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Trace theory applied to composite analysis: A comparison with micromechanical models

Lucas L. Vignoli [a,*], Ranulfo M.C. Neto [a], Marcelo A. Savi [b], Pedro M.C.L. Pacheco [c], Alexander L. Kalamkarov [d]

[a] *Center for Technology and Applications of Composite Materials, Department of Mechanical Engineering, Universidade Federal do Rio de Janeiro, Macaé, RJ, Brazil*

[b] *Center for Nonlinear Mechanics, COPPE, Department of Mechanical Engineering Universidade Federal do Rio de Janeiro, 21.941.972, P.O. Box 68.503, Rio de Janeiro, Brazil*

[c] *Department of Mechanical Engineering, Centro Federal de Educação Tecnológica Celso Suckow da Fonseca CEFET/RJ, Rio de Janeiro, RJ, Brazil*

[d] *Department of Mechanical Engineering, Dalhousie University Halifax, Nova Scotia, B3H 4R2, Canada*

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ABSTRACT

Composite materials design is a challenging topic, and its first step is the estimation of the macromechanical elastic properties. Usually, this analysis is carried out applying micromechanical models that use constituent properties as input. Alternatively, according to Tsai's trace theory, the trace of stiffness matrix is invariant. In other words, it is independent of fibers and matrices used, as well as their volume fractions. The main advantage of trace-based approach is that the properties can be normalized by trace and only one test is required to compute all elastic properties, resulting in considerable saving of cost and time. On the other hand, the original proposal is limited to in-plane properties of carbon fiber laminates. In the present study, the trace-based methodology is compared with a set of 138 experimental data compiled from the literature for carbon fiber reinforced polymers (CFRP) and glass fiber reinforced polymers (GFRP). An extension of trace theory is proposed to compute the out-of-plane shear modulus. Trace theory is compared with 10 micromechanical models establishing a comparative discussion. Results show that average error of Tsai's trace approach is smaller than 20% for all the properties for both fiber types. Especially for the in-plane Poisson's ratio, trace-based estimation results in a smaller error than all other micromechanical models. The main goal of this paper is to compare Tsai's trace approach with classical micromechanical models. Although other models are useful in some cases, Tsai's trace approach is advantageous due to a considerable reduction of the design cost and time.

1. Introduction

The investigation of invariants is not new in composite materials design, see, e.g., Ref. [1]. Tsai & Melo [2] proposed that the trace of the stiffness tensor works as a material property for CFRP. The main idea is that properties normalized by this trace are constant for plane stress condition. The main advantage of this approach is in reduction of the design cost by avoiding the extensive experimental material characterization. Instead of tests to measure the longitudinal and transversal elastic moduli, E_1 and E_2, the in-plane Poisson's ratio, ν_{12}, and the in-plane shear modulus, G_{12}, just a simple uniaxial test is required to measure E_1 and the other properties can be computed using the normalized relation by the trace. This theory can be also applied to multidirectional laminates using the classical laminate theory. Tsai et al. [3] pointed out that the extension for three-dimensional elasticity may be carried out with expected larger variations.

Ha & Cimini Jr [4] presented a review of this theory and evaluated 44 laminates, including 20 with carbon fibers, 10 with aramid fibers and 14 with glass fibers. Among these laminates, the carbon fiber laminates resulted in the best fit. Guedes [5] also investigated the application of trace theory for glass fiber laminates. A general overview of this theory can be found in Tsai et al. [6].

Arteiro et al. [7] presented a micromechanical investigation of the trace theory using the Halpin-Tsai model. According to the authors, the variation of the longitudinal elastic modulus normalized by the trace is smaller than 6% for fiber volume fractions between 50% and 70%

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* Corresponding author.
E-mail addresses: ll.vignoli@mecanica.coppe.ufrj.br (L.L. Vignoli), ranulfocarneiro@macae.ufrj.br (R.M.C. Neto), savi@mecanica.coppe.ufrj.br (M.A. Savi), pedro.pacheco@cefet-rj.br (P.M.C.L. Pacheco), alex.kalamkarov@dal.ca (A.L. Kalamkarov).

