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Nonlinear prediction of time series obtained from an experimental pendulum

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ABSTRACT

Time series is a sequence of observations of one or a few time variable of a dynamical system. Linear analysis assumes that the intrinsic dynamics of the system is related to the fact that small causes lead to small effects. On the other hand, nonlinear data set may be related to irregular data with purely deterministic inputs. Nonlinear time series analysis is of special interest of several areas. Time series prediction is an area of this general topic that has the objective of estimating future values from a known time series, called past, without any knowledge of the governing equations of phenomena. This article considers the analysis of some prediction techniques applied to time series obtained from an experimental nonlinear pendulum. Noise suppression is not contemplated and all signals are analyzed without filtering. Periodic and chaotic signals are analyzed employing three different predictors: simple nonlinear, polynomial and radial basis functions. The influence of state space reconstruction is exploited showing that it is an important task to be taking into account in prediction problems.

1. Introduction.

The most direct link between chaos theory and the real world is the analysis of time series from real systems in terms of nonlinear dynamics. Time series is a sequence of observations of one or a few time variable of the system [1]. Usually, it is related to a nonlinear dynamical system which experimental analysis furnishes a scalar sequence of measurements.

Linear methods interpret regular structure in a data set meaning that the intrinsic dynamics of the system is related to the fact that small causes lead to small effects. Nonlinear, chaotic systems, however, can produce irregular data with purely deterministic inputs [1]. Chaotic behavior presents sensitive dependence on initial conditions and long-term unpredictability.

Nonlinear analysis involves different tools from linear ones. State space reconstruction and the determination of dynamical invariants are some of these tools. Lyapunov exponents and attractor dimension are some examples of dynamical invariants that could be used to identify chaotic behavior. The signs of the Lyapunov exponents provide a qualitative picture of the system's dynamics and any system containing at least one positive exponent presents chaotic behavior. The attractor dimension, on the other hand, counts the effective number of degrees of freedom in a dynamical system and its strangeness is related to situations where a noninteger number describes the system dimension.

An area related to time series analysis is the prediction which objective is estimating future values from a known time series, called past, $S_n, n = 1, ..., N$. Therefore, it is necessary to estimate future time series, S_{N+1} , S_{N+2} , ... , S_{N+p} , employing some prediction technique that results in an estimated series: P_{N+1} , P_{N+2} , ... , P_{N+p} .

Figure 1 shows a schematic plot related to the prediction problem. From a known time series, called past, some predictor evaluates future values of time series, prediction. These estimated values can be compared with future values associated with the original series in order establish a prediction accuracy, defining the predictor error.

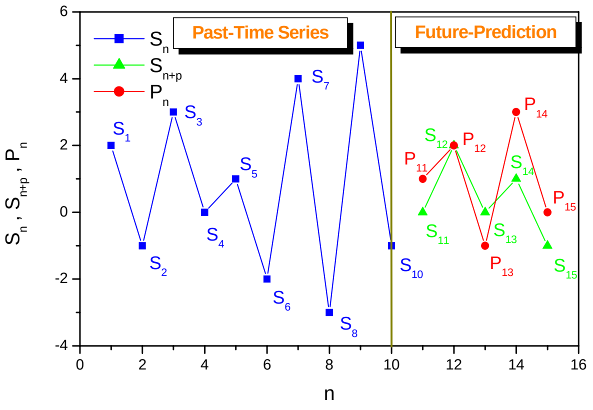


Figure 1. Time series prediction.

Lyapunov exponents give an indication of how far into the future reliable predictions can be made. On the other hand, information dimension gives indication of how complex must be the system model [2]. The accuracy of the predictor is evaluated from an error value that can be defined with different ways. In this text, a normalized root mean square error, e_{RMS}, is considered:

$$e_{RMS} = \frac{1}{\sigma}\sqrt{\frac{1}{p}\sum_{n=N+1}^{N+p}(S_n - P_n)^2} \qquad (1)$$

where p is the number of steps in future and σ is the standard deviation of the signal. Usually, there are two kinds of error that could be evaluated from predictions: *in-sample* and *out-of-sample* errors.

Gershenfeld and Weigend [3] says that the beginning of the modern study of time series prediction is due to Yule [2], which presents, for the first time, the autoregressive technique applied to a state variable in order to estimate the future behavior of a dynamical system.

Nowadays, there are several areas with special interest in time series prediction. Among them, one could mention engineering [5–7], economics [8–9], marketing [10], weather forecasting [11–12] and medicine [13]. Prediction can be imagined even in arts. Dirst and Weigend [14] employ these techniques in order to analyze an unfinished music of Johann Sebastian Bach (1685-1750).

In general, techniques for time series prediction may be classified in *linear* and *nonlinear* methods. Other classification reported in literature considers *local* and *global* methods [2]. References [1, 15–19] provide an overview of the main aspects related to nonlinear time series analysis and prediction.

Linear prediction methods include *linear zeroth order*, *moving average*, *autoregressive* and *autoregressive moving average* [1, 18–19]. Nonlinear methods are based on the state space reconstruction [1–2, 15, 19, 21–22] and include techniques such as: *simple nonlinear*, *polynomial* and *radial basis functions*. Recently, *neural networks* are being used as a good alternative in prediction problems [1–2, 15, 21]. *Fuzzy Logic* and *Logic Rules* are also new improvements included in prediction techniques [10–11].

This article considers the analysis of some prediction techniques applied to time series obtained from an experimental nonlinear pendulum [23]. Noise suppression is not contemplated and all signals are analyzed without filtering. Periodic and chaotic signals are contemplated employing three different predictors: simple nonlinear, polynomial and radial basis functions. The influence of state space reconstructions is analyzed.

