Phablo R. Carvalho and Marcelo A. Savi*

An investigation of synchronization robustness considering randomness and asymmetries

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Abstract: Synchronization phenomena are related to several natural systems defining patterns of interactions. This paper deals with a synchronization robustness investigation evaluating pattern formation on a network of pendulum-chart oscillators receiving energy from a base excitation. Random aspects are investigated establishing the sensitivity to parameter changes and initial conditions. System asymmetries are analysed allowing the investigation of different kinds of dynamical patterns. Results show that asynchronous regions can change due to random effects. The asynchronous region reduces with the dissipation increase and the chimera state can occur under parametric asymmetry. Energetic argues are employed to explain the pattern robustness with respect to randomness and asymmetries.

Keywords: asymmetry; chimera; oscillators; randomness; robustness; synchronization.

1 Introduction

Synchronization is a pattern that emerges from the interaction of different dynamical systems. Chimera state is the coexistence between coherent and incoherent behaviors in a symmetrical network of identical oscillators [1, 2], characterizing a symmetry break, typically associated with a certain asymmetry in the initial conditions [3]. Although the first description of this pattern occurred during the 2000s [4], the experimental verification is due to about one decade later [5–9]. There are several natural systems that present chimera characteristics as ventricular fibrillation [10], power grid [11], social systems [12, 13], neural networks [14, 15], metamaterials [16] and earthquake sequences [17].

A general and comprehensive overview of chimeras is presented by Parastesh et al. [18]. The chimera state has been analysed by the influence of the initial conditions [19] and the strength of the coupling [20]. Different aspects of the random effects were evaluated concerning the distribution of randomness in the connections inside the network [14, 15, 21–24], the initial conditions [25–27], distribution of natural frequencies [28] and excitation [29–32]. The positive feedback in the D-dimensional Kuramoto model causes the transition between coherent and incoherent states that emerge as rhythmic dynamics [33].

The robustness of synchronization phenomena has been analysed by several research efforts that treat different aspects, including randomness and asymmetry. Considering decentralized networks, as random networks [34, 35], the synchronized behavior is preponderant with the increase of the network size; however, the larger the network is more susceptible to perturbation it is [23]. Random excitation tends to promote

***Corresponding author**: **Marcelo A. Savi**, Universidade Federal do Rio de Janeiro, COPPE – Department of Mechanical Engineering, Center for Nonlinear Mechanics, 21.941-972 Rio de Janeiro, RJ, Brazil, E-mail: savi@mecanica.coppe.ufrj.br. https://orcid.org/0000-0001-5454-5995
Phablo R. Carvalho, Universidade Federal do Rio de Janeiro, COPPE – Department of Mechanical Engineering, Center for Nonlinear Mechanics, 21.941-972 Rio de Janeiro, RJ, Brazil, E-mail: pr.carvalho@mecanica.coppe.ufrj.br

situations where the oscillators present a robust synchronization [29]. However, the randomness contributes to increase the instability region on some self-sustained electrical systems with non-identical parameters [30]. With respect to pattern formation, the noise can be used as an adjustable parameter in order to control the behavior of some systems [31]. The addition of noise to the system excitation is favorable to the formation of chimera states [36]. Besides, the literature points that the increase of dissipation promotes a reduction of the chimera state [37] and the propension of the system to synchronize [29].

Another interesting investigation is the modeling of cardiac pacemaker cells considering a random coupling network. This network system has the characteristic to increase the emergence of a mutual synchronization [21], which is a desirable behavior. This con guration represents a typical biological behavior where the local coordination among the cells di erentiates some specialized nodes [14].

This paper investigates the robustness of synchronization phenomena, including chimera state. A mechanical system composed by a three pendulum-chart system with symmetrical coupling is of concern. This system was previously investigated by Carvalho and Savi [38] using an energetic approach and a time series strategy to identify dynamical patterns. A hypothesis is also presented to explain the formation of the chimera state. The present paper investigates the robustness of the synchronization phenomena developing a parametric analysis that considers the e ects of randomness of external excitation, dissipation and initial conditions. In addition, the in uence of asymmetries is also of concern. The analysis is developed evaluating the energy level needed for the persistence of the dynamical patterns. Based on that, it is possible to evaluate if the randomness interferes on the relation among synchronous and asynchronous behaviors. Furthermore, the level of parametric uncertainty related to the chimera state considering asymmetry robustness has been investigated.