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considering carbon fiber laminates. Note that for the traditional micromechanical models, assuming plane stress conditions, there are six constituents' properties required: fiber longitudinal and transversal elastic moduli, E_1^f and E_2^f, fiber in-plane shear modulus, G_{12}^f, fiber in-plane Poisson's ratio, ν_{12}^f, matrix elastic modulus, E^m, and matrix Poisson's ratio, ν^m. For 3D characterization, fiber out-of-plane shear modulus, G_{23}^f, is also necessary.

Vignoli et al. [8] presented an overview comparing 10 micromechanical models with 188 experimental data from 25 references and concluded that the asymptotic homogenization with the square symmetry and a novel modification of the rule of mixture resulted in the closest prediction in comparison with the experimental data.

All the previous publications are related to the traditional unidirectional laminae. An overview of micromechanics of nano- and micro-fiber reinforced composites is presented in Ref. [9]. Other references deal with nano reinforcements [10,11] and coated fibers laminae [12].

Based on these results, the present paper aims to investigate the estimation of Tsai's trace theory compared with a set of 138 experimental data. Results from classical micromechanical models discussed in Vignoli et al. [8] are also presented for the comparison.

2. Trace theory

Tsai & Melo [2] proposed an approach based on the plane stress stiffness matrix, **C**, using its trace, $tr(\mathbf{C})$, and the normalized components, C_{ij}^*, to compute the effective elastic properties of laminae as follows:

$$C_{ij} = C_{ij}^* tr(\mathbf{C}) \tag{1}$$

where $C_{11}^* = 0.8815$, $C_{22}^* = 0.0499$, $C_{12}^* = 0.0164$ and $C_{33}^* = 0.0342$ are experimentally calibrated.

Assuming a uniaxial test to measure E_1, the trace can be computed with the following equation:

$$tr(\mathbf{C}) = \frac{E_1}{E_1^*} \tag{2}$$

where $E_1^* = 0.8581$ is proposed based in experimental results.

The key point of the trace methodology is to evaluate Eqs. (1) and (2) and to compute $tr(\mathbf{C})$using only one experimentally measured property once the normalized properties C_{ij}^* are calibrated from experimental data. Despite that more experimental data are required to calibrate C_{ij}^*, a reduced number of experimental tests are necessary. It is points out in Ref. [2] that these properties can be employed for any CFRP.

Recently, Arteiro et al. [13] presented a general overview of the trace theory and its applications. The estimation of effective in-plane elastic properties of the lamina is computed using

$$E_2 = E_2^* tr(\mathbf{C}) \tag{3}$$

$$G_{12} = G_{12}^* tr(\mathbf{C}) \tag{4}$$

$$\nu_{12} = \nu_{12}^* \tag{5}$$

where $E_1^* = 0.880$, $E_2^* = 0.052$, $G_{12}^* = 0.031$ and $\nu_{12}^* = 0.320$. Note that the value of the in-plane Poisson's ratio is assumed independently of $\boldsymbol{tr}(\mathbf{C})$.

According to Vignoli et al. [8], E_1 can be estimated using the classical rule of mixture with an average error smaller than 5%. Based on this result, the application of rule of mixtures is proposed in order to avoid the mechanical test required by Tsai's trace theory

$$E_1 = E_1^f V_f + E^m(1 - V_f) \tag{6}$$

Substituting Eqs. (2) and (6) in the Eqs. (3)–(5), the following expressions are obtained:

$$E_2 = [E_1^f V_f + E^m(1 - V_f)]\frac{E_2^*}{E_1^*} \tag{7}$$

$$G_{12} = [E_1^f V_f + E^m(1 - V_f)]\frac{G_{12}^*}{E_1^*} \tag{8}$$

Note that if Eq. (6) is used to compute E_1, just the ratio between the normalized property and E_1^* appears in the Eqs. (7) and (8). By assuming that a similar equation can be obtained for the out-of-plane shear modulus, the following expression is proposed to extend this theory for the 3D laminae:

$$G_{23} = [E_1^f V_f + E^m(1 - V_f)]\frac{G_{23}^*}{E_1^*} \tag{9}$$

3. Comparative analysis

A comparison between the trace theory and some other micromechanical models, namely with the rule of mixture (ROM); Halpin-Tsai (HT); modified Halpin-Tsai (HTm); Chamis (Ch); generalized self-consistent method (GSCM); bridging (Br); Mori-Tanaka (MT); asymptotic homogenization with square symmetry (AHs); asymptotic homogenization with hexagonal symmetry (AHh); modified rule of mixture (ROMm), with the experimental data is conducted by considering the average errors, following the procedure applied in Refs. [8,14,15]. The idea is to show the estimation of the parameters E_2^*/E_1^*, G_{12}^*/E_1^*, G_{23}^*/E_1^* and $\nu_{12}^* = \nu_{12}$. Several references are compiled defining a set of experimental data listed in Ref. [8], as well as a detailed discussion of the micromechanical models. Data is divided into cases of CFRP and GFRP due to the specificity of the trace theory.

Two different approaches are used to evaluate the average properties of trace-based estimations. First, the normalized properties proposed in Refs. [4,9] are applied. Alternatively, the modified values of these normalized properties are proposed, calibrating E_2^*/E_1^*, G_{12}^*/E_1^*, G_{23}^*/E_1^* and $\nu_{12}^* = \nu_{12}$ to minimize the errors in comparison with the set of experimental data compiled in this study. From now on, the first approach will be referred to by ***tr*** and the second one (modified trace) by ***tr-m***.

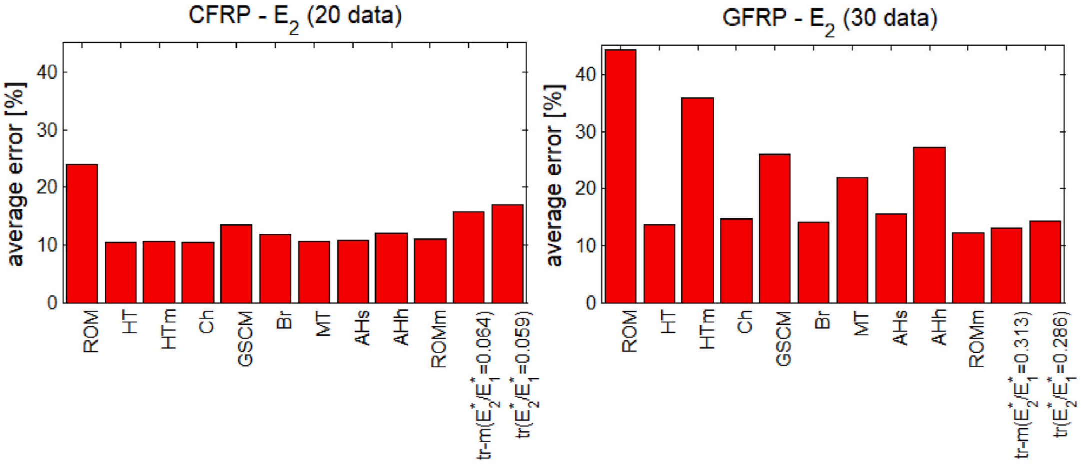
The average error for $E_2$is shown in the Fig. 1.

The first point to highlight is that the trace approach can be also applied for GFRP with an error range similar to CFRP. Despite that the errors from trace-based estimations are higher than from some micromechanical models, its simple implementation represents good alternative for composite design, mainly when all constituent properties are not available. Additionally, the estimation ***tr*** shows a very close average error to the proposed modified version.

