Nonlinear dynamics of an adaptive origami-stent system

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ABSTRACT

Origami is an ancient art of the paper folding that has been the source of inspiration in many engineering designs due to its intrinsic ability of changing shape and volume. Self-expandable devices have been created based on origami concept together with smart materials. Shape memory alloys belong to this class of materials and provide high forces and large displacements by varying their temperature. This work deals with the nonlinear dynamics of an origami-stent, a cylindrical shaped origami structure, which has the capacity of changing its radius. The actuation is provided by antagonistic torsional shape memory alloy wires placed in the origami creases. The mathematical model assumes a polynomial constitutive model to describe the shape memory alloy thermomechanical behavior. Geometric assumptions establish a one-degree of freedom model with constitutive and geometric nonlinearities. Numerical simulations are carried out considering different thermomechanical loadings that represent operational conditions. The system presents complex responses including chaos.

1. Introduction

Origami, the art of the paper folding, has been inspiring many innovative designs of engineering systems. This Japanese art creates a three-dimensional geometry by following a sequence of folds in a sheet of paper. Some of these sequences create a geometry able to contract-expand itself with an expressive change in its shape. Such behavior is of great interest to self-expanding adaptive systems.

The capacity of compacting structures, the lightness and the ability to present synchronized movements is of special interest of aerospace, robotic and medical applications [4,16]. In aerospace applications, the need and transportation restrictions motivate the use of packed configurations and, once arrived at the desired place, be deployed to the expanded configuration. Miura [15] developed a foldable solar panel based on origami concept. In medical applications, origami has inspired the design of new medical apparatus especially dedicated to minimally invasive surgery.

Shape memory alloys (SMAs) are widely employed as actuators due to their capacity of generate high forces and large displacements when subjected to thermomechanical fields that yield phase transformations. The association of SMA actuators with origami structures is promising being the focus of some research efforts [7,8,17]. Lee et al. [13] developed an origami-wheel actuated by SMA connected to a bias linear spring, being able to alter its radius in a robotic vehicle. The waterbomb pattern is employed for the origami design as employed on several applications since it allows significant shape changes [6,14]. Le et al. [12] also used a waterbomb origami pattern in an aerial drone employed to landing in water. Salerno et al. [18] presented an SMA origami device that takes advantage of geometric relations of the origami movements to increase the maneuver capacity.

Origami system is a slender structure with complex geometry that makes stability analysis an essential part of the design. Therefore, dynamical behavior of origami systems is an important issue to be investigated and only few works are available in literature. Basically, this analysis combines geometric and constitutive nonlinearities, providing a rich dynamics with operations close to the stability limits. In this regard, Fonseca et al. [5] presented a dynamical analysis of an origami-wheel with shape memory actuators. A single degree of freedom system is employed as an archetypal model of the origami structure. Results show a rich response, including chaos.

This work investigates the nonlinear dynamics of an origami-stent, a self-expandable cylindrical origami being originally presented by Kuribayashi et al. [11]. The stent is usually employed as a tubular medical device used to protect weakened arterial walls in the human body, but it can also be applied to increase stiffness of several systems including pipes and oil drilling well. Basically, origami-stent has a cylindrical shape and the folds allow the change of its radius (Fig. 1).

A shape memory alloy origami-stent system is analyzed considering torsion SMA wires (TSW) as actuators. A polynomial constitutive model describes thermomechanical behavior of SMAs. Origami geometric relations are employed to propose a one-degree of freedom (1-DOF) system.

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that represents the system dynamics. Numerical simulations are carried out showing rich responses due to situations related to different thermo-mechanical operational excitations.

After this introduction, this paper is organized as follows. Initially, a geometric analysis is performed, establishing geometric relations and the kinematics of the origami system. Dynamical model is then proposed considering a single-degree of freedom system. Afterward, numerical simulations are discussed considering different thermal and thermo-mechanical loadings. Concluding remarks are presented in the sequence.

2. Geometric analysis

The origami-stent is a self-expandable cylindrical origami constructed by following the folding pattern showed in Fig. 2-a, where the continuous lines mean mountain folds (folds outwards the paper plane) and the dashed lines mean valley folds (folds inwards the paper plane). This pattern is composed by the repetition of rectangular components as showed in Fig. 2-b, which is known as waterbomb base, shifted half an element in comparison with those on their side. The rectangular element has a length L and an angle \( a_0 \), and therefore, \( L_2 = L/\tan(a_0) \) and \( a_0 = 45^\circ \) means it is a square element. A single waterbomb base folded is presented in Fig. 2-c.