2. State Space Reconstruction.

The basic idea of the state space reconstruction is that a signal contains information about unobserved state variables that can be used to predict the present state. Therefore, a scalar time series, S_n, may be used to construct a vector time series that is equivalent to the original dynamics from a topological point of view. The state space reconstruction needs to form a coordinate system to capture the structure of orbits in state space, which could be done using lagged variables, $S_{n+\tau}$, where τ is the time delay. Then, it is possible to use a collection of time delays to create a vector in a D_e-dimensional space,

$$\mathbf{U}_n = \left(S_{n-(De-1)\tau}\ , S_{n-(De-2)\tau}\ , \ldots, S_{n-\tau}\ , S_n\right) \qquad (2)$$

The literature reports many methods employed to determine time delay. The mutual information method [24] presents good results, which disseminate its use. The determination of embedding dimension, D_e, also involves different methods. The false nearest neighbors (*FNN*) [25] is a good alternative with this aim.

2.1 - Method of Average Mutual Information

Fraser and Swinney [24] establishes that the time delay τ corresponds to the first local minimum of the average mutual information function $I(\tau)$, which is defined as follows,

$$I(\tau) = \sum_{n=1}^{N-\tau} P(S_n, S_{n+\tau}) \log_2 \left[\frac{P(S_n, S_{n+\tau})}{P(S_n)\, P(S_{n+\tau})} \right] \qquad (3)$$

where $P(S_n)$ is the probability of the measure S_n, $P(S_{n+\tau})$ is the probability of the measure $S_{n+\tau}$, and $P(S_n, S_{n+\tau})$ is the joint probability of the measure of S_n and $S_{n+\tau}$ [24]. The average mutual information is really a kind of generalization to the nonlinear phenomena from the correlation function in the linear phenomena. When the measures S_n and $S_{n+\tau}$ are completely independent, $I(\tau) = 0$. On the other hand, when S_n and $S_{n+\tau}$ are equal, $I(\tau)$ is maximum. Therefore, plotting $I(\tau)$ versus τ it is possible to identify the best value for the time delay which is related to the first local minimum.

2.2 - Method of False Nearest Neighbors

The false nearest neighbors algorithm (*FNN*) was originally developed for determining the number of time delay coordinates needed to recreate autonomous dynamics, but it is extended to examine the problem of determining the proper embedding dimension.

In an embedding dimension that is too small to unfold the attractor, not all points that lie close to one another will be neighbors because of the dynamics. Some will actually be far from each other and simply appear as neighbors because the geometric structure of the attractor has been projected down onto a smaller space [25].

In order to use the method of false nearest neighbors, a D-dimensional space is considered where the point $\mathbf{U}_n$ has rth nearest neighbors, $\mathbf{U}_n^r$. The square of the Euclidean distance between these points is,

$$r_D^2(n,r) = \sum_{k=0}^{D-1} \left[S_{n+k\tau} - S_{n+k\tau}^{r} \right]^2 \qquad (4)$$

Now, going from dimension D to D+1 by time delay, there is a new coordinate system and, as a consequence, a new distance between $\mathbf{U}_n$ and $\mathbf{U}_n^r$. When these distances change from one dimension to another, these are false neighbors. A natural criterion for catching embedding errors is that the increase in distance between $\mathbf{U}_n$ and $\mathbf{U}_n^r$ is large when going from dimension D to D+1. The increase in distance can be stated with distance equations and some criteria must be established to designate the existence of false neighbors. Reference [25] establishes proper criteria for this aim.

3. Prediction Methods.

This section presents a brief discussion on prediction methods. Basically, three different techniques are considered: simple nonlinear, polynomial and radial basis functions.

3.1 Simple Nonlinear Prediction

Simple nonlinear prediction is based on the state space reconstruction. After the reconstruction, in order to predict a time Δn (Δn = 1, ..., p) ahead N, it is necessary to define a parameter ε that is related to the size of the neighborhood $V_\varepsilon(\mathbf{U}_N)$ around point $\mathbf{U}_N$. Therefore, for all points $\mathbf{U}_n$ closer than ε to $\mathbf{U}_N$ $(\mathbf{U}_n \in V_\varepsilon(\mathbf{U}_N))$ look up the individual prediction $S_{n+\Delta n}$. The prediction $P_{N+\Delta n}$ is then calculated from the average of the individual predictions $S_{n+\Delta n}$ [1].

$$P_{N+\Delta n} = \frac{1}{\left|V_\varepsilon(\mathbf{U}_N)\right|} \sum_{\mathbf{U}_n \in V_\varepsilon(\mathbf{U}_N)} S_{n+\Delta n} \tag{5}$$

where $\left|V_\varepsilon(\mathbf{U}_N)\right|$ denotes the number of elements of the neighborhood $V_\varepsilon(\mathbf{U}_N)$.

Figure 2 presents a schematic representation of the simple nonlinear prediction applied to a time series with 10 elements and $D_e = 2$. For a parameter ε, points U_2, U_4, U_5, U_7 and U_8 are inside the neighborhood and hence, the first prediction, P_{11}, is evaluated from the average of the values S_3, S_5, S_6, S_8 and S_9.

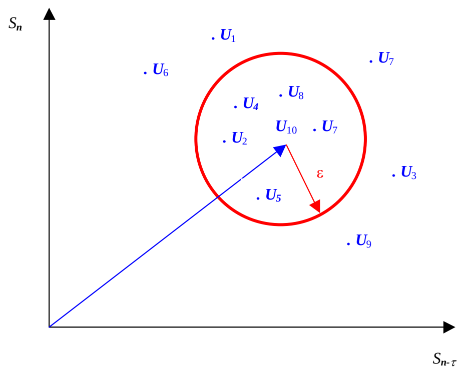


Figure 2. Simple nonlinear prediction.

The simple nonlinear predictor may be understood as a local method since it uses only a limited number of neighboring samples to perform the prediction.