The average errors for G_{12} are shown in the Fig. 2. CFRP estimations using ***tr*** and ***tr-m*** also have errors slightly higher than the traditional micromechanical models. On the other hand, for GFRP, ***tr-m*** estimation resulted in the smallest error among all models. This result indicates the possibility of application of normalized properties for the GFRP design.

The average error for ν_{12}.is shown in the Fig. 3. For this property, considering both laminae types, trace-based estimations demonstrate the most notable predictions. There are many factors that may influence the estimation of ν_{12} that usually are not considered by micromechanical models (at least for those reviewed in Ref. [8]), like interface and interphase properties. Somehow, these factors are intrinsically contemplated by trace-based approach, one it is empirically calibrated according to the experimental data.

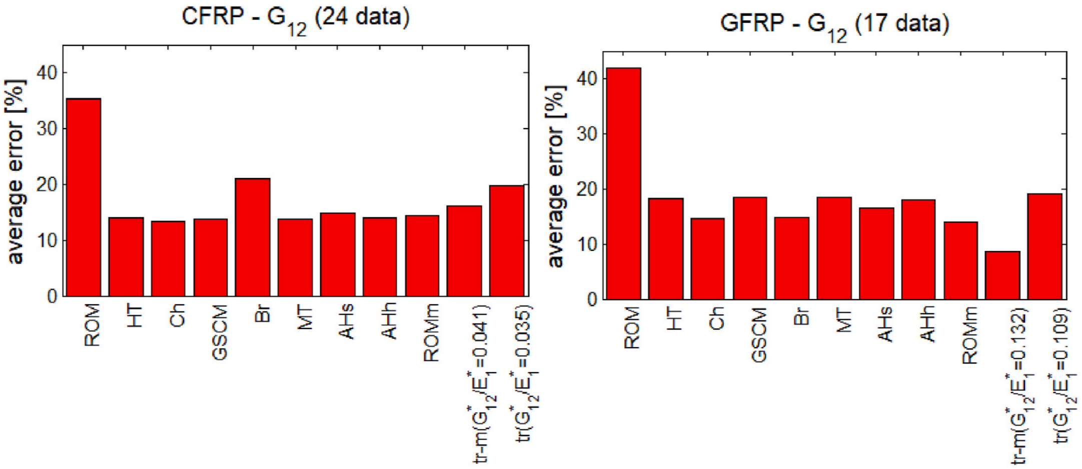
Finally, Fig. 4 shows the average error for G_{23}. This result indicates the possibility of extending the trace estimation for all the elastic properties. Despite that only 5 experimental data were used, the error of ***tr-m*** for GFRP is smaller than 1%, highlighting the efficiency of this methodology.



(a)

(b)

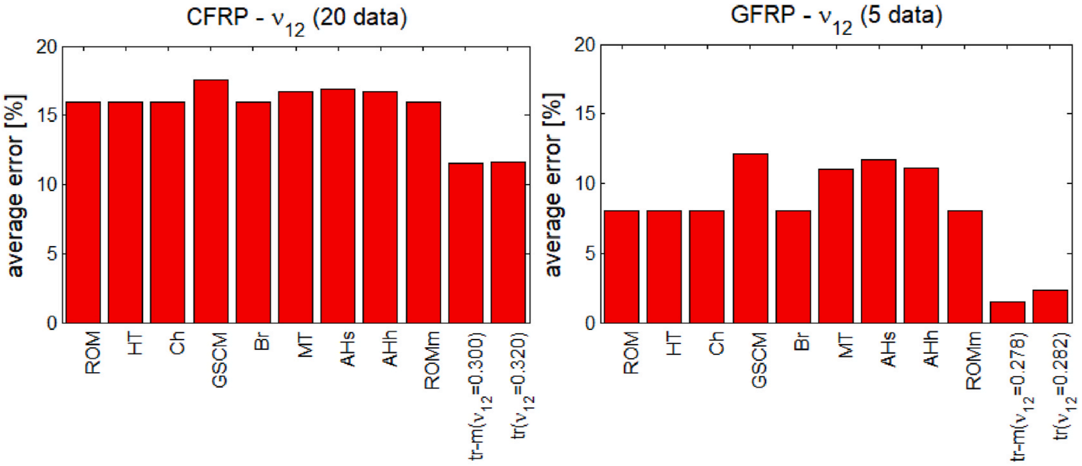
Fig. 1. Results for the transversal elastic modulus, E_2: (a) CFRP; (b) GFRP.



