springs.

The dynamical analysis of intelligent systems and structures that use SMA as actuators involves multi-degree of freedom systems. High-dimensional dynamical systems show intricate behavior either for temporal or spatial evolution properties. In the past, most of the work on chaotic dynamics has been concentrated on temporal behavior of low-dimensional systems. Recently, spatiotemporal chaos has attracted much attention due to its theoretical and practical applications [Lai & Grebogi, 1999; Shibata, 1998; Barreto *et al.*, 1997; Thompson & Van der Heijden, 1997; Umberger *et al.*, 1989]. The present contribution concerns with the nonlinear dynamics of shape memory systems considering single and two-degree of freedom oscillators. Equations of motion are formulated using polynomial constitutive model to describe the restitution force of the oscillator. The prospect of chaotic behavior is of concern and, since the equations of motion of the two-degree of freedom oscillator are associated with a five-dimensional system, the analysis is performed by considering two oscillators, both with single-degree of freedom, connected by a spring-dashpot system. With this assumption, it is possible to analyze the transmissibility of motion between the two oscillators. Results show some relation between the transmissibility of order and chaos with temperature. The existence of hyperchaos is another interesting characteristic of these systems. Despite the deceiving simplicity of the model used, the authors agree that this analysis may contribute to the understanding of the nonlinear dynamics of shape memory systems.

2. Polynomial Constitutive Model

Shape memory and pseudoelastic effects may be modeled either by microscopic or macroscopic point of view. Constitutive equations may be formulated within the framework of continuum mechanics and the thermodynamics of irreversible processes, by considering thermodynamic forces, defined from the Helmholtz free energy, ψ, and thermodynamic fluxes, defined from the pseudo-potential of dissipation, ϕ [Lemaitre & Chaboche, 1990].

The formulation of phenomenological constitutive models to describe SMAs behavior is based on different assumptions on the free energy and the pseudo-potential of dissipation [Savi & Braga, 1993a]. There are many different works dedicated to the constitutive description of the thermomechanical behavior of shape memory alloys, however, this is not a well established topic [James, 2000; Birman, 1997; Bertram, 1982; Souza *et al.*, 1998; Auricchio & Lubliner, 1997; Auricchio & Sacco, 1997; Auricchio *et al.*, 1997; Tanaka & Nagaki, 1982; Liang & Rogers, 1990; Brinson, 1993; Boyd & Lagoudas, 1994; Ivshin & Pence, 1994; Fremond, 1987, 1996; Abeyaratne *et al.*, 1994].

Polynomial constitutive model is based on Devonshire theory proposed by Falk [1980]. This is a one-dimensional model which represents the shape memory and pseudoelastic effects considering a polynomial free energy that depends on the temperature and on the one-dimensional strain, E, i.e. $\psi = \psi(E, T)$. No other internal variables are considered and this characteristic makes this model a simple alternative to describe SMAs behavior.

The form of the free energy is chosen in such a way that the minima and maxima points represent stability and instability of each phase of the SMA. As it is usual on one-dimensional models proposed for SMAs [Savi & Braga, 1993a], three phases are considered: Austenite (A) and two variants of martensite ($M+$, $M-$). Hence, the free energy is chosen such that for high temperatures it has only one minimum at vanishing strain, representing the equilibrium of the austenitic phase. At low temperatures, martensite is stable, and the free energy must have two minima at nonvanishing strains. At intermediate temperatures, the free energy must have equilibrium points corresponding to both phases. These restrictions are satisfied by the following polynomial expression:

$$\rho\psi(E, T) = \frac{1}{2}a(T - T_M)E^2 - \frac{1}{4}bE^4 + \frac{1}{6}eE^6 \tag{1}$$

where a, b and e are positive constants, while T_M is the temperature below which the martensitic phase is stable and ρ is the mass density. If T_A is defined as the temperature above which the austenite is stable, and the free energy has only one minimum at zero strains, it is possible to write the following condition,

$$T_A = T_M + \frac{1}{4}\frac{b^2}{4ae} \tag{2}$$

Therefore, the constant e may be expressed in terms of other constants of the material. By definition [Savi & Braga, 1993a], the stress–strain relation is given by,

$$\sigma = \rho\frac{\partial\psi}{\partial E} = a(T - T_M)E - bE^3 + eE^5 \tag{3}$$

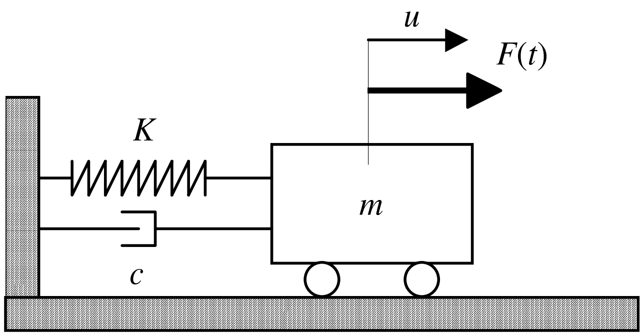


Fig. 2. Single-degree of freedom shape memory oscillator.

It should be emphasized that this model considers one free energy, with no extra internal variable, to represent phase transformations on SMAs with an austenitic phase and two variants of martensite. This model describes both the shape memory and pseudoelastic effects qualitatively well in a simple way. The absence of experimental data evaluating material constants is one of the drawbacks to its use.

3. Single-Degree of Freedom Oscillator

In order to perform the dynamical analysis of mechanical systems with SMA elements, a single-degree of freedom shape memory oscillator, depicted in Fig. 2, is considered. It consists of a mass, m, supported by a SMA element and a linear damper with coefficient c, being harmonically excited by a force $F = \overline{F}\sin(\Omega t)$.