3.2 Polynomial Prediction

A general procedure for prediction is to adjust general functions $\Phi_j\,(j=1,\ldots,\alpha)$ to the last m values of the time series.

$$P_{n+1} = \sum_{j=1}^{\alpha} a_j\, \Phi_j(S_n) = a_1\, \Phi_1(S_n) + \ldots + a_\alpha\, \Phi_\alpha(S_n) \tag{6}$$

Parameters a_j (j = 1, …, α) are defined in order to minimize the in-sample error [1]. The minimization of this error furnishes:

$$\frac{\partial e_{RMS}}{\partial a_i} = 0 \quad (i=1,\ldots,\alpha) \tag{7}$$

which follows the linear system,

$$\sum_{j=1}^{\alpha} C_{ij}\, a_j = \sum_{n=N-m+1}^{N} S_{n+1}\, \Phi_i \quad (i=1,\ldots,\alpha) \tag{8}$$

where C_{ij} is the covariant matrix given by,

$$C_{ij} = \sum_{n=N-m+1}^{N} \Phi_i\, \Phi_j \quad (i,\, j=1,\ldots,\alpha) \tag{9}$$

A recursive procedure allows one to estimate predicted values of the time series. Polynomial function is a good alternative for the time series modeling: $\Phi_j(S_n) = (S_n)^j$, $j = 0,\ldots,\alpha$, that is,

$$P_{n+1} = \sum_{j=0}^{\alpha} a_j\, (S_n)^j = a_0 + a_1\, S_n + \ldots + a_\alpha\, (S_n)^\alpha \tag{10}$$

Polynomial order is an important task in this method. Gershenfeld and Weigend [3] says that lower degrees is related to robust predictions, however, it is not possible to capture all details of the data set structure. On the other hand, higher order polynomials tend to capture noise characteristics of the signal, which is related to a phenomenon known by *overfitting*. The polynomial predictor may be understood as a global method since it is a generalization of Taylor series expansion to perform the prediction.

3.3 Radial Basis Functions Prediction

Radial basis functions constitute another alternative to the known functions employed to the predictor presented in the preceding section. With this aim, the predictor may be written as follows,

$$P_{n+1} = \sum_{j=1}^{\alpha} a_j\, \Phi_j(\mathbf{U}_n) \tag{11}$$

where Φ_j are radial basis functions. Usually, bell-shaped functions are defined with respect to k centers on the attractor, $\mathbf{y}_j$, with r_j width. Therefore,

$$\Phi_j(\mathbf{U}_n) = \exp\left(-\frac{(\mathbf{U}_n - \mathbf{y}_j)^2}{r_j^{\,2}} \right) \qquad (12)$$

Parameters a_j are defined from an error minimization process. It is reported in literature that these functions are convenient to model time series related to complex behaviors, as strange attractors. However, there is a raise in the number of parameters, which is associated with higher computational effort.

Lillekjendlie *et al.* [17] considers this technique as a *semi-local* method, which combines the smoothness of global predictors and the localized dependence on new information of local predictors. This article employs the algorithm due to Hegger *et al.* [22] to evaluate predictions considering 10 centers uniformly distributed over the domain.

4. Experimental Signals.

The experimental data related to the nonlinear pendulum response is obtained from the apparatus depicted in Figure 3 [23]. The pendulum is constructed by a disc with a lumped mass (1) and is connected to a rotary motion sensor (3). An adjustable magnetic device (2) regulates the dissipation of the system. A motor-string-spring device (4-5) provides the excitation of the pendulum. The motor (5), *PASCO ME-8750*, has the following characteristics: 12V DC, 0.3-3Hz and 0-0.3A. The signal measurement is done with the aid of two transducers. The rotary motion sensor (3), *PASCO encoder CI-6538*, has 1440 orifices and a precision of 0.25^0. The magnetic transducer (6) is employed in order to generate a frequency signal associated with the forcing frequency of the motor, which is used to construct the Poincaré map of the signal. The apparatus is connected with an A/D interface, *Science Workshop Interface 500* (*CI-6760*), where the sampling frequency varies from 2Hz to 20kHz. The interface oversamples the signal 8 times for frequencies below 100Hz and a single time for higher sampling rates. Furthermore, this interface does not have any anti-aliasing filters and a 9V AC-DC adapter provides power supply.

All signals are analyzed with the aid of the *Science Workshop Data Acquisition*, which allows one to evaluate angular velocity and angular position. Noise suppression is not contemplated and all signals are stored without filtering. In order to perform the analysis of the nonlinear pendulum, one considers a time series which is a sequence of angular position, θ, measured from the experiment: $S_n = \theta_n$, n = 1, 2,..., N.

Figure 4 presents two different signals obtained from the cited experimental apparatus [23]: A period-2 and a chaotic signal. The determination of delay parameters is done employing the algorithm of Hegger *et al.* [22]. Figure 5 presents results related to these determinations for the periodic signal. Time delay and embedding dimension are, respectively, $\tau = 7\Delta t$ and $D_e = 3$.

Figure 6 presents time delay parameters analysis for the chaotic signal. Now, time delay and embedding dimension are $\tau = 6\Delta t$ and $D_e = 3$.