Geometric relations are now in focus and some hypotheses are established for this aim. The first one is the symmetry, assuming that all elements behave in the same way and the element itself behaves symmetrically. Under this assumption, one-quarter of the cell element is representative of the general cell behavior. The second assumption is that fold occurs only in the creases and therefore, the element facets remain straight. Under these assumptions, it is possible to analyze the whole origami from a single cell element.

The deployment of the origami goes through two stages [10]. On the first stage, the element unfolds until it reaches a flattened configuration and the nodes A, C and \( A_2 \) are co-linear. On the second stage, the crease \( AA_2 \) moves outwards in the radial direction (Fig. 3).

The cell element geometry is characterized by three angles: \( \theta \), \( \varphi \) and \( \beta \). These angles are coupled in such a way that each one can be described as a function of the others. Hence, it is possible to elect \( \theta \) as the basic angle. It is noticeable that \( \theta < 90^\circ \) characterizes the first stage while \( \theta > 90^\circ \) characterizes the second stage.

On the first stage, \( \varphi \) can be expressed by considering the positions of the points A and B on the \((x,y,z)\) system with origin on C: \( A = (L_2 \sin(\varphi), 0, L_2 \cos(\varphi)) \) and \( B = (L_2, L \sin(\theta), L \cos(\theta)) \). Since the distance AB is equal to \( L \), the following equation can be written:

\[
\cos(a_0) = \cos(a_0) \sin(\varphi) + \sin(a_0) \cos(\varphi) \cos(\theta) \tag{1}
\]

By solving Eq. (1), the following equation is obtained:

\[
\sin(\varphi) = \frac{1 - \tan^2(a_0) \cos^2(\theta)}{1 + \tan^2(a_0) \cos^2(\theta)} (\sin(\varphi) - 1) = 0 \tag{2}
\]

Based on that, there are two possible solutions for \( \varphi \). Since this angle cannot be constant on the first stage, the following equation is established to define \( \varphi \) during the first stage:

\[
\varphi = \arcsin\left(\frac{1 - \tan^2(a_0) \cos^2(\theta)}{1 + \tan^2(a_0) \cos^2(\theta)}\right) \tag{3}
\]

In order to analyze the angle \( \beta \) on the first stage, consider the triangle \( A_0B_0C \) (Fig. 4). Since \( A_0C = L \cos(\varphi) \) and \( D_0C = L \cos(\theta) \), it is possible to write \( A_0D_0 = L \cos(\varphi) - L \cos(\theta) \). On the other hand, \( AD = L \cos(\theta) \) and, since the angle \( \angle ADD_0 \varphi \), the distance \( A_0D_0 = L \cos(\theta) \sin(\varphi) \). Therefore, the following relation is written for the first stage:

\[
\beta = \arccos\frac{\cos(\varphi) \tan(a_0) - \cos(\theta)}{\sin(a_0)} \tag{4}
\]

In order to obtain a function of \( \beta \) that depends only of \( \theta \), the angle \( \varphi \) in Eq. (4) is substituted by Eq. (3) and it is found that:

\[
\beta = \arccos(\cos(\theta)) = \theta \tag{5}
\]

On the second stage, Fig. 3 shows that the angle \( \varphi \) remains constant, being equal to 90°. This can be also pointed out by considering the second term on the left side of Eq. (2). Besides, since both facets that contains the crease BC remains on the same plane, \( \beta = 180^\circ - \theta \).

An analysis of the origami-stent radii is now of concern. Let \( O \) be the central point of the origami-stent. Fig. 5 shows two radii that define origami geometry: an internal radius, \( R_i \) defined as OC; and an external radius, \( R_e \) defined as OB. The angle \( a \) defined as \( \angle A_0B_0C \) is related to the number of cell elements circumferentially distributed, \( n \), and by the hypotheses already assumed, it is equal to all elements. Therefore, this angle is such that \( n(2a) = 360^\circ \). Hence:

\[
a = 180^\circ/n \tag{6}
\]

Reminding that \( \theta \) is the angle \( \angle A_0B_0C \), it is possible to see from Fig. 5 that \( R_e \sin(a) = L \sin(\theta) \) and \( R_e \cos(a) = R_i + L \cos(\theta) \), furnishing the following relations:

\[
R_e = \frac{L \sin(\theta)}{\sin(a)} \tag{7}
\]