Shape memory behavior is described considering polynomial constitutive model. Therefore, the restoring force is given by,

$$K = K(u, T) = \overline{a}(T - T_M)u - \overline{b}u^3 + \overline{e}u^5 \quad (4)$$

where

$$\overline{a} = \frac{aA}{L};\ \overline{b} = \frac{bA}{L^3};\ \overline{e} = \frac{eA}{L^5} \quad (5)$$

and variable u represents the displacement associated with the SMA element, L is its length while A is its area. By establishing the equilibrium of the system, equations of motion are written as follows

$$\begin{aligned} y_0' &= y_1 \\ y_1' &= \delta\sin(\overline{\omega}\tau) - \xi y_1 - (\theta - 1)y_0 + \beta y_0^3 - \varepsilon y_0^5 \end{aligned} \quad (6)$$

where the following definitions are considered:

$$\begin{gathered} \omega_0^2 = \frac{aAT_M}{mL};\quad \tau = \omega_0 t;\quad (\ )' = d(\ )/d\tau; \\ y_0 = u/L;\quad y_1 = u'/L;\quad \overline{\omega} = \Omega/\omega_0; \\ \delta = \frac{\overline{F}}{mL\omega_0^2};\quad \xi = \frac{c}{m\omega_0};\quad \beta = \frac{bA}{mL\omega_0^2}; \\ \varepsilon = \frac{eA}{mL\omega_0^2};\quad \theta = \frac{T}{T_M} \end{gathered} \quad (7)$$

Numerical simulations are performed employing a fourth-order Runge–Kutta method for numerical integration and time steps less than $\Delta\tau = 2\pi/200\overline{\omega}$ present good results. The characterization of chaotic motion is done regarding Lyapunov exponents, and its estimation employs the algorithm proposed by Wolf *et al.* [1985].

In all simulations one considers a unitary mass and $\overline{\omega} = 1$, $\xi = 0.1$, $\beta = 1.3\text{e}3$ and $\varepsilon = 4.7\text{e}5$. Notice that $\theta_A = 1 + \beta^2/4\varepsilon$, and therefore, $\theta_A = 1.9$.

3.1. *Free vibration*

In this section, the free response of the shape memory oscillator is discussed. This is done by letting δ vanish in the nondimensional equation of motion (6). The system has different equilibrium points depending on temperature. Denoting by $(\overline{y}_0, \overline{y}_1)$ a point that makes the right-hand sides of equations of motion vanish, the following possibilities are found,

$$\begin{gathered} \overline{y}_0 = 0 \quad \text{and} \quad \overline{y}_1 = 0 \\ \overline{y}_0 = \pm\sqrt{\frac{2(\theta_A - 1)}{\beta}\left[1 \pm \sqrt{\frac{\theta_A - \theta}{\theta_A - 1}}\right]} \quad \text{and} \quad \overline{y}_1 = 0 \end{gathered} \quad (8)$$

Of these five possibilities, only those that correspond to real numbers have physical meaning. Stability of these equilibrium configurations may be determined by the behavior of the system in their neighborhood. An analysis of the eigenvalues of the Jacobian matrix of the system reveals its local stability. Therefore,

(a) $\theta \le 1$, the system has three fixed points: The origin of the phase space is a saddle point. The other two fixed points are centers when $\xi = 0$ and stable spirals when $\xi > 0$. This is consistent with the low temperature behavior of SMA, where two martensitic phases are stable.

(b) $1 < \theta < \theta_A$, the system has five fixed points: The system has two saddle points in the phase space. The remaining three fixed points are centers when $\xi = 0$ and stable spirals when $\xi > 0$. The existence of three stable fixed points is explained by the stability of both martensitic phases and austenite in this range of temperature.

(c) $\theta = \theta_A$, the system has three fixed points: The origin is a center when $\xi = 0$ and stable spirals when $\xi > 0$. The other two fixed points are saddles.

(d) $\theta > \theta_A$, the system has only one fixed point: The origin is the only fixed point and it is either a center or a stable spiral, again depending on whether the system is dissipative or not. Under this temperature range, austenite is the only stable phase in the stress-free SMA.

In order to illustrate the free response of the oscillator, a nondissipative system ($\xi = 0$) is considered. Results from simulations are presented in the form of phase portraits. Figure 3 presents the free response of the system at a temperature where the martensitic phase is stable ($\theta = 0.7$). There are, in this case, three equilibrium points. From these, two are stable while the other one is unstable. Now, by considering a higher temperature, where austenitic phase is stable in the alloy ($\theta = 3.5$), the system presents only one stable equilibrium point.

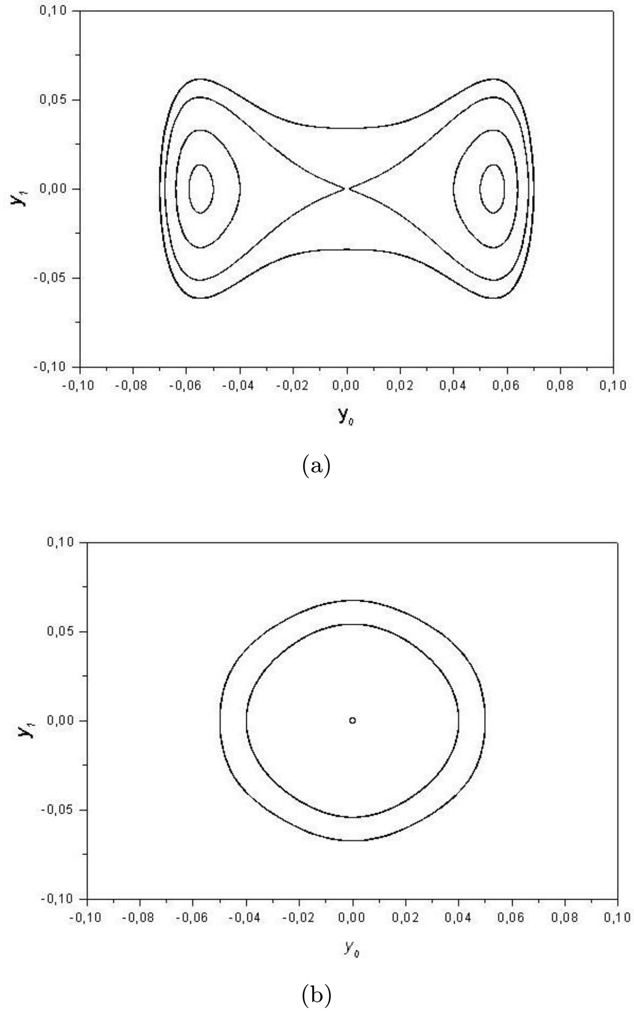


Fig. 3. Phase portrait. (a) $\theta = 0.7$; (b) $\theta = 3.5$.