After this introductory section, this paper is organized in seven sections. Section 2 presents the mathematical formulation and the methodology to evaluate the level of synchrony of the dynamical patterns. Section 3 treats the excitation e ects. Afterward, dynamical patterns are evaluated under di erent conditions, with the focus on some system perturbations; Section 4 presents the in uence of the dissipation; Section 5 analyses the robustness of the system collective behaviors by perturbations and initial condition in uence; Section 6 investigates the in uence of asymmetries on the system response. Finally, Section 7 presents conclusions and nal remarks.

2 Mechanical model

Consider a mechanical system composed by three identical pendulum-chart systems, symmetrically coupled and excited by a base movement represented by a harmonic function $\mu_b = u \sin(t)$, Figure 1. The charts are

Figure 1: Mechanical system composed by three identical pendulum-chart systems.

represented by masses M_i and respective displacements x_i. Each pendulum has mass m_i and their motions are the angles $_i$ which is attached by a viscous damping with coe cient C. Connecting each pendulum-chart to the wall and base has one pair of spring and dampers, respectively k_j and C_j ($j = 1, 2, f, 9$).

The equations of motion (1)..(6) are obtained considering Lagrange equations, being expressed as follows [38].

$$M_1 + m_1 \; 1Š \cos^2 \; _1 \; x_1 Š \frac{C_1}{l} \; _1 \cos \; _1 Š m_1 g \sin \; _1 \cos \; _1 Š m_1 l \; _1^2 \sin \; _1$$
$$+ \; C_1 + C_4 + C_5 + C_9 \; x_1 Š C_5 x_2 Š C_9 x_3 + \; k_1 + k_4 + k_5 + k_9 \; x_1 Š k_5 x_2 Š k_9 x_3$$
$$= uk_1 \sin \; t + uC_1 \; \cos \; t \tag{1}$$

$$M_2 + m_2 \; 1Š \cos^2 \; _2 \; x_2 Š \frac{C_2}{l} \; _2 \cos \; _2 Š m_2 g \sin \; _2 \cos \; _2 Š m_2 l \; _2^2 \sin \; _2$$
$$+ \; C_2 + C_5 + C_6 + C_7 \; x_2 Š C_5 x_1 Š C_7 x_3 + \; k_2 + k_5 + k_6 + k_7 \; x_2 Š k_5 x_1 Š k_7 x_3$$
$$= uk_2 \sin \; t + uC_2 \; \cos \; t \tag{2}$$

$$M_3 + m_3 \; 1Š \cos^2 \; _3 \; x_3 Š \frac{C_3}{l} \; _3 \cos \; _3 Š m_3 g \sin \; _3 \cos \; _3 Š m_3 l \; _3^2 \sin \; _3$$
$$+ \; C_3 + C_7 + C_8 + C_9 \; x_3 Š C_7 x_2 Š C_9 x_1 + \; k_3 + k_7 + k_8 + k_9 \; x_3 Š k_7 x_2 Š k_9 x_1$$
$$= uk_3 \sin \; t + uC_3 \; \cos \; t \tag{3}$$

$$m_1 l^2 \; _1 + C_1 \; _1 + m_1 g l \sin \; _1 + \frac{m_1 l \cos \; _1}{(M_1 + m_1 (1Š \cos^2 \; _1))}$$
$$\times \; \frac{C_1}{l} \; _1 \cos \; _1 + m_1 g \sin \; _1 \cos \; _1 + m_1 l \; _1^2 \sin \; _1 Š \; C_1 + C_4 + C_5 + C_9 \; x_1$$
$$+ C_5 x_2 + C_9 x_3 Š \; k_1 + k_4 + k_5 + k_9 \; x_1 + k_5 x_2 + k_9 x_3 + uk_1 \sin \; t + uC_1 \; \cos \; t \; = 0 \tag{4}$$

$$m_2 l^2 \; _2 + C_2 \; _2 + m_2 g l \sin \; _2 + \frac{m_2 l \cos \; _2}{(M_2 + m_2 (1Š \cos^2 \; _2))}$$
$$\times \; \frac{C_2}{l} \; _2 \cos \; _2 + m_2 g \sin \; _2 \cos \; _2 + m_2 l \; _2^2 \sin \; _2 Š \; C_2 + C_5 + C_6 + C_7 \; x_2 + C_5 x_1 + C_7 x_3$$
$$Š \; k_2 + k_5 + k_6 + k_7 \; x_2 + k_5 x_1 + k_7 x_3 + uk_2 \sin \; t + uC_2 \; \cos \; t \; = 0 \tag{5}$$