(a)

(b)

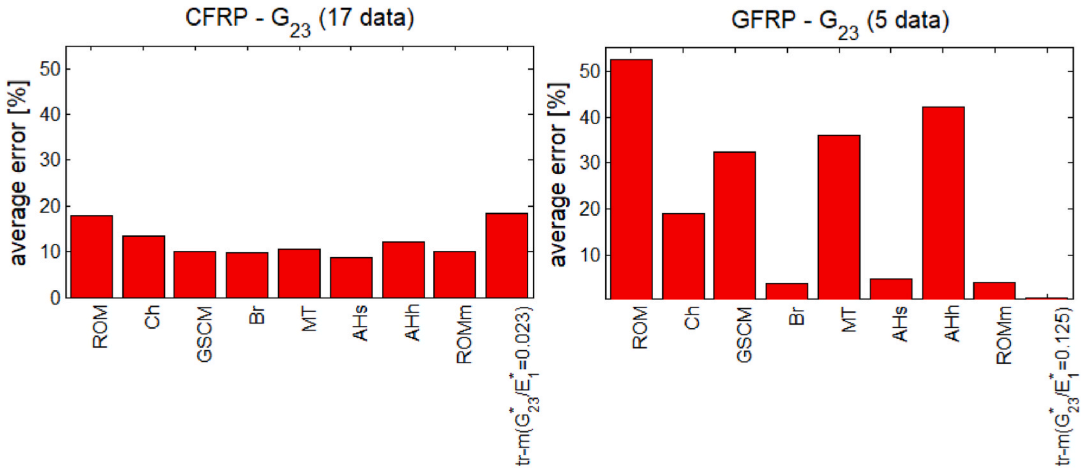
Fig. 2. Results for the in-plane shear modulus, G_{12}: (a) CFRP; (b) GFRP.



(a)

(b)

Fig. 3. Results for the in-plane Poisson's ratio, ν_{12}: (a) CFRP; (b) GFRP.



(a) (b)

Fig. 4. Results for the out-of-plane shear modulus, G_{23}: (a) CFRP; (b) GFRP.

4. Conclusions

This study presents a comparison between traditional micromechanical models and the recently proposed Tsai trace-based approach. A comparison with the same set of experimental data compiled in Ref. [8] is carried out. Trace approach presents interesting estimations requiring fewer experimental data. The extension of this theory for application in GFRP and estimation of G_{23} is verified. The authors believe that in an ideal approach, the Asymptotic Homogenization method with square unit cell (***AHs***) and modified rule of mixture (***ROMm***) are the best options for design. However, as pointed out in Ref. [2], the main motivation of trace theory is to decrease the cost of composite design with a reliable estimation, which is demonstrated in this investigation.

CRediT authorship contribution statement

Lucas L. Vignoli: Conceptualization, Methodology, Investigation, Formal analysis, Writing – review & editing. **Ranulfo M.C. Neto:** Methodology, Investigation, Writing – review & editing. **Marcelo A. Savi:** Conceptualization, Methodology, Supervision, Writing – review & editing. **Pedro M.C.L. Pacheco:** Methodology, Supervision, Writing – review & editing. **Alexander L. Kalamkarov:** Methodology, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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