3.2. *Forced vibration*

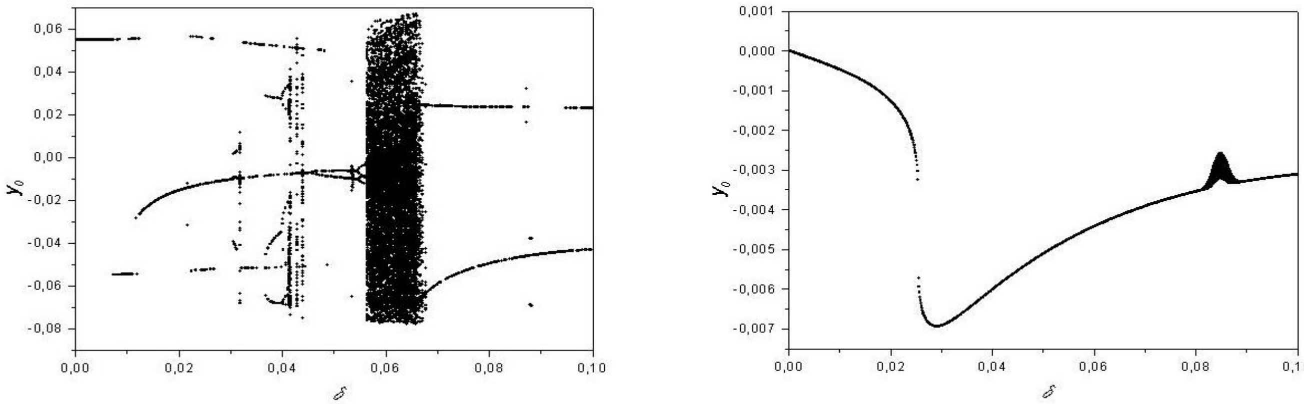
The behavior of the forced system is far more complex. In this section, different kinds of the shape memory oscillator response are shown. In order to start the analysis, bifurcation diagrams are presented (Fig. 4), showing the stroboscopically sampled displacement values, y_0, under the slow quasi-static increase of the driving force amplitude, θ, and different temperatures. Notice that there are parameter values associated with a cloud of points, which are related to chaotic motion.

At this point, different responses are contemplated. Assuming $\theta = 3.5$ and $\delta = 0.06$, the system presents a period-1 motion. Figure 5 shows the phase space and the Poincaré section associated with this motion. Regarding the same forcing parameter and a lower temperature, $\theta = 0.7$, where the martensitic phase is stable, the motion becomes chaotic. The phase space and the Poincaré section associated with this motion are presented in Fig. 6. Under this condition, a strange attractor is identified and Lyapunov spectrum estimated by the algorithm due to Wolf *et al.* [1985] is $\lambda_i = (+0.28, -0.42)$, presenting one positive exponent. Decreasing the forcing amplitude parameter to $\delta = 0.038$, a period-3 motion is observed (Fig. 7) and, once again, a period-1 response occurs when $\delta = 0.02$ (Fig. 8).

4. Two-Degree of Freedom Oscillator

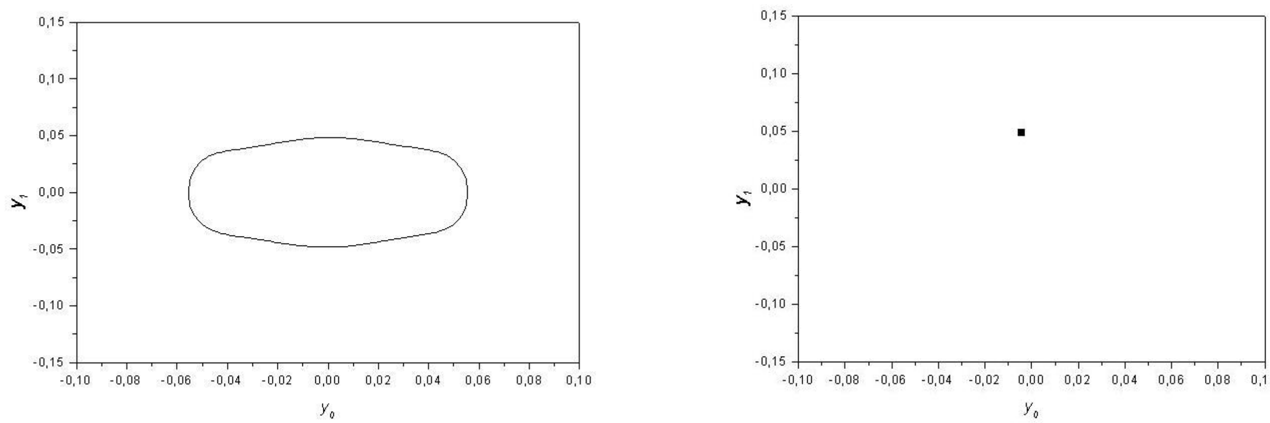
In this section, a two-degree of freedom oscillator, depicted in Fig. 9, is considered. It consists of two masses, m_i ($i = 1, 2$), supported by SMA elements and linear dampers with coefficient c_i ($i = 1, 2, 3$). The system is harmonically excited by two forces $F_i = \overline{F}_i \sin(\Omega_i t)$ ($i = 1, 2$).