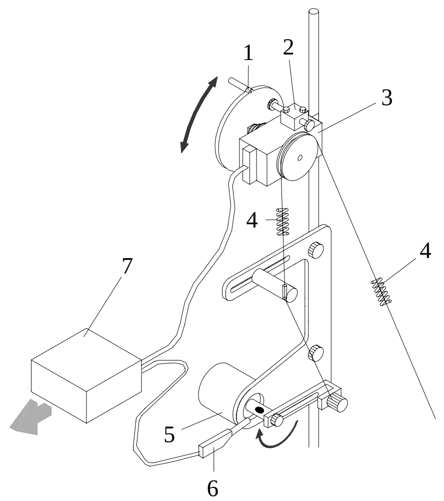
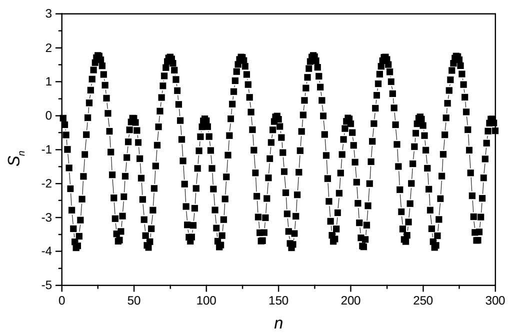


Figure 3. Experimental apparatus of the nonlinear pendulum: (1) Disc with lumped mass; (2) Magnetic damping device; (3) Rotary motion sensor: PASCO CI-6538; (4) Spring; (5) DC Motor: PASCO ME-8750; (6) Magnetic transducer: TEKTRONIX; (7) Science workshop interface: PASCO CI-6760.



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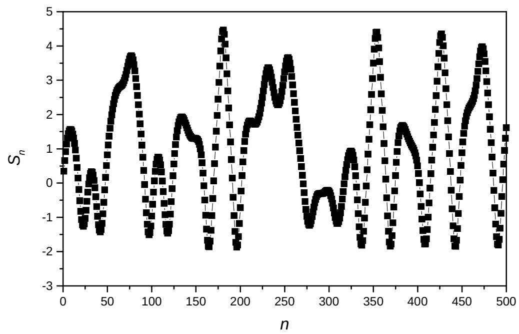
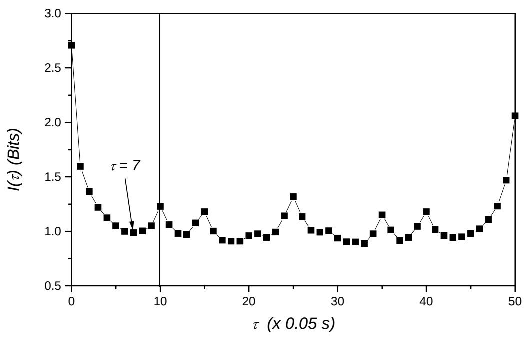


(b)

Figure 4. Experimental signals.
(a) Periodic; (b) Chaotic.



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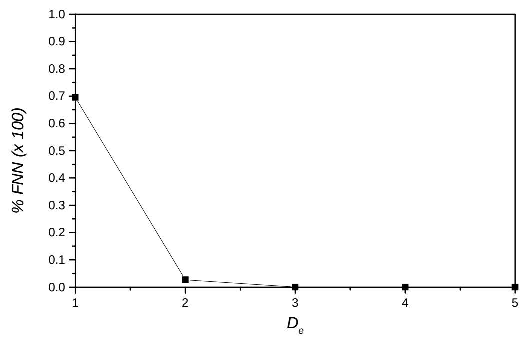
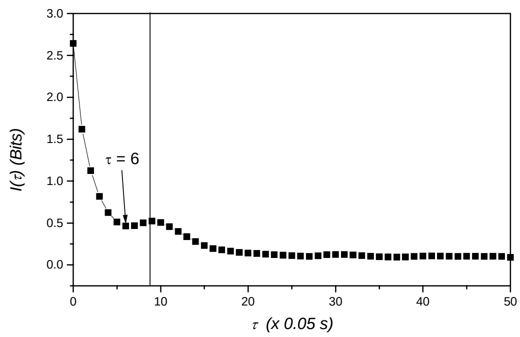


(b)

Figure 5. Delay parameters for periodic signal (N = 37090): (a) Mutual information *versus* τ; (b) False neighbors *versus* D_e.



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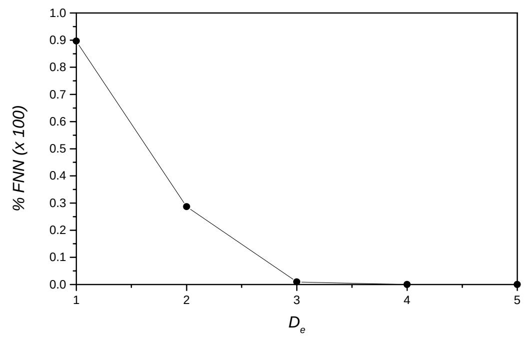


(b)

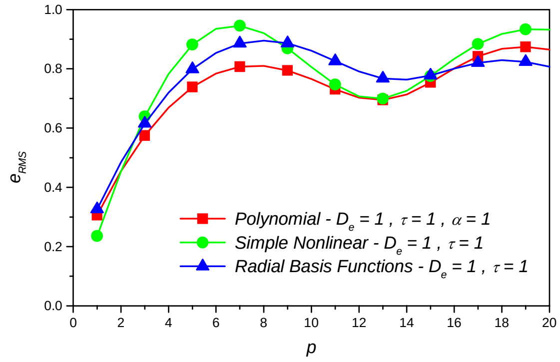
Figure 6. Delay parameters for chaotic signal (N = 29589): (a) Mutual information *versus* τ; (b) False neighbors *versus* D_e.

The following sections discuss prediction techniques applied to both periodic and chaotic experimental time series. In all Figures, symbols are used to identify techniques.

5. Prediction in Periodic Signal.

The analysis of periodic signal assumes $N = m = 37090$. Initially, a scalar prediction ($D_e = 1$ and $\tau = \Delta t$) is considered, and then these parameters are varied in order to analyze the effect of state space reconstruction in predictions. The analysis is presented contemplating two situations: short-term prediction, arbitrarily defined for $p \leq 20$, and long-term prediction, defined for $p > 20$. Furthermore, results are analyzed in terms of RMS in-sample errors.

Figure 7 shows scalar predictions for the three methods discussed here. In general, polynomial predictor presents better results in short-term. For the first point, however, simple nonlinear predictor presents the better prediction. For long-term prediction, simple nonlinear prediction presents better results, despite the irregular behavior of the response. It should be pointed out that this irregularity is related to inadequate delay parameters [22].