$$m_3 l^2 \; _3 + C_3 \; _3 + m_3 g l \sin \; _3 + \frac{m_3 l \cos \; _3}{M_3 + m_3 \; 1Š \cos^2 \; _3}$$
$$\times \; \frac{C_3}{l} \; _3 \cos \; _3 + m_3 g \sin \; _3 \cos \; _3 + m_3 l \; _3^2 \sin \; _3 Š \; C_3 + C_7 + C_8 + C_9 \; x_3 + C_7 x_2 + C_9 x_1$$
$$Š \; k_3 + k_7 + k_8 + k_9 \; x_3 + k_7 x_2 + k_9 x_1 + uk_3 \sin \; t + uC_3 \; \cos \; t \; = 0 \tag{6}$$

As discussed in Carvalho & Savi [38], an energetic approach is interesting to analyse the pattern formation, establishing the energy exchanges among the system elements. Thereby, it is important to de ne the energy that is transmitted by the springs among the charts. Additionally, it is interesting to de ne the amount of

energy received and accumulated in each oscillator by the springs that connects the charts to the xed wall and to the base with an excitation. Under these assumptions, the ux of energy $E_{F,c}$ is considered as the time mean of the ratio of these two forms of energy in each chart, that is, the transmitted energy for the received, as shown in the sequence.

$$E_{F,c} = \frac{1}{t_f} \int_0^{t_f} \frac{k_5 (x_2 \check{S} x_1)^2 + k_9 \ x_3 \check{S} x_1 \ ^2}{k_1 x_1^2 + k_4 x_1^2} + \frac{k_5 (x_2 \check{S} x_1)^2 + k_7 \ x_3 \check{S} x_2 \ ^2}{k_2 x_2^2 + k_6 x_2^2} + \frac{k_7 \ x_3 \check{S} x_2 \ ^2 + k_9 \ x_3 \check{S} x_1 \ ^2}{k_3 x_3^2 + k_8 x_3^2} \, dt \tag{7}$$

In order to investigate the randomness in uence on the system response, three di erent approaches are assumed, considering random excitation parameters: excitation frequency, ; excitation amplitude, u; and harmonic excitation with noise, r. Therefore, the base excitation can be expressed by $u_b = u \sin \ t + r(0, \ )$, where $u = u \ \bar{u}, \ ), \ = \ \bar{}, \ $ and $r = r(0, \ )$ are represented by normal distributions with average values, $\bar{u}, \ \bar{}$ and zero, with standard deviation . In addition, random e ects are also included on the dissipation parameter $C = C \ \bar{C}, \ )$ and on the initial conditions.

Numerical simulations are carried out using the fourth order Runge…Kutta method adopting the following parameters: $k_{4,6,8} = 0$ N/m, $k_{5,7,9} = 50$ N/m, $g = 9.81$ m s^2, $l = 0.2$ m, $M = 1$ kg, $m = 1$ kg, $C_{1,2,3,5,7,9} = 0.56$ N s/m, $C_{4,6,8} = 0$ N s/m and $C \ = 0.01$ N s/m. Initial conditions of the pendula always consider null velocity and displacement. Parameters $k_{1,2,3}, \ $, u and the initial conditions of displacements for the charts are altered accordingly.

Dynamical patterns are identi ed considering time series analysis. The procedure monitors the response of each oscillator, establishing a di erence-curve that is representative of the dynamical pattern. In this regard, signal of each oscillator is compared with the others, de ning the kind of response [38]. Dynamical patterns are classi ed considering six kinds of responses: in-phase synchronization, observed when all the oscillators move identically; anti-phase synchronization, present when at least one oscillator moves in opposite phase to the others; frequency lock behavior, de ned by the identity of frequency among the observed oscillators despite di erent orbits; long period synchronization, observed for the presence of an oscillator in which the frequency is a multiple of the frequency of the others; chimera state, de ned by the coexistence of synchronized and desynchronized behaviors in systems of identical oscillators symmetrically coupled; and desynchronized behavior, characterized by the independent motion of each oscillator.

3 Excitation effects

Randomness• in uence on the system response is evaluated by considering three di erent random sources on excitations: excitation frequency, ; excitation amplitude, u; and harmonic excitation with noise, r.