As discussed in the previous section, shape memory behavior is described by considering



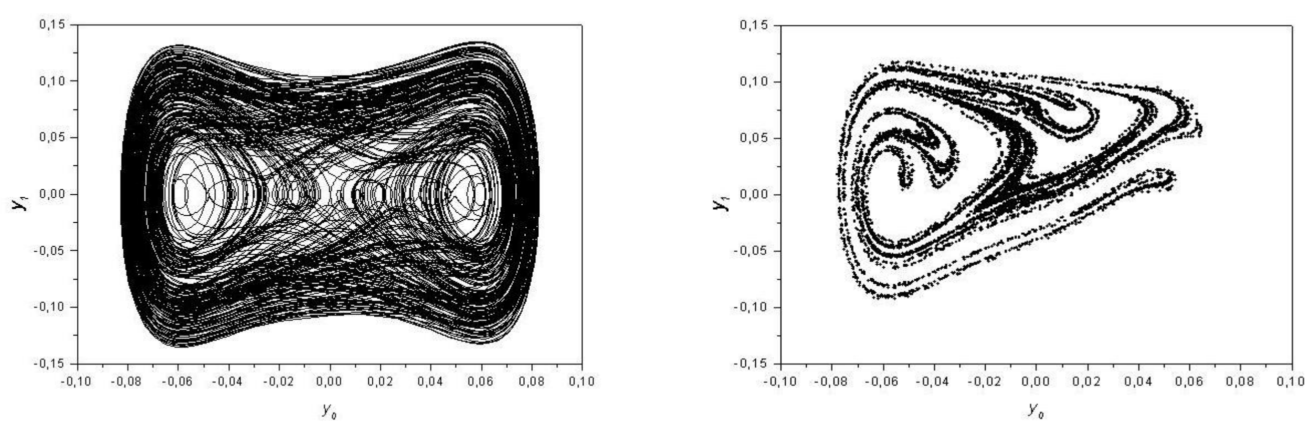
(a) (b)

Fig. 4. Bifurcation diagrams. (a) $\theta = 0.7$; (b) $\theta = 3.5$.



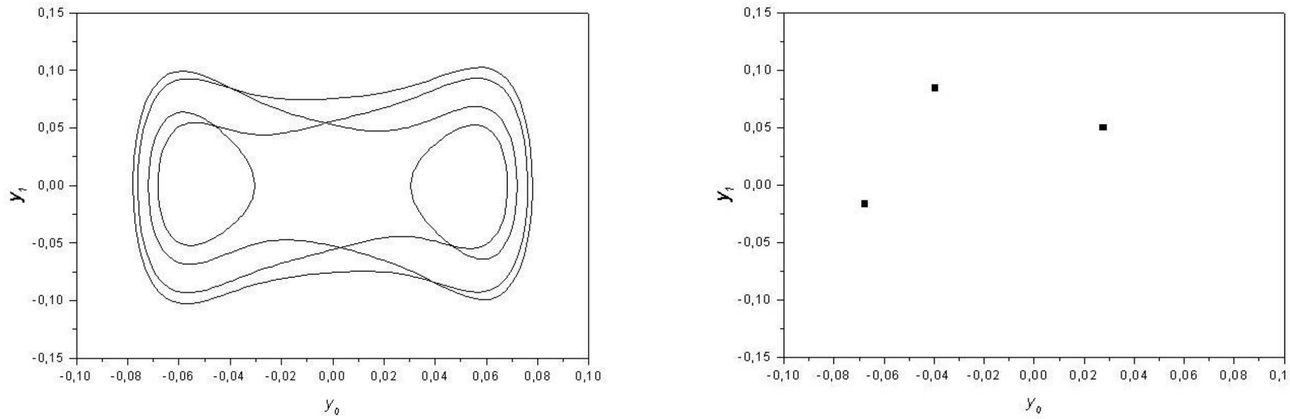
(a) (b)

Fig. 5. Periodic motion. $\theta = 3.5$ and $\delta = 0.06$. (a) Phase space; (b) Poincaré section.



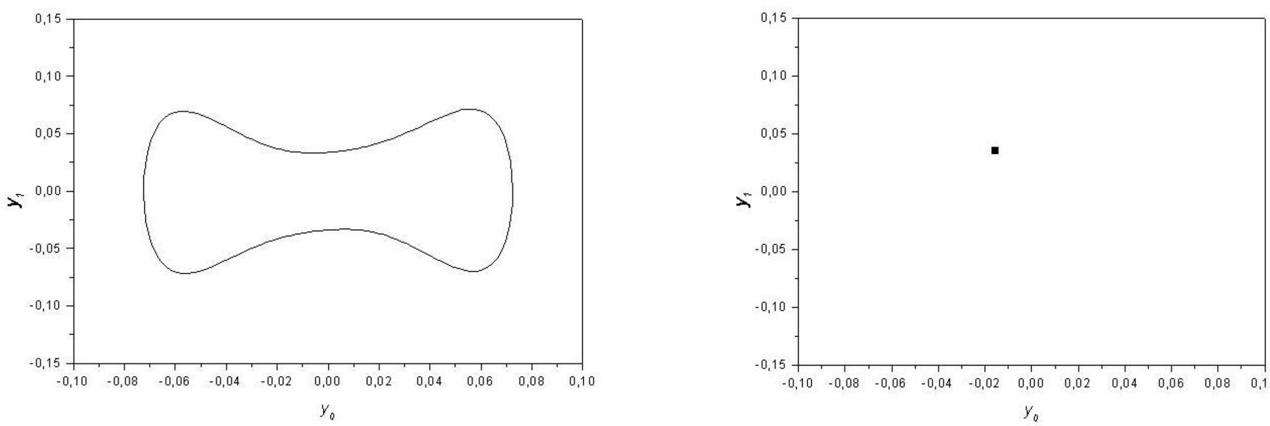
(a) (b)

Fig. 6. Chaotic motion: $\theta = 0.7$ and $\delta = 0.06$. (a) Phase space; (b) Strange attractor.



(a) (b)

Fig. 7. Period-3 motion: $\theta = 0.7$ and $\delta = 0.038$. (a) Phase space; (b) Poincaré section.



(a) (b)

Fig. 8. Period-3 motion: $\theta = 0.7$ and $\delta = 0.02$. (a) Phase space; (b) Poincaré section.