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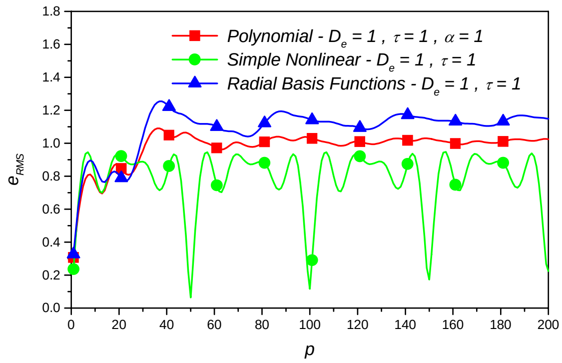
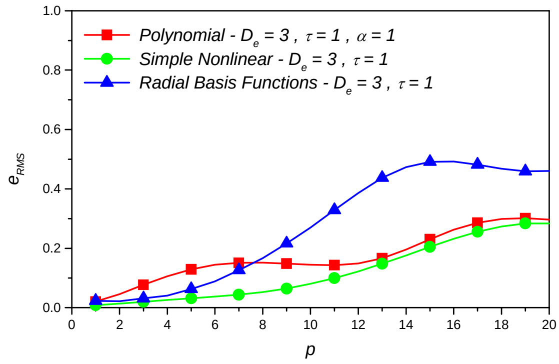


(b)

Figure 7. RMS error for periodic signal: $N = m$ =37090, D_e = 1 and $\tau = 1\Delta t$; (a) p = 1 to 20; (b) p = 1 to 200.

The forthcoming analysis considers different delay parameters, evaluating the influence of state space reconstruction. Changing only the time delay, τ, results are not altered. The next step considers the variation of embedding dimension, D_e, which performs a great influence in prediction results (Figure 8). For the first prediction, methods present similar responses. The next predictions show that simple nonlinear predictor presents better results either in short-term or long-term prediction. Notice, however, that prediction related to the simple nonlinear predictor continues to present an irregular behavior associated with inadequate delay parameters.



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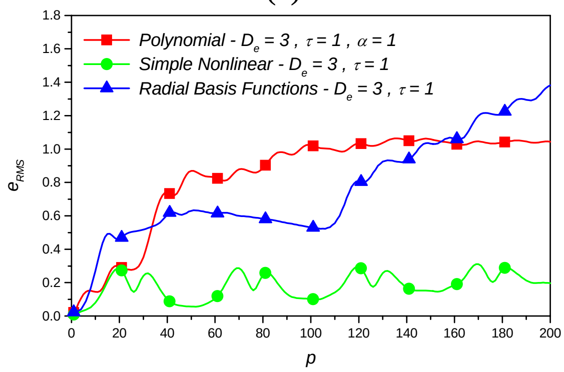


(b)

Figure 8. RMS error for the periodic signal: $N = m$ = 37090, D_e = 3 and $\tau = 1\Delta t$; (a) p = 1 to 20; (b) p = 1 to 200.

The calculated delay parameters are now considered (D_e = 3 and $\tau = 7\Delta t$). Simple nonlinear prediction presents better results and errors vary in a regular way, indicating that delay parameters are adequate. Adopting an admissible error, for example $e_{RMS} \leq 0.20$, it is possible to establish a comparison among techniques. Polynomial and radial basis functions needs to have $p \leq 4$ while simple nonlinear predictor presents acceptable errors until long-term (Figure 9).

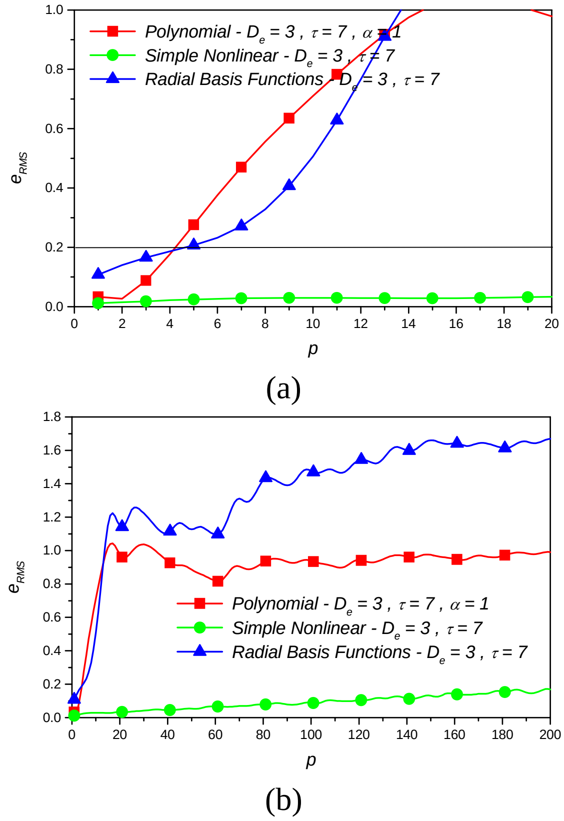


Figure 9. RMS error for periodic signal: $N = m$ = 37090, $D_e = 3$ and $\tau = 7\Delta t$;
(a) p = 1 to 20; (b) p = 1 to 200.