Initially, consider the case where a random excitation frequency is of concern, adopting an excitation frequency $= \ \bar{}, \ $ and di erent standard deviations in order to represent distinct levels of randomness. Figure 2 shows the amount of energy that the system receives by the base excitation represented by the sum of the potential energy of the chart•s ensemble, which is the sum of the denominators of Eq. (7). The curve for $= 0$ is the energy distribution of the system without randomness. The increase of the standard deviation causes a reduction of the system energy near the rst resonance peak. This is a consequence of a larger band of frequencies which excites the system. Based on that, the larger the standard deviation, smaller the energy concentrated at the resonance frequency.

The randomness of the excitation amplitude is now of concern, considering $u = u \ \bar{u}, \ $. By employing the same strategy to analyse the system, results are presented in Figure 3 showing an opposite in uence compared with the previous case: the increase of the total energy for higher standard deviations. This e ect is equivalent to add a di erent level of excitation to the system.

Figure 2: Random excitation frequency showing the energy level received by the system to a different standard deviation level: higher standard deviations reduce the amount of energy due the larger contribution of frequencies with respect to the resonance.

Figure 3: Random amplitude of excitation showing the energy level received by the system to a different standard deviation level: the increase of the standard deviation provides more energy to the system for all the frequencies and the nominal frequency remains constant.

An analysis considering random excitation incorporating a noise over the system base excitation is now in focus using an excitation of the form: $u_b = u \sin(\ \ t) + r(0,\ \ )$, where r is the noise. Figure 4 presents results of this case that are similar to the one with random excitation amplitude. Nevertheless, an increase of the energy level for high frequencies, higher than 30 rad/s is noticeable.

A parametric analysis is carried out in the sequence considering the in uence of the frequency, , and the sti ness of the springs that connects the charts to the base excitation, $k = k_1 = k_2 = k_3$. This analysis allows one to compare pattern diagrams related to di erent random levels. Figure 5 shows pattern diagrams considering the in uence of the random excitation frequency, for di erent randomness levels represented by the standard deviation, . Figure 5(a) shows the system response without randomness, = 0, exhibiting an exchange among in-phase and anti-phase behaviors, forming a pattern similar to a fractal basin and a chimera state region around 10 rad/s. This region is named as the asynchronous region, presenting both chimera state and desynchronized behaviors. The asynchronous region area is reduced with the increase of the randomness level. Figure 5(b) presents results for = 0.02 showing that in-phase and anti-phase remain the same. By increasing the randomness to = 0.03, Figure 5(c), there is a reduction of the asynchronous region that does not exist for = 0.1, Figure 5(c). This change occurs due the randomness distribution of energy in a band which has the width proportional to the standard deviation, as shown in Figure 2. Therefore, the increase of the standard deviation tends to reduce the energy received and the system has no su cient energy to develop asynchronous behaviors.

Figure 4: Excitation with harmonic and random components showing the energy level received by the system from each noise intensities: the increase of the standard deviation provides more energy to the system since randomness is added to the harmonic excitation, being more evident outside the resonant peaks.

Figure 5: Pattern diagram considering the influence of the random term at the excitation frequency showing that the asynchronous region reduces proportionally to the increase of standard deviation: (a) = 0, (b) = 0.02, (c) = 0.03 and (d) = 0.1.

The same analysis is now performed by considering an energetic approach, de ned by Eq. (7), Figure 6. Observe that the energy ux limits are Š60 to 30 for all the energetic diagrams in order to establish a comparison among them and this ratio represents the intensity of the synchronous (for negative) and asynchronous behaviors (for positive values) in which the transition is the frontier around zero. The case without randomness

Figure 6: Energetic diagram in logarithmic scale considering the influence of the random term at the excitation frequency showing a distinction between asynchronous … positive ratios and synchronous behaviors … negative ratios; and the transitions of in-phase and anti-phase for higher standard deviations: (a) = 0, (b) = 0.02, (c) = 0.03 and (d) = 0.1.

is presented in Figure 6(a) showing that asynchronous states are related to high level of the energy ux [7] while synchronized states are associated with low energy level. The increase of the randomness level reduces the asynchronous region, with the same trend observed in Figure 5. The increase of the randomness level makes the di erence between in-phase and anti-phase distinguishable due to the small amount of energy between them. It should be pointed out that the higher level of energy is associated with resonant conditions highlighted in Figure 2, which is the sum of the denominators of Eq. (7).