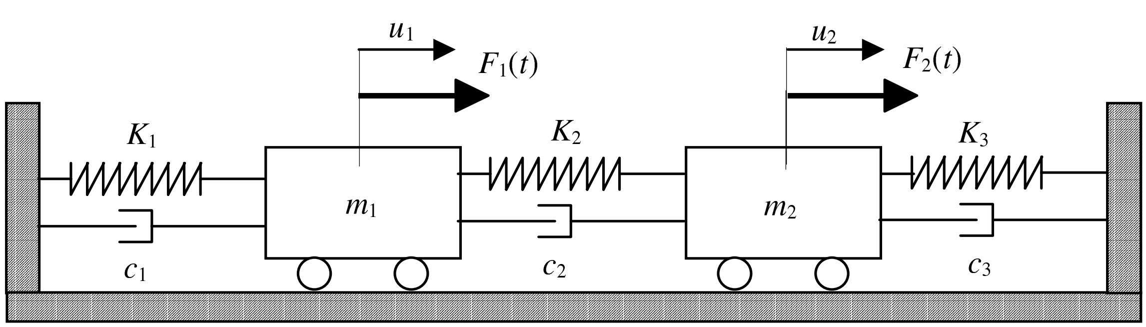


Fig. 9. Two-degree of freedom shape memory oscillator.

polynomial constitutive model. Hence, establishing the equilibrium of the system, equations of motion are written as follows

$$
\begin{aligned}
y_0' &= y_1 \\
y_1' &= \delta_1 \sin(\overline{\omega}_1\tau) - (\xi_1 + \xi_2\alpha_{21}\mu)y_1 \\
&\quad + \xi_2\alpha_{21}\mu y_3 - [(\theta_1 - 1) + \alpha_{21}^2\mu(\theta_2 - 1)]y_0 \\
&\quad + \alpha_{21}^2\mu(\theta_2 - 1)y_2 + \beta_1 y_0^3 - \varepsilon_1 y_0^5 \\
&\quad - \beta_2\alpha_{21}^2\mu(y_2 - y_0)^3 + \varepsilon_2\alpha_{21}^2\mu(y_2 - y_0)^5 \\
y_2' &= y_3 \qquad (9) \\
y_3' &= \alpha_{21}^2\delta_2 \sin(\overline{\omega}_2\tau) + \xi_2\alpha_{21}y_1 \\
&\quad - (\xi_2\alpha_{21} + \xi_3\alpha_{21}\alpha_{32})y_3 + \alpha_{21}^2(\theta_2 - 1)y_0 \\
&\quad - [\alpha_{21}^2(\theta_2 - 1) + \alpha_{21}^2\alpha_{32}^2(\theta_3 - 1)]y_2 \\
&\quad + \beta_2\alpha_{21}^2(y_2 - y_0)^3 - \varepsilon_2\alpha_{21}^2(y_2 - y_0)^5 \\
&\quad + \beta_3\alpha_{21}^2\alpha_{32}^2 y_2^3 - \varepsilon_3\alpha_{21}^2\alpha_{32}^2 y_2^5
\end{aligned}
$$

where the following definitions are adopted,

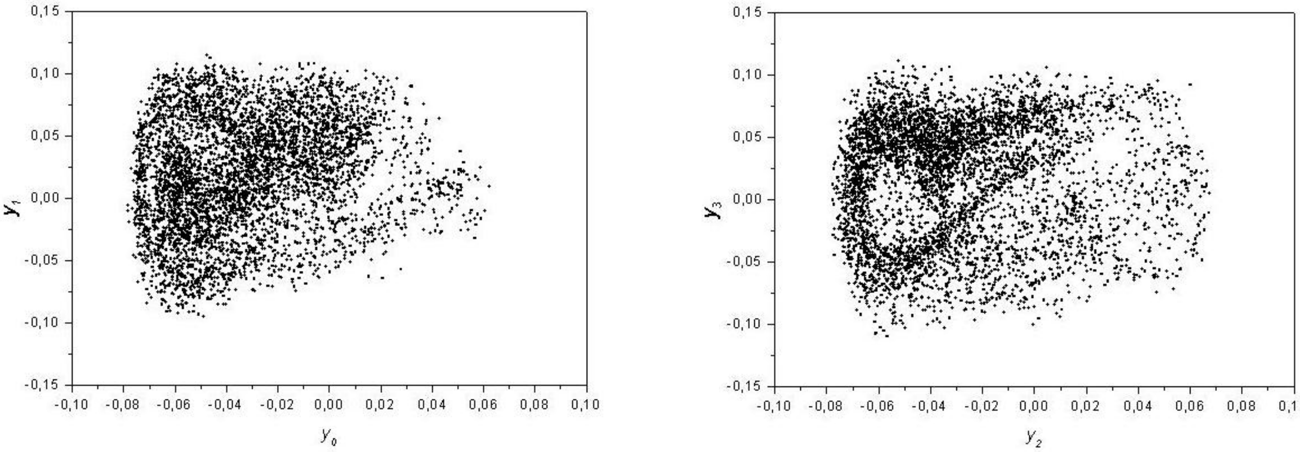
$$
\begin{gathered}
\omega_1^2 = \frac{a_1 A T_{M_1}}{m_1 L}\,; \quad \omega_2^2 = \frac{a_2 A T_{M_2}}{m_2 L}\,; \quad \omega_3^2 = \frac{a_3 A T_{M_3}}{m_2 L}\,; \\
\tau = \omega_1 t\,; \quad (\ )' = d(\ )/d\tau\,; \quad y_0 = u_1/L\,; \\
y_1 = u_1'/L\,; \quad y_2 = u_2/L\,; \quad y_3 = u_2'/L\,; \\
\overline{\omega}_1 = \Omega_1/\omega_1\,; \quad \overline{\omega}_2 = \Omega_2/\omega_1\,; \\
\theta_i = T_i/T_{M_i}\ (i = 1,\ 2,\ 3)\,; \\
\delta_1 = \frac{\overline{F}_1}{m_1 L\omega_1^2}\,; \quad \delta_2 = \frac{\overline{F}_2}{m_2 L\omega_2^2}\,; \qquad (10) \\
\xi_1 = \frac{c_1}{m_1\omega_1}\,; \quad \xi_2 = \frac{c_2}{m_2\omega_2}\,; \quad \xi_3 = \frac{c_3}{m_2\omega_3}\,; \\
\alpha_{21} = \frac{\omega_2}{\omega_1}\,; \quad \alpha_{32} = \frac{\omega_3}{\omega_2}\,; \quad \mu = \frac{m_2}{m_1}\,; \\
\beta_1 = \frac{b_1 A}{m_1 L\omega_1^2}\,; \quad \beta_2 = \frac{b_2 A}{m_2 L\omega_2^2}\,; \quad \beta_3 = \frac{b_3 A}{m_2 L\omega_3^2}\,; \\
\varepsilon_1 = \frac{e_1 A}{m_1 L\omega_1^2}\,; \quad \varepsilon_2 = \frac{e_2 A}{m_2 L\omega_2^2}\,; \quad \varepsilon_3 = \frac{e_3 A}{m_2 L\omega_3^2}
\end{gathered}
$$