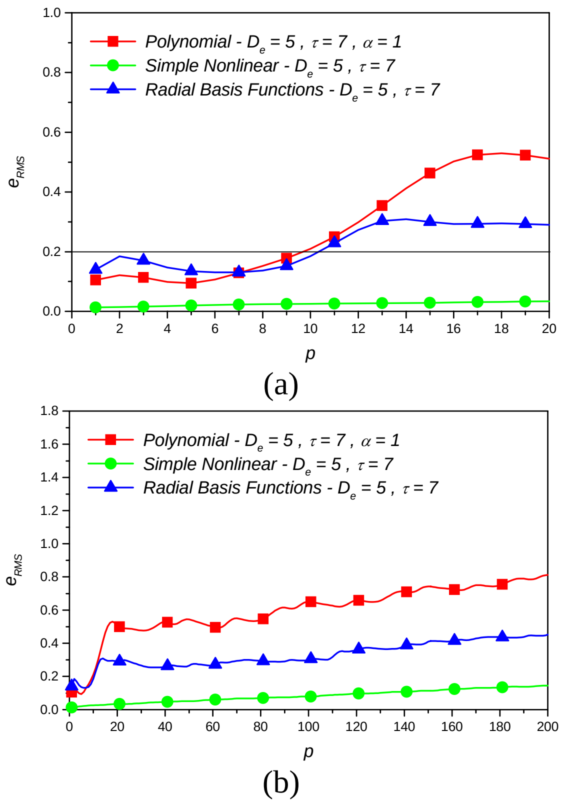


Figure 10. RMS error for periodic signal: $N = m$ = 37090, $D_e = 5$ and $\tau = 7\Delta t$; (a) p = 1 to 20; (b) p = 1 to 200.

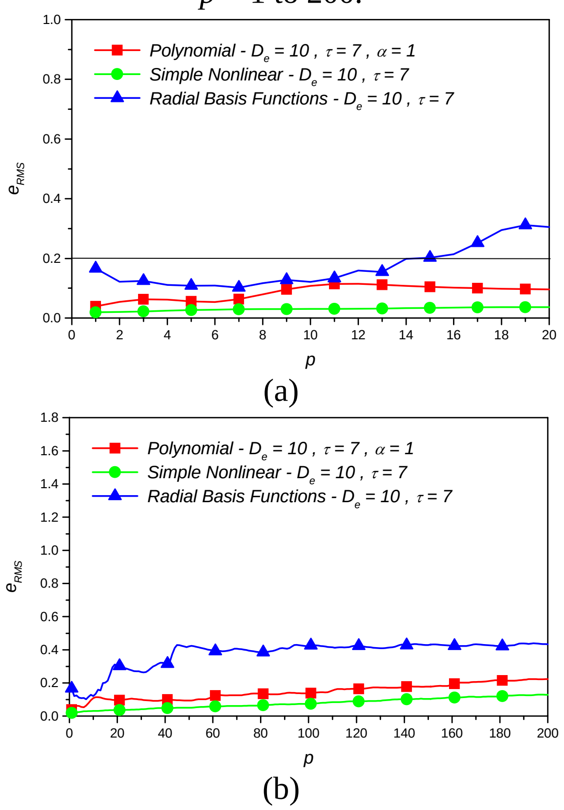
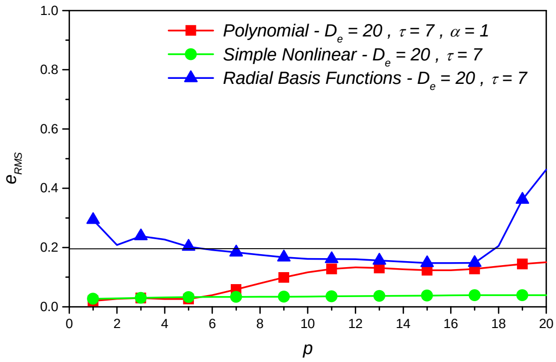


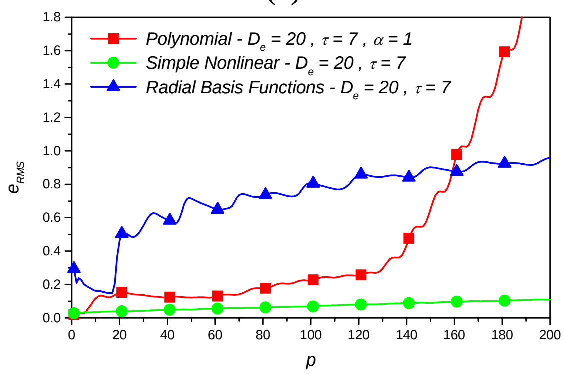
Figure 11. RMS error for periodic signal: $N = m$ =37090, $D_e = 10$ and $\tau = 7\Delta t$; (a) p = 1 to 20; (b) p = 1 to 200.

In order to analyze the influence of embedding dimension, the forthcoming analysis consider the calculated time delay ($\tau = 7\Delta t$) and different values of the embedding dimension. Observing Figures 10 and 11, it is clear that predictions obtained from polynomial and radial basis functions predictors are improved with the increase of embedding dimension. On the other hand, results from simple nonlinear predictor are not significantly altered with these variations.

Increasing a little more the embedding dimension (D_e = 20), polynomial predictor present better results in short-term prediction, however, these results tends to become unstable in long-term predictions. On the other hand, radial basis functions predictors present worse results, which are explained by the fact that the data set becomes scarce in higher dimension and hence most points in the state space have no basis function cover [17]. As in the previous examples, simple nonlinear predictor is not significantly altered with these variations (Figure 12).



(a)



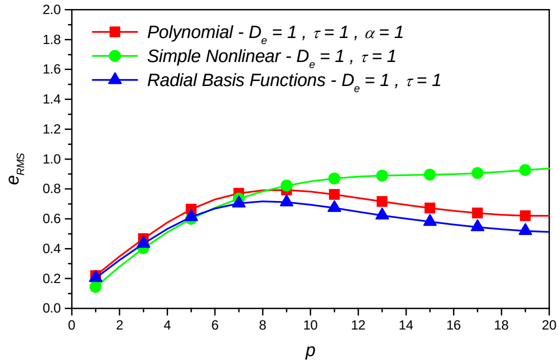
(b)

Figure 12. RMS error for periodic signal: $N = m$ =37090, D_e = 20 and $\tau = 7\Delta t$;
(a) p = 1 to 20; (b) p = 1 to 200.