Fig. 2. Origami-stent folding pattern: (a) waterbomb folding pattern where continuous line represents mountains and dashed line represents valleys; (b) plane view of the single cell element; (c) three-dimensional view of the single cell element.
In summary, the origami geometry can be described with the following set of equations:

\[ \beta = \begin{cases} \theta & \text{if } \theta_{\min} \leq \theta < 90^\circ \\ 180^\circ - \theta & \text{if } 90^\circ \leq \theta \leq \theta_{\max} \end{cases} \quad (10) \]

\[ \varphi = \begin{cases} \sin \left( \frac{1 - \tan^2(\alpha_0)\cos^2(\theta)}{1 + \tan^2(\alpha_0)\cos^2(\theta)} \right) & \text{if } \theta_{\min} \leq \theta < 90^\circ \\ \frac{90^\circ}{2} & \text{if } 90^\circ \leq \theta \leq \theta_{\max} \end{cases} \quad (11) \]

Based on the geometric relations of the origami-stent, Fig. 6 illustrates the general behavior of an origami with square elements \( \left( \alpha_0 = 45^\circ \right) \). Fig. 6-a shows the evolution of the angles \( \beta \) and \( \varphi \) as a function of the angle \( \theta \). Note that at \( \theta = 90^\circ \), both angles \( (\beta \) and \( \varphi \) ) have a transition configuration between the different stages. Fig. 6-b shows internal and external radii divided by the length \( L \) for different values of \( n \) (the number of elements circumferentially distributed). Besides the limit angles, \( \theta_{\min} \) and \( \theta_{\max} \) the number \( n \) influences the radius maximum values. The maximum value of the external radius \( R_e \) occurs at \( \theta = 90^\circ \). Fig. 6-b also shows the degree of compactness of the structure. For \( n = 6 \), \( R_e \) can double its size, while for \( n = 10 \), \( R_e \) can triplicate its size. Hence, the greater is the value of elements \( n \), the more compact the origami is able to be. Nevertheless, the increase of the number of elements makes harder the origami manufacturing process.

### 2.1 Kinematic analysis

Kinematics of the origami-stent system is now treated. Due to symmetry assumptions, it is possible to analyze one-quarter of the cell element. A global coordinate frame \( F(OXYZ) \) is placed on the center axis of the origami-stent with origin on \( O \) (see Fig. 5). A local coordinate frame \( F(OCZ) \) is placed due to a translation from \( O \) to \( C \) (see Fig. 5). Hence, the coordinate frame \( F \) is translated from the coordinate frame \( F \) through \( Z \) direction of a distance equal to \( OC \), that is, \( R_e \).

Basically, kinematic analysis corresponds to two triangles which movements are geometrically coupled to each other. It is assumed that each one of these triangles rotates presenting inertia. The triangle \( \triangle BB_p \) rotates of an angle \( \theta \) while the triangle \( \triangle ABC \) rotates of an angle \( \varphi \) and then of an angle \( \beta \) (see Fig. 4). Besides that, translation motion associated with each triangle has a lumped mass placed in its centroid: \( m_1 \) on the centroid of the triangle \( \triangle ABC \) and \( m_2 \) on the centroid of the triangle \( \triangle BB_p \).
Therefore, five coordinate frames are considered: F and P; Q, R and S. The coordinate frames Q, R and S have origin on C, presenting movements related to each geometric angle, φ, β and θ, respectively (Fig. 7). The triangle ABC rotates about φ and, then, rotates about β. Q(Cx(1), y(1), z(1)) rotates about φ in the x(1)-axis direction of P while R(Cx(2), y(2), z(2)) rotates about β in the z(2)-axis direction of Q. The triangle BB_A C rotates θ. S(Cx(3), y(3), z(3)) rotates about θ on the x(3)-axis of P. Since the coordinate frame P does not rotates about the global coordinate frame F (it only translates), the rotation matrices are the following:

\[ F_T^Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix} \]

\[ Q_T^R = \begin{bmatrix} \cos(\beta) & \sin(\beta) & 0 \\ -\sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ F_T^S = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \]

The triangle inertia tensors are the following:

\[ I_{ABC} = m_{ABC} \begin{bmatrix} \frac{1}{6}L_1^2 + \frac{1}{2}L_2^2 & 0 & 0 \\ 0 & \frac{1}{2}L_1^2 & -\frac{1}{4}L_1L_2 \\ 0 & -\frac{1}{4}L_1L_2 & \frac{1}{6}L_2^2 \end{bmatrix} \]

\[ I_{BB_A C} = m_{BB_A C} \begin{bmatrix} \frac{1}{6}L_2^2 + \frac{1}{2}L_1^2 & 0 & 0 \\ 0 & \frac{1}{2}L_1^2 & -\frac{1}{4}L_1L_2 \\ 0 & -\frac{1}{4}L_1L_2 & \frac{1}{6}L_2^2 \end{bmatrix} \]