Again, numerical simulations are performed employing a fourth-order Runge–Kutta method for numerical integration and time steps less than $\Delta\tau = 2\pi/200\overline{\omega}_1$ present good results. In all simulations, similar mechanical properties are regarded for the three spring-dashpot systems. A unitary mass is assumed and $\overline{\omega}_1 = \overline{\omega}_2 = 1$, $\xi_1 = \xi_2 = \xi_3 = 0.1$, $\beta_1 = \beta_2 = \beta_3 = 1.3\text{e}3$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 4.7\text{e}5$. These informations allow one to conclude that $\alpha_{21} = \alpha_{32} = \mu = 1$, and $\theta_{A_1} = \theta_{A_2} = \theta_{A_3} = 1.9$.

Since equations of motion are associated with a five-dimensional system, the analysis is performed by considering two oscillators, both with single-degree of freedom, connected by a spring-dashpot system. With this assumption, it is possible to analyze the transmissibility of motion between the two oscillators, constructing a phase subspace for each mass. This transmissibility is evaluated studying different temperatures on the connection system, which causes different patterns on each phase subspace.

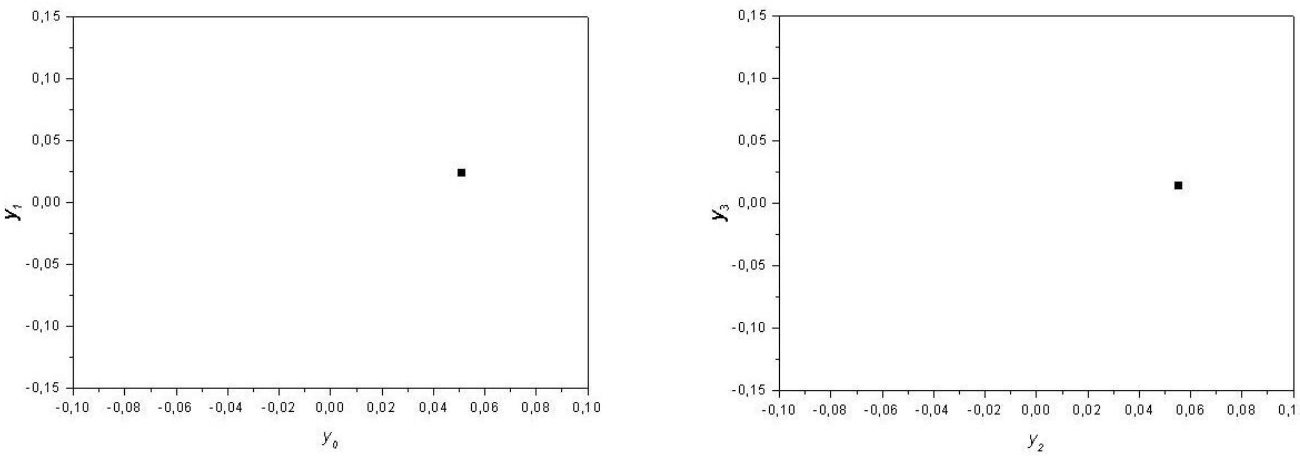
At first, consider uncoupled systems excited in such form that there is a chaotic motion on mass m_1 ($\delta_1 = 0.06$ and $\theta_1 = 0.7$) and a null forcing amplitude parameter on mass m_2 ($\delta_2 = 0$ and $\theta_3 = 0.7$). Therefore, Poincaré section related to mass m_1, subspace $y_0 - y_1$, presents a strange attractor shown in Fig. 6. On the other hand, Poincaré section related to mass m_2, subspace $y_2 - y_3$, is a point. Next, systems are coupled introducing a connection with $\theta_2 = 0.7$ where martensitic phase is stable. Under this condition, a cloud of points is transmitted to the phase subspace associated with mass m_2, while the strange attractor associated with mass m_1 changes its pattern (Fig. 10). Lyapunov spectrum estimated by the algorithm due to Wolf *et al.* [1985] is $\lambda_i \equiv (+0.50,\ +0.08,\ -0.34,\ -0.82)$, presenting two positive exponents. The occurrence of two or more positive Lyapunov exponents in a dynamical system is called hyperchaos [Moon, 1992; Rossler, 1979]. This situation means that two or more directions in the phase space suffer stretching under the dynamical process. When the temperature of the connection is altered, $\theta_2 = 3.5$, austenitic phase becomes stable. Under this new condition, the transmissibility is quite different from the previous one (Fig. 11). Notice that the strange attractor of mass m_1 becomes a point and similar situation occurs on the phase space associated with mass m_2. Now, Lyapunov spectrum is $\lambda_i \equiv (-0.09,\ -0.11,\ -0.16,\ -0.20)$ where there is no positive exponent, meaning a periodic motion.

A different excitation condition is now in focus. Hence, consider an excitation that causes chaotic motions on both masses ($\delta_1 = \delta_2 = 0.06$ and $\theta_1 = \theta_3 = 0.7$) of two uncoupled systems. Introducing a connection with $\theta_2 = 0.7$, martensitic phase is stable. The response of this system is similar to the case where $\delta_1 = 0.06$ and $\delta_2 = 0$ associated with Fig. 10, that is, a cloud of points is transmitted to the phase space associated with mass m_2, while the strange attractor associated with mass m_1 changes its pattern (Fig. 12). Lyapunov spectrum is $\lambda_i \equiv (+0.54,\ +0.17,\ -0.37,\ -0.92)$ presenting