6. Prediction in a Chaotic Signal.

The analysis of chaotic signal considers $N = m$ = 29589. The sequence of analysis is similar to the one presented for the periodic signal. Initially, a scalar prediction (D_e = 1 and $\tau = \Delta t$) is considered, and then these parameters are varied in order to analyze the influence of state space reconstruction in the predictions. Again, the analysis is presented in order to contemplate short-term and long-term predictions, showing the RMS in-sample errors.

Scalar predictions (D_e = 1 and $\tau = \Delta t$) show that all techniques has similar behavior for $p \leq 7$. In general, short-term predictions present less error values employing radial basis functions and polynomials predictors (Figure 13).



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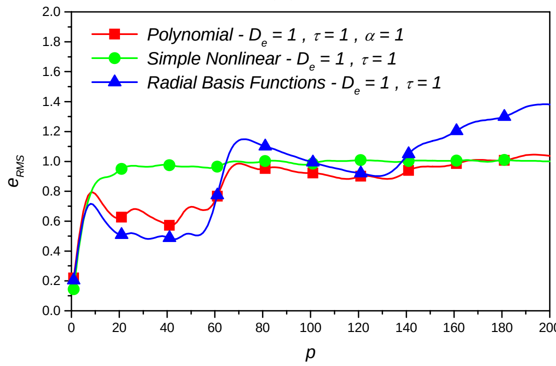
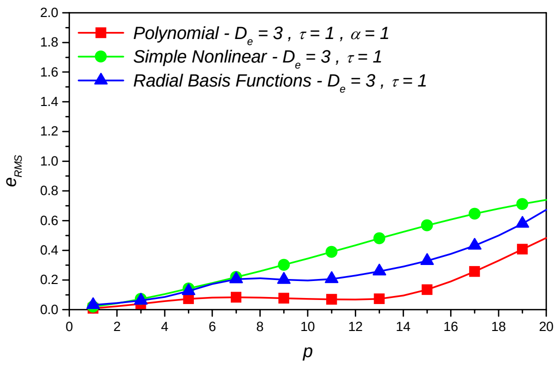


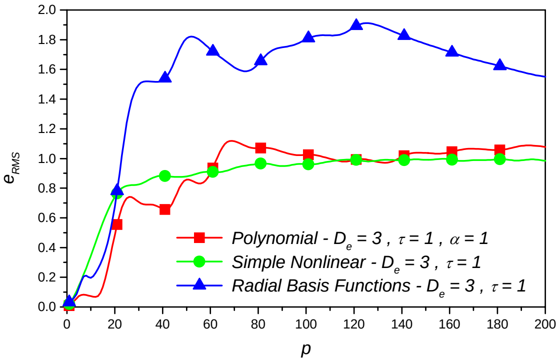
(b)

Figure 13. RMS error for chaotic signal: $N = m$ = 29589, D_e = 1 and $\tau = 1\Delta t$;
(a) p = 1 to 20; (b) p = 1 to 200.

The alteration of only time delay has small influence in predictions. Considering D_e = 3 and $\tau = \Delta t$, however, predictions present better results (Figure 14). In short-term predictions, polynomial predictor present good results when compared to the other techniques, also presenting stable results in long-term.

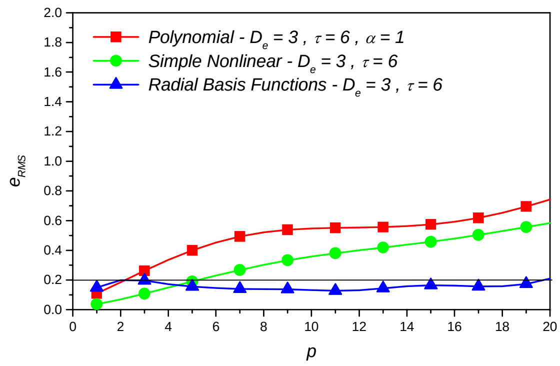


(a)

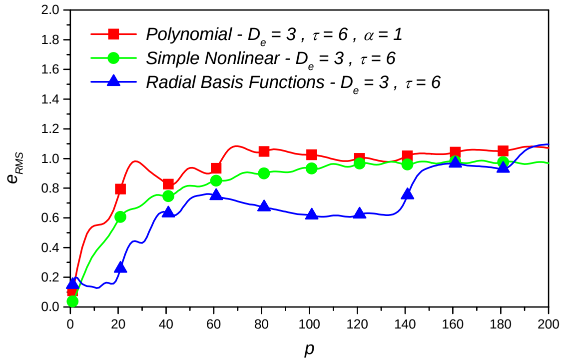


(b)

Figure 14. RMS error for chaotic signal: $N = m =$ 29589, $D_e = 3$ and $\tau = 1\Delta t$; (a) p = 1 to 20; (b) p = 1 to 200.



(a)

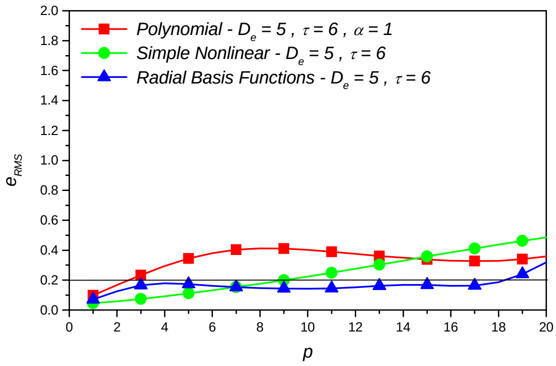


(b)

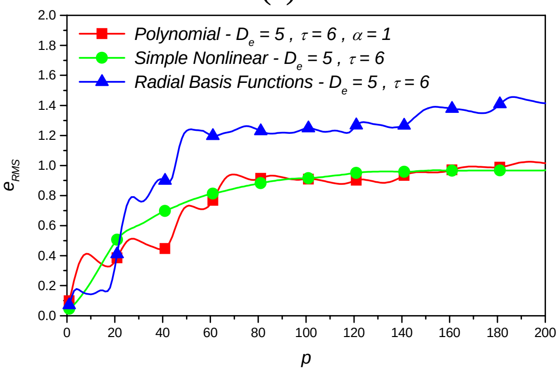
Figure 15. RMS error for chaotic signal: $N = m =$ 29589, $D_e = 3$ and $\tau = 6\Delta t$; (a) p = 1 to 20; (b) p = 1 to 200.