After these definitions, it is defined triangle angular velocities with respect to the global frame:

\[ \omega_{ABC} = \begin{bmatrix} \dot{\phi} \\ \frac{\dot{\sin(\phi)} \dot{\beta}}{\cos(\phi)} \end{bmatrix} \]
Fig. 8. Kinetic energy of origami-stent square element with unitary length and unitary masses and six elements (n = 6).

\[ \mathbf{F}_{\omega}_{BB_{2}C} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]  
(20)

The lumped mass linear velocities \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) are given by the position time derivative, reminding that all angles are described as a function of \( \theta \).

\[
F_{V_1} = \frac{d}{dt} \left[ \mathbf{F} \mathbf{T}^O \mathbf{T}^R \mathbf{r}_{1} \right] = \\
= \begin{bmatrix}
\frac{1}{2} L_1 \cos(\beta) \theta \\
-\frac{1}{2} L_1 \sin(\beta) \cos(\phi) \theta - \left( \frac{1}{2} L_1 \cos(\beta) \sin(\phi) - \frac{1}{2} L_1 \cos(\phi) \right) \phi \\
\frac{1}{2} L_1 \sin(\beta) \sin(\phi) \theta - \left( \frac{1}{2} L_1 \cos(\beta) \cos(\phi) + \frac{1}{2} L_2 \sin(\phi) \right) \phi + R_L
\end{bmatrix}
\]  
(21)

\[
F_{V_2} = \frac{d}{dt} \left[ \mathbf{F} \mathbf{T}^S \mathbf{r}_{2} \right] = \\
= \begin{bmatrix}
\frac{1}{2} L_1 \cos(\theta) \theta \\
-\frac{1}{2} L_1 \sin(\theta) \theta + R_L
\end{bmatrix}
\]  
(22)

3. Dynamical model

The dynamical analysis of the origami-stent system assumes kinematic relations in order to write an equivalent 1-DOF system to represent the origami dynamics. Equations of motion are formulated by considering energetic approach, electing angle \( \theta \) as a generalized coordinate. Using the transformation matrices to describe all terms in the system \( F \), the kinetic energy \( E \) is given by:

\[
E = \frac{1}{2} m_1 \mathbf{v}_1 \cdot \mathbf{v}_1 + \frac{1}{2} m_2 \mathbf{v}_2 \cdot \mathbf{v}_2 + \frac{1}{2} (\mathbf{\omega}_{ABC})^T \mathbf{F} I_{ABC} \mathbf{\omega}_{ABC} + \\
\frac{1}{2} (\mathbf{\omega}_{BB_{2}C})^T \mathbf{F} I_{BB_{2}C} \mathbf{\omega}_{BB_{2}C} 
\]  
(23)

Fig. 8 shows the kinetic energy \( E \) as a function of \( \theta \) and its time derivative, \( \dot{\theta} \), for a square element, considering \( n = 6 \), unitary length \( L \) and unitary masses. Fig. 9 shows the projection of this energy for different values of \( \eta \), in the left as a function of \( \omega = \theta \) and in the right as a function of \( \theta \) for a unitary velocity \( \dot{\theta} = 1/s \). It should be pointed out that the energy is a nonsmooth function at \( \theta = 90^\circ \) due to changes of the angles \( \beta \) and \( \phi \) (see Fig. 6).

The actuation of the origami-stent is provided by a Torsion SMA Wire (TSW) with length \( L_2 \) and radius \( r_S \) \([9]\). Two TSWs are antagonistically placed on each side of a cell element on the crease AG (Fig. 10) and these actuators promote the origami configuration changes (opening and closing).

The SMA thermomechanical behavior is described by a polynomial constitutive model \([3,19]\) that establishes a stress-strain-temperature \( \sigma = \gamma + T \) relation. This constitutive model considers that the austenitic phase is stable at high temperatures, and two variants of the martensitic phase, induced by positive and negative stress fields, stable at low temperatures.

\[
\sigma = a_1(T - T_M) + a_2 \gamma^3 + a_3 \gamma^5 
\]  
(24)

where \( a_1, a_2 \) and \( a_3 \) are material parameters and \( T_M \) is the temperature below which the martensitic phase is stable. Considering \( T_A \) as the temperature above which the austenitic phase is stable, it is possible to write the following relationship: \( a_1 = \frac{1}{4} \frac{\alpha^2}{\kappa_T(T_A - T_M)} \).