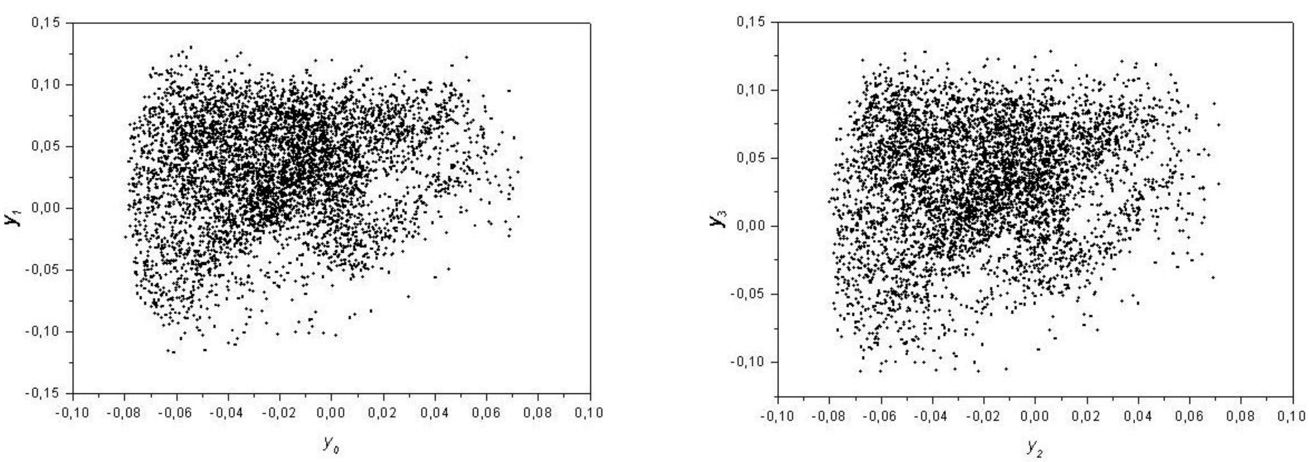
(a) (b)

Fig. 10. Phase space. $\delta_1 = 0.06$, $\delta_2 = 0$; $\theta_1 = \theta_2 = \theta_3 = 0.7$.



(a) (b)

Fig. 11. Phase space. $\delta_1 = 0.06$, $\delta_2 = 0$; $\theta_1 = \theta_3 = 0.7$; $\theta_2 = 3.5$.

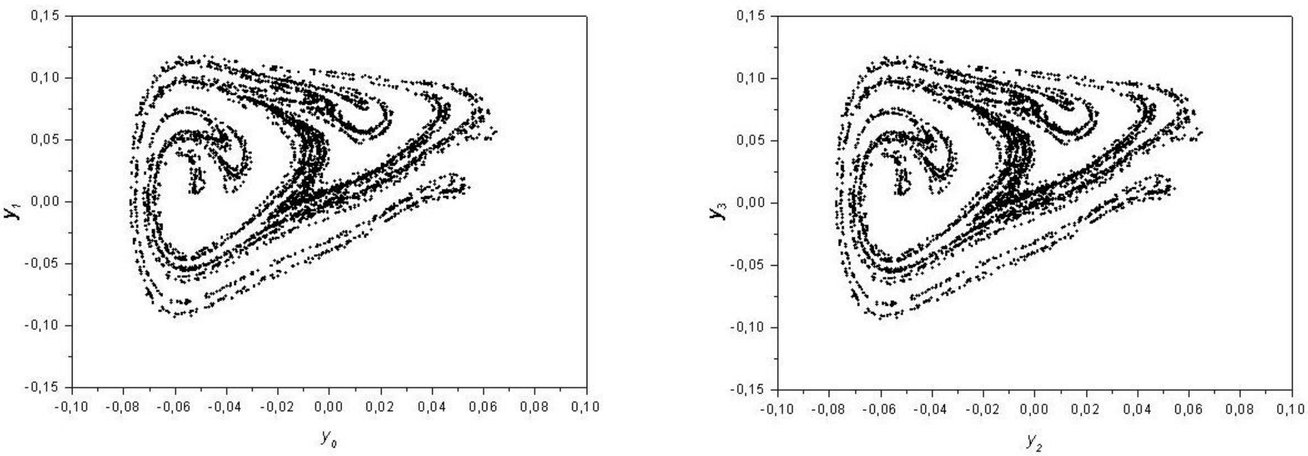


(a) (b)

Fig. 12. Phase space. $\delta_1 = 0.06$, $\delta_2 = 0.06$; $\theta_1 = \theta_2 = \theta_3 = 0.7$.

two positive exponents, meaning that there is hyperchaos. Now, a different connection temperature is considered, $\theta_2 = 3.5$, meaning that austenitic phase becomes stable. Under this new condition, the transmissibility is quite different from the previous one (Fig. 13). Notice that there are strange attractors related to both masses, showing the existence of chaos. This conclusion is confirmed by Lyapunov spectrum, $\lambda_i \equiv (+0.30, -0.14, -0.29\ -0.45)$, which presents only one positive exponent.

At this point, further figures are used to illustrate these behaviors. With this aim, consider a 3D plot ($y_0 - y_1 - y_2$) of the five-dimensional phase space of the previous example. Figure 14 shows the case where $\delta_1 = \delta_2 = 0.06$, and $\theta_1 = \theta_2 = \theta_3 = 0.7$, representing a martensitic connection. This 3D



(a) (b)

Fig. 13. Phase space. $\delta_1 = 0.06$, $\delta_2 = 0.06$, $\theta_1 = \theta_3 = 0.7$; $\theta_2 = 3.5$.