The randomness e ect of the excitation amplitude, u, is now in focus. Figure 7 shows the pattern diagram where it is possible to observe the asynchronous region together with the synchronous region with in-phase and anti-phase behaviors. It is noticeable that the increase of the randomness level enlarges the size of the asynchronous band. In addition, it promotes an erosion of the synchronous region that is evident in Figure 7(c) and (d) where in-phase and anti-phase behaviors become homogeneously distributed. Figure 8 presents the energetic diagram showing the same trends for an energetic perspective.

The random excitation, r, is now of concern together with the harmonic excitation. Figure 9 shows the pattern diagram for four random levels. This case presents the same asynchronous region associated with the erosion of the in-phase and anti-phase behaviors with the increase of the standard deviation. Figure 10 presents the energetic perspective of the same cases. Once again, it is possible to observe similar synchronization regions and the erosion of the synchronous behaviors.

Figure 7: Pattern diagram considering the influence of the random amplitude of excitation showing that the asynchronous region increases for higher standard deviations, while the pattern between in-phase and anti-phase is diffused. (a) = 0, (b) = 0.1, (c) = 1 and (d) = 2.

4 Dissipation effects

The in uence of the dissipation is now evaluated on the pattern formation considering a random dissipation parameter. This analysis provides a comprehension about the robustness of the synchronization phenomena. It is adopted that u = 0.2 m and random dissipation connecting the charts, considering a normal distribution around the mean damping $C_{1,2,3,5,7,9} = C_{1,2,3,5,7,9}(\overline{C}_{1,2,3,5,7,9}, \ )$, where the average value is the nominal one, $\overline{C}_{1,2,3,5,7,9} = 0.56$ Ns m. Random values are assumed to be always positive and hence, all negative values are vanished. Figure 11 presents pattern diagram for di erent standard deviations. Figure 11(a) shows a situation without random e ects, = 0. Figure 11(b) considers = 0.1 that causes the suppression of the anti-phase behavior and the asynchronous region increases the bandwidth near sti ness of 20 N/m. The same characteristics are observed considering = 1. It should be pointed out that the increase of the random level causes the suppression of the anti-phase behavior and the reduction of the proportion of desynchronized behavior inside the asynchronous region.

In order to have a better comprehension of the in uence of the dissipation, the limit cases of the random variation is analysed in Figure 12: C = 0 and C = 1.68 N s m. Figure 12(a) shows the response of the non-dissipative system. Note a di usion of the synchronous region with respect to in-phase and anti-phase behaviors. Moreover, the asynchronous bandwidth is larger showing a second band around 3 rad/s and others asynchronous bands for superior frequencies. The transition between the in-phase and anti-phase behaviors

Figure 8: Energetic diagram in logarithmic scale considering the influence of the random amplitude of excitation showing an erosion of the pattern between in-phase and anti-phase at the low energetic ratio areas: (a) = 0, (b) = 0.1, (c) = 1 and (d) = 2.

disappears for a larger dissipation level, Figure 12(b) showing that the anti-phase behavior does not occur. This is due to the higher dissipation level which decreases the level of energy, reducing the propension of anti-phase behavior [38] since it needs a higher level of synchrony. Additionally, there is less desynchronized behavior in the asynchronous region.

5 Initial condition effects and perturbations

In order to investigate the robustness of the system response with respect to initial conditions, random initial conditions are analysed. Figure 13 shows a pattern diagram for di erent levels of randomness, starting with a deterministic response, Figure 13(a) without randomness, and with di erent values of standard deviations, = 0.01 .. Figure 13(b) and = 0.1 .. Figure 13(c) Results show a clear erosion of the patterns, generating a pattern diagram with a fractal-like structure. This means that synchronous and asynchronous behaviors have a high sensitivity to initial conditions.

Results show that di erent initial conditions can change the orbit and the pattern behaviors. Another approach evaluates the response of the system to a destabilization. In this regard, displacement and velocity of the rst chart in a time instant during the steady state regime is imposed. The idea is to observe if the system changes the collective behavior due to perturbation.

Figure 9: Pattern diagram considering the influence of the frequency excitation with noise showing that the asynchronous region increases for higher standard deviations, while the pattern between in-phase and anti-phase is diffused in lower intensity for the same standard deviation: (a) = 0, (b) = 0.01, (c) = 1 and (d) = 3.