Aguir et al. \([1]\) and Enemark et al. \([2]\) showed that helical spring force-displacement-temperature curve is similar to the stress-strain-temperature curve assuming homogeneous phase transformation through the spring element. Under this assumption, it is possible to use the same arguments to define an torsional actuator torque-displacement-temperature relation that is formally similar to the stress-strain-temperature:

\[
M = \frac{F}{r_L^2} \left[ a_1(T - T_M) \left( \frac{2 L_s}{L_s} \theta - 2 \frac{r_s}{L_s} \theta \right) \right. \\
- a_2 \left( \frac{r_s}{L_s} \theta - 2 \frac{r_s}{L_s} \theta \right)^3 + a_3 \left( \frac{L_s}{L_s} \theta - 2 \frac{r_s}{L_s} \theta \right)^3 \right] 
\]  
(25)

where \( \theta \) is the angle where the TSW is free of stress. Note that this expression represents constitutive relation of both actuators, being expressed by \( M_{TSW_1} \) and \( M_{TSW_2} \).

On the origami-stent system, the generalized forces \( Q_\alpha \) are related to the TSW generalized force \( M_{SMA} \), which due to symmetric considerations is given by: \( M_{SMA} = (M_{TSW_1} + M_{TSW_2})/2 \). An external generalized force \( Q_{ext} \) is applied at the point \( C \), z-direction. Hence, the expression of the virtual work \( \delta W \), neglecting gravitational effect, is given by:

\[
\delta W = Q_\theta \delta \theta = -M_{SMA} \delta \theta + Q_{ext} \delta R_L 
\]  
(26)

Note that \( R_L = R_L(\theta) \), \( \delta R_L = \partial R_L / \partial \theta \delta \theta \), and therefore, \( Q_\theta = -M_{SMA} \delta \theta + Q_{ext} \delta R_L \). Two extra generalized forces are considered in order to have a complete description of the origami-stent system. The first one is related to a linear viscous dissipation of coefficient \( \zeta \) representing all dissipation processes, including the hysteresis phenomenon: \( \delta W_\zeta = \zeta \delta \theta \delta \theta \).

The second one is related to origami geometric restrictions that limit \( \theta \) to vary in the range \( [\theta_{min}, \theta_{max}] \) Eqs. (8) and (9), expressed as a moment of the form:

\[
M_{lim}(\theta) = \begin{cases} 
\tilde{k}_{lim}(\theta - \theta_{min}) & \text{if } \theta < \theta_{min} \\
0 & \text{if } \theta_{min} \leq \theta \leq \theta_{max} \\
\tilde{k}_{lim}(\theta - \theta_{max}) & \text{if } \theta > \theta_{max}
\end{cases} 
\]  
(27)

where \( \tilde{k}_{lim} \) limits the system response and represents an equivalent elastic spring stiffness.

Equation of motion is then obtained from Lagrange’s equation.

\[
\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{\theta}} \right) - \left( \frac{\partial E}{\partial \theta} \right) = \delta Q_\theta = -\zeta \dot{\theta} - M_{SMA}(\theta) - M_{lim}(\theta) + Q_{ext} \frac{\partial R_L}{\partial \theta} 
\]  
(28)

The kinetic energy \( E \) is a nonlinear expression in \( \theta \) multiplied by the velocity and can be written as \( E = g(\theta)\dot{\theta} \). Details about this function are presented in Appendix. Substituting this expression in the left hand side of Lagrange’s Eq. (28), it gives a term \( g_1(\theta) \dot{\theta} \) and a term \( g_2(\theta) \dot{\theta}^2 \) (see Appendix for more details). Therefore, the equation of motion is written as follows:

\[
g_1(\theta) \dot{\theta} + g_2(\theta) \dot{\theta}^2 = -\zeta \dot{\theta} - M_{SMA}(\theta) - M_{lim}(\theta) + Q_{ext} \frac{\partial R_L}{\partial \theta} 
\]  
(29)

In order to write dimensionless equations, it is defined a dimensionless time \( \tau = o_{REF} t \), where \( o_{REF} \) is a reference frequency, defined from the linear part of the restitution force:

\[
o_{REF} = \sqrt{\frac{J_1 T_M}{m_1 L_s^2}} 
\]  
(30)
Hence, after dividing Eq. (29) by $m_\text{l}L^2\omega_{\text{REF}}^2$, the following set of equations of motion is written:

$$
\begin{align*}
\theta' &= \nu \\
\theta'' &= -f_1(\theta)\nu^2 - f_2(\theta)\xi \nu - f_3(\theta)H_M(\theta) + f_5(\theta)\left(\frac{1}{L^2} \right)D_{\text{ext}}
\end{align*}
$$