Calculated delay parameters are now focused (D_e = 3 and $\tau = 6\Delta t$), Figure 15. Radial basis functions predictor presents better results. The regular behavior of simple nonlinear predictor indicates that delay parameters are adequate to describe the dynamics of the system [22]. Adopting an admissible error $e_{RMS} \leq 0.20$, it is possible to see in short-term predictions that radial basis functions predictor is always inside this region while polynomial predictor needs to have $p \leq 2$ and simple nonlinear predictor, $p \leq 5$.

The forthcoming analysis considers the calculated time delay ($\tau = 6\Delta t$) altering the embedding dimension. Initially, D_e = 5 is assumed (Figure 16), and short-term predictions presents better results. Considering D_e = 10 (Figure 17), results are more or less the same for the polynomial and simple nonlinear predictors, however, radial basis functions predictor presents worse results. Assuming D_e = 20, radial basis functions predictor present even worse results. This behavior is explained because basis functions cannot capture the attractor structure in this dimension (Figure 18).

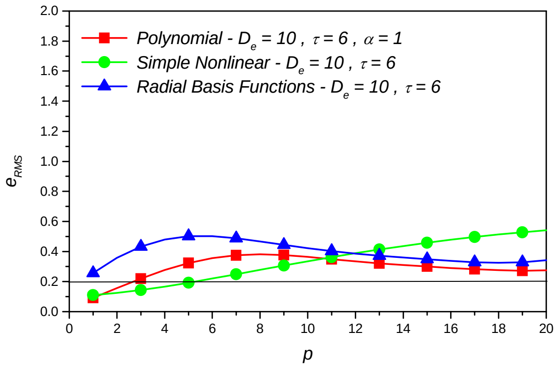


(a)

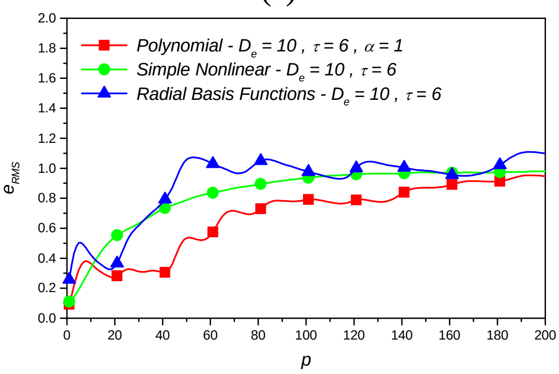


(b)

Figure 16. RMS error for chaotic signal: $N = m = 29589$, D_e = 5 and $\tau = 6\Delta t$; (a) p = 1 to 20; (b) p = 1 to 200.

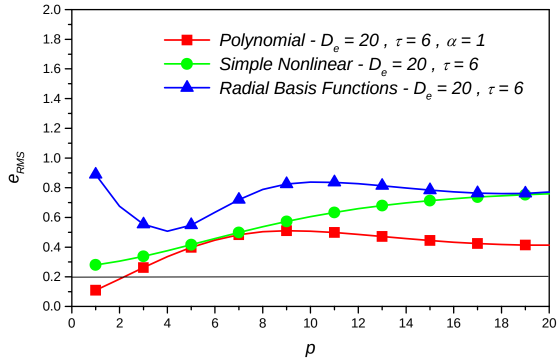


(a)

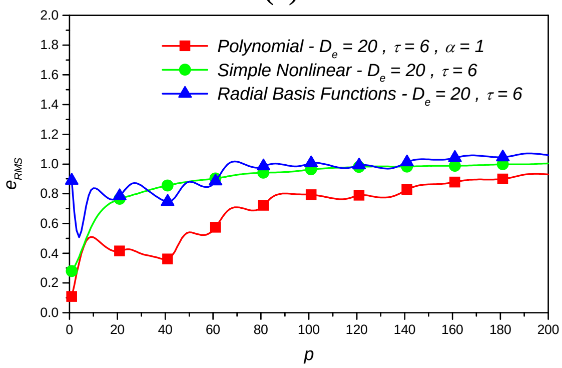


(b)

Figure 17. RMS error for chaotic signal: $N = m = 29589$, D_e = 10 and $\tau = 6\Delta t$; (a) p = 1 to 20; (b) p = 1 to 200.



(a)



(b)

Figure 18. RMS error for chaotic signal: $N = m = 29589$, D_e = 20 and $\tau = 6\Delta t$; (a) p = 1 to 20; (b) p = 1 to 200.

7. Conclusions

This article presents an overview of some prediction techniques applied to an experimental nonlinear pendulum. In particular, the analyses of simple nonlinear, polynomial and radial basis functions methods are carried out. Periodic and chaotic signals are analyzed. In general, a comparison among these techniques points that simple nonlinear predictor presents better results in periodic signal while radial basis predictor has better predictions in chaotic signals. Nevertheless, it should be pointed out that simple nonlinear predictor present stable results in any kind of signal and it is a good alternative when there is less information about the signal characteristics. State space reconstruction shows to be a very important task in predictions. The calculated delay parameters tend to be the most stable alternative to be used in state space reconstruction.

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