Figure 14 shows the chimera state system response considering u = 0.1 m, = 9.5 rad/s, $k_{1,2,3}$ = 18 N/m and initial condition of displacement of $x(0)$ = 1 m for the rst chart and null to the others oscillators. A perturbation is imposed to the system by assuming ve times the displacement and velocity at the time instant 530 s. After the dramatic perturbation, the system passes through a transient response and in about 20 s, it recovers the chimera pattern.

Figure 15 shows the response with u = 1 m, = 10 rad/s, $k_{1,2,3}$ = 19 N/m and null initial conditions which means that the system is in a synchronized chaotic state. The perturbation imposes an increase of 10% of the displacement at time instant of 500 s. This perturbation alters the system response and, after time instant 530 s, they are in a desynchronized state. Therefore, the perturbation changes the pattern response from synchronized to a desynchronized state.

A situation where an in-phase synchronized response occurred is with u = 0.2 m, = 5.7 rad/s, $k_{1,2,3}$ = 12.2 N/m and initial condition of displacement of $x(0)$ = 1 m for the rst chart and null to the others oscillators is now of concern (Figure 16). A perturbation is applied considering ve times the displacement and velocity at time instant 882 s. Note that, even with a dramatic perturbation, the system returns to the same pattern after a transient response, around 910 s.

Except for the synchronized chaotic behavior, Figure 15, the others present a stable response, which means that the perturbation does not change the dynamical pattern. For the synchronized chaotic behavior, even small perturbations change the system response.

Figure 10: Energetic diagram in logarithmic scale considering the influence of the excitation frequency with noise showing the erosion progression of the pattern among in-phase and anti-phase with the increase of the standard deviation: (a) for a = 0, (b) for = 0.01, (c) for = 1 and (d) for = 3.

6 Asymmetry effects

System asymmetry is now of concern considering situations where parameter values vary in such way that it breaks the system symmetry. The idea is to evaluate the in uence of some parameter change close to a symmetrical con guration. In other words, a perturbation diagram analysing the pattern of the responses in a vicinity of the symmetrical parameter space is built. Therefore, the central point of the diagram represents the symmetry condition and the increase of the asymmetry is represented by the distance from the central symmetrical point. The analysis veri es the symmetry deviations for the charts and for the pendula using a diagram for each one, establishing the robustness due to asymmetries.

Consider an in-phase synchrony response under symmetric condition: $u = 0.2$ m, $k_3 = 122$ N m, = 5.7 rad s; k_1 and k_2 are de ned by each point of Figure 17 that shows the perturbation diagram related to k_1 Š k_2 for charts and pendula, evaluating the asymmetrical robustness. Note that symmetric condition, represented by the central point, is related to gray color that occupies a neighborhood that de nes the robustness where the system is still presenting the same dynamical pattern. It is possible to quantify the size of this region that is around 0.01 N/m (or a parameter perturbation of 0.8%) for the pendulum (left panel) and around ten times bigger to the charts (right panel). Pendulum symmetry break is much more dramatic,

Figure 11: Pattern diagrams for excitation amplitude of $u = 0.2$ m and dissipation $C_{1,2,3,5,7,9} = C_{1,2,3,5,7,9}(\overline{C}_{1,2,3,5,7,9}, \quad)$ with $\overline{C}_{1,2,3,5,7,9} = 0.56$ Ns m. (a) $= 0$, (b) $= 0.1$ and (c) $= 1$. For higher standard deviation the anti-phase behavior and the desynchronized state reduce due to more stable states in-phase and chimera state, respectively.

Figure 12: Pattern diagram for excitation amplitude of $u = 0.2$ m and constant damping of $C_1 = C_2 = C_3 = C_5 = C_7 = C_9 = C$; (a) $C = 0$ and (b) $C = 1.68$ N s m. The dissipation tends to homogenize the patterns that are reduced to in-phase and chimera states.

Figure 13: Pattern diagram considering random initial conditions with = 7.7 rad s, u = 0.2 m and $k_{1,2,3}$ = 10.8 rad/s. (a) = 0, (b) = 0.01 and (c) = 0.1. It is noticeable that the erosion of the fractal-like structure is proportional to the standard deviation.

Figure 14: Response of the system in chimera state behavior with a destabilization of five times the variables of state to 530 s with the system recovering the chimera state.

giving space for chimera state (red) and desynchronized responses (yellow). On the other hand, charts present a small region of frequency locked response.