(31)

where $\nu = \partial y/\partial \tau$, and therefore $\theta = \theta'\omega_{\text{REF}}$ and $\theta = \theta''\omega_{\text{REF}}^2$. Besides, the following dimensionless terms are employed:

$$
\begin{align*}
f_1(\theta) &= \frac{\xi f_2(\theta)}{\xi_L} \\
f_2(\theta) &= \frac{m_\text{l}L^2}{\xi_L} \\
H_M &= \frac{M_{\text{SM}}(\theta) + M_{\text{IOH}}(\theta)}{m_\text{l}L^2\omega_{\text{REF}}^2} \\
D_{\text{ext}} &= \frac{Q_{\text{ext}}}{m_\text{l}L^2\omega_{\text{REF}}} \\
\xi &= \frac{\xi}{m_\text{l}L^2\omega_{\text{REF}}}
\end{align*}
$$

(32) \quad (33) \quad (34) \quad (35) \quad (36)

Fig. 11 presents dimensionless functions $f_1(\theta)$ and $f_2(\theta)$ for a square element and equal masses. The discontinuity of the function $f_1(\theta)$ at $\theta = 90^\circ$ is due to the nonsmoothness of the kinetic energy. These functions indicate the origami-stent nonlinearities.

4. Numerical simulations

Dynamical behavior of the origami-stent system actuated by two antagonistic TSWs is now analyzed. Numerical simulations are carried out by employing the fourth-order Runge–Kutta method. Different thermal and thermomechanical loads are contemplated to study the system behavior under distinct operational conditions.

Table 1 presents the origami-stent parameters employed in all simulations considering a square element. Besides, a dissipation coefficient $\xi = 0.3$ is employed. Under these conditions, the reference frequency is $\omega_{\text{REF}} = 62.09$ rad/s. Temperature $T_s$ represents a reference temperature where the origami-stent actuator does not apply any moment at a chosen angle of $\theta = 30^\circ$ (closed configuration). Thereby, one actuator has $\theta_1 = 30^\circ$ The other one has $\theta_1 = 90^\circ$ since this assures that the origami can reach the opened configuration of maximum radius ($\theta = 90^\circ$) by changing the TSW temperature. This set of parameters can represent a human body stent. Note that the length $L$ of the cell element is 6.6 mm. Hence, for $n = 6$, the external radius $R_e = 13.2$ mm for $\theta = 90^\circ$ (Eq. (6)), which is close to the usual radius of aortal stent grafts.

Fig. 12 presents the torque of the TSWs as a function of $\theta$ at the temperature $T_s$ and at $T = 310K$ ($T > T_s$). TSW-1 is located at $\theta_1 = 90^\circ$ with a temperature $T_1$ while TSW-2 is at $\theta_1 = 30^\circ$ with a temperature $T_2$. 

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Table 1
Origami-stent parameters.

<table>
<thead>
<tr>
<th>$m_i = m_s$ (kg)</th>
<th>$m_{abc} = m_{abc}$ (kg)</th>
<th>$L$ (m)</th>
<th>$r_i$ (m)</th>
<th>$L_a$ (m)</th>
<th>$k_{ex}$ (N m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^{-5}$</td>
<td>$5 \times 10^{-3}$</td>
<td>$6.6 \times 10^{-3}$</td>
<td>$8.0 \times 10^{-5}$</td>
<td>$2.2 \times 10^{-5}$</td>
<td>$1.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>$a_1$ (MPa/K)</td>
<td>$a_2$ (MPa)</td>
<td>$a_3$ (MPa)</td>
<td>$T_a$ (K)</td>
<td>$T_a$ (K)</td>
<td>$T_a$ (K)</td>
</tr>
<tr>
<td>$1.0 \times 10^6$</td>
<td>$1.4 \times 101$</td>
<td>$2.2 \times 101$</td>
<td>$287.15$</td>
<td>$309.1$</td>
<td>$293.15$</td>
</tr>
</tbody>
</table>

Fig. 12. Torque of the TSWs at different temperatures: (a) reference temperature $T_0$; (b) $T = 310$ K.

Fig. 13. Actuator temperature influence on the generalized forces: (a) TSW-1 and TSW-2 at reference temperature with three vanishing points; (b) TSW-1 at high temperature with one vanishing point at 30°; (c) TSW-2 at high temperature with one vanishing point at 90°; (d) TSW-1 and TSW-2 at high temperature with three vanishing points.