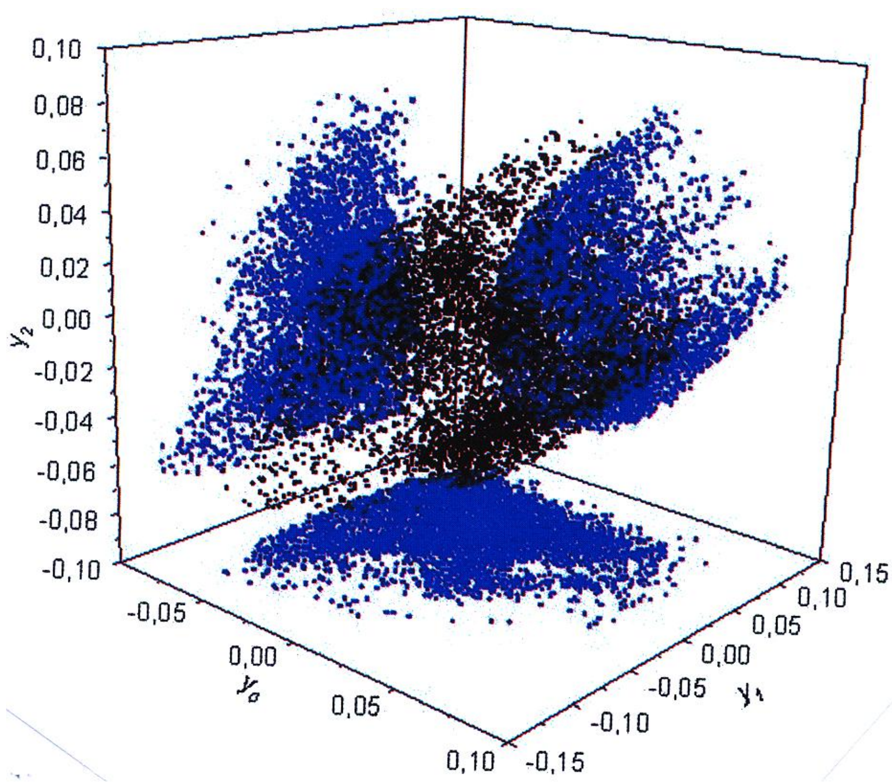


Fig. 14. Poincaré section in the space y_0–y_1–y_2 for $\delta_1 = \delta_2 = 0.06$, and $\theta_1 = \theta_2 = \theta_3 = 0.7$.

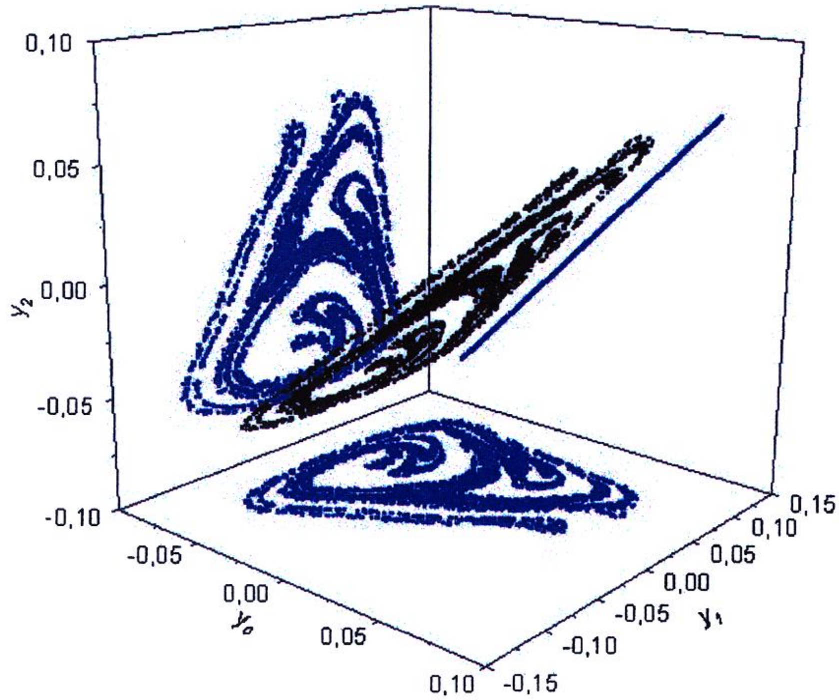


Fig. 15. Poincaré section in the space y_0–y_1–y_2 for $\delta_1 = \delta_2 = 0.06$, $\theta_1 = \theta_3 = 0.7$ and $\theta_2 = 3.5$.

projection shows a cloud of points and allows one to observe the phase subspace of mass m_1 (Fig. 12) projected on the y_0–y_1 plane. Observing the projections on planes, it is not possible to see a cantor-like structure. The same behavior is observed when the 3D plot considers other variables, y_0–y_1–y_3, for example. When an austenitic connection is conceived assuming $\delta_1 = \delta_2 = 0.06$, $\theta_1 = \theta_3 = 0.7$ and $\theta_2 = 3.5$, the 3D plot (y_0–y_1–y_2) shows a strange attractor with a typical structure (Fig. 15). Projection on the y_0–y_1 plane allows one to observe the strange attractor associated with mass m_1, presented in Fig. 13.

The preceding examples show that austenitic connection, which occurs in higher temperatures ($\theta_2 = 3.5$, for example), tends to preserve order in contrast to the situation where a martensitic connection is considered ($\theta_2 = 0.7$, for example). This conclusion passes from the understanding that there is an inherent order associated with the pattern of the strange attractor.

Notice that Figs. 10 and 12, which are related to martensitic connections, present chaotic behavior where attractors have different patterns, showing no order in these motions. On the other hand, Figs. 11 and 13, which are related to austenitic connections, present respectively, a periodic and a chaotic motion. Both situations preserve order in the sense that chaotic motion present a strange attractor with a typical structure.

A further comment associated with the transmissibility of motion is related to Lyapunov spectra. When an austenitic connection is regarded, order is preserved and Lyapunov spectrum associated with chaotic motion has only one positive exponent. On the other hand, when a martensitic connection is conceived, Lyapunov spectrum associated with chaotic motion has more than one positive exponent, characterizing hyperchaos. Therefore, one can infer that hyperchaos is related to the mechanism that breaks the order.

5. Conclusions

This contribution reports on the chaotic response of shape memory systems where the restitution force is described by polynomial constitutive model. Two different systems are considered: a single and a two-degree of freedom oscillator. Since equations of motion of the two-degree of freedom oscillator are associated with a five-dimensional system, the analysis is performed considering two oscillators, both with single-degree of freedom, connected by a spring-dashpot system. With this assumption, it is possible to analyze the transmissibility of motion between two oscillators. Results show that variations of the connection temperature alter the transmissibility of motion between both masses. An austenitic connection tends to preserve order in contrast to the situation where a martensitic connection is considered. Furthermore, hyperchaos is related to the mechanism that breaks this order. The authors agree that, despite the deceiving simplicity of the model used, similar behavior may be expected in other shape memory systems.

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