Concerning external excitation, a harmonic external force is considered of amplitude $\mu$ and frequency $\Omega$.

$$D_{ext} = \mu \sin(\Omega t)$$  \hfill (37)

In addition, temperature has fluctuations represented by:

$$T = T_i + \mu_T \sin(\Omega_T \tau)$$  \hfill (38)

where $T_i$ is the temperature around which the temperature $T$ oscillates.

4.1. Thermal loadings

The actuator temperature influence on the system dynamical response is investigated considering different thermal loadings. An overview of the origami-stent equilibrium configurations can be provided by the analysis of the moments represented by $H_M$ (Eq. (34)). Fig. 13 presents these moments for different pairs of temperatures and $n = 6$. In the reference configuration (Fig. 13-a), $T_1 = T_2 = T_0$, the sum of the moments vanishes at three angles: 30°, 60° and 90°. If the temperature $T_1$ is greater than $T_0$ ($T_1 = 310$ K) while $T_2 = T_0$, only 30° is an equilibrium point (Fig. 13-b). On the other hand, if the temperature $T_2$ is greater than $T_0$ ($T_1 = 310$ K) while $T_1 = T_0$, only 90° is an equilibrium point (Fig. 13-c). If the temperatures of both TSWs are greater than $T_0$ ($T_1 = T_2 = 310$ K), there are again three points where the sum of the moments is zero (Fig. 13-d). Therefore, by controlling the temperature one can establish the configuration (opened or closed) of the origami-stent, defining the equilibrium point structure.

Free vibration analysis is now of concern identifying the temperature influence. Figs. 14 and 15 present the dynamical behavior of the system for different initial conditions and temperatures. The temperature of one actuator is modified while the other remains constant at the reference temperature. Fig. 14 presents the phase space at different temperatures for the TSW-1 while TSW-2 temperature is $T_0$. The sum of the moments points up to three equilibrium points at $T_1 = T_0$, and the phase space...
Fig. 14. Origami free vibration response at different temperatures with 6 elements (n = 6). (a) Sum of the moments at T₂ = T₀ and different values of T₁. (b) Phase space at T₁ = T₀. (c) Phase space at T₁ = 306 K. (d) Phase space at T₁ = 310 K.

Fig. 15. Origami free vibration response at different temperatures with 6 elements (n = 6). (a) Sum of the moments at T₁ = T₀ and different values of T₂. (b) Phase space at T₂ = T₀. (c) Phase space at T₂ = 306 K. (d) Phase space at T₂ = 310 K.

shows that two of them are stable while the other one is unstable. By increasing the temperature T₁ (T₁ = 306 K), 30° remains a stable equilibrium point while the other stable equilibrium point is, now, smaller than 90°. When T₁ > T_A (T₁ = 310 K), only 30° is an equilibrium point, meaning that the structure admits only the closed configuration.

Fig. 15 presents the phase space at different temperatures for the TSW-2 while the temperature of the TSW-1 is T₀. By increasing the temperature T₂ (T₂ = 306 K), 90° remains a stable equilibrium point while the other stable equilibrium point is now, greater than 30°. When T₂ > T_A (T₂ = 310 K), only 90° is an equilibrium point, and the structure only admits an opened configuration. It is important to highlight that the origami-stent is initially in a reference configuration where the structure is closed (θ = 30°). The opened configuration can be reached either by temperature variation or by imposing proper initial conditions, as one can see in Fig. 15-a (or Fig. 14-a). This reveals an initial condition dependence that is now investigated using different thermal loads.

From the reference condition, when temperature T₂ increases up to 310 K, the origami-stent opens (Fig. 16). System response presents an oscillation before reaching θ = 90°. If the temperature T₂ decreases to T₀ again, the origami-stent remains in the opened configuration. By in-
increasing temperature $T_1$, 90° is not an equilibrium point anymore and the origami-stent closes. By decreasing $T_1$ to $T_0$, the structure remains closed.

A different thermal loading is now investigated. By increasing $T_2$ up to 310 K and subsequently increasing $T_1$ up to 310 K (Fig. 17), the structure remains opened but not completely, since the equilibrium point is around 82°. By decreasing the temperature $T_2$, the origami-stent closes and by decreasing $T_1$ there is no more changes.

In contrast, if $T_1$ is first increased (Fig. 18), there is no structure change. By increasing $T_2$ subsequently, there is a small increase of the angle $\theta$ and it stabilizes around 38°. If $T_1$ decreases, the stent-origami opens and, subsequently, if $T_2$ decreases, it remains opened.

Equilibrium configuration small increase can be obtained increasing both temperatures $T_1$ and $T_2$ simultaneously as presented in Fig. 19. Note that the angle $\theta$ is changed to 38°.

Until now, all simulations consider the same number of elements, $n = 6$. The change of the number of elements can influence the dynamical response, which is now investigated. The first point that should be observed is that when $n$ changes, the limit angles change; both $\theta_{\text{min}}$ and $\theta_{\text{max}}$ tend to reduce for greater values of $n$. For $n = 6$, $\theta_{\text{min}} = 30°$ and $\theta_{\text{max}} = 105°$. For $n > 6$, $\theta_{\text{min}} < 30°$ and the origami is not completely closed at the reference condition and $\theta_{\text{max}} < 105°$. Besides that, Fig. 9 shows that the amount of kinetic energy is larger for greatest values of $n$. Fig. 20 presents the origami response when temperature $T_2$ increases from the reference condition causing the opening of the origami. Different values of $n$ are treated. Although the actuation response occurs at the same time, the stabilization takes more time to occur for origami with larger number of elements due to the greater amount of kinetic energy.

Since the origami system has strong temperature dependent behavior, it is important to evaluate the effect of thermal oscillations on system response. Figs. 21–23 present the influence of thermal oscillations on temperatures $T_1$ and $T_2$ considering $n = 6$. Fig. 21 presents the opening process changing temperature $T_2$. Afterward, both temperatures oscillate at the opened configuration. A variation of temperature $T_2$ does not alter the response, but a variation of the temperature $T_1$ causes small oscillations. The TSW-2 is at a high temperature and 90° is a free-stress point (see Fig. 12) that does not change with the thermal variation. Based on that, the sum of system moments is not altered and, therefore, its equilibrium point does not change. However, the TSW-1 is at the ref-

Fig. 16. Opening-closing process induced by temperature variations ($n = 6$): (a) thermal load; (b) time history of the deployment angle $\theta$.

Fig. 17. Opening-closing process induced by temperature variations ($n = 6$): (a) thermal load; (b) time history of the deployment angle $\theta$.

Fig. 18. Opening process induced by temperature variations ($n = 6$): (a) thermal load; (b) time history of the deployment angle $\theta$. 

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Fig. 19. Opening process induced by temperature variations \( n = 6 \): (a) thermal load; (b) time history of the deployment angle \( \theta \).

Fig. 20. Comparison of the opening process considering different values of the number of elements, \( n \). (a) thermal load; time history of the deployment angle \( \theta \) for: (b) \( n = 6 \); (c) \( n = 8 \); (d) \( n = 10 \).

Fig. 21. Opening process with thermal oscillation \( n = 6 \): (a) thermal load \( (\mu_T = 2.0 \text{ K and } \Omega_T = 0.1) \); (b) time history of the deployment angle \( \theta \).

Temperature and thermal fluctuation alters the free-stress points, except the one at 30°. Hence, the sum of the moments is altered and its equilibrium point oscillates while the temperature \( T_1 \) oscillates. Fig. 22 shows a similar opening condition but with a situation where both \( T_1 \) and \( T_2 \) are increased. Under this assumption, a variation of temperature \( T_2 \) also induces oscillations and the origami response is a combination of both influences. Since both actuators have \( T > T_A \), a variation of any actuator temperature is responsible to alter the sum of the moments of the system and, as a consequence, there is an oscillatory response. Besides, frequency thermal frequency phase is analyzed. It is noticeable that when actuator thermal variation is out of phase, the oscillatory response has higher amplitude.
Fig. 22. Opening process with thermal oscillation \( n = 6 \): (a) and (c) thermal load \((\mu_T = 2.0 \text{ K and } \Omega_T = 0.1)\); (b) and (d) time history of the deployment angle \( \theta \).

Fig. 23. Closed configuration with thermal oscillation \( n = 6 \). (a) Thermal load \((\mu_T = 2.0 \text{ K and } \Omega_T = 0.1)\); (b) time history of the deployment angle \( \theta \).

Fig. 23 presents a situation where both actuators have \( T < T_A \), characterizing a closed configuration. Under this condition, only the variation of \( T_2 \) affects the dynamic response. In contrast of the case of Fig. 21, small fluctuation of \( T_1 \) does not alter actuator dynamical response since \( 30^\circ \) is an actuator free-stress point (see Fig. 12) that does not change with a thermal variation. On the other hand, TSW-2 has \( 90^\circ \) as a free-stress point that does not change, while the other points change with the temperature and, hence, thermal fluctuation changes the sum of the moments, affecting the system equilibrium point.

The influence of the number of elements in a case with thermal fluctuation is presented in Fig. 24. Note that the increase of the number of elements, \( n \), tends to cause a greater change on system response.

4.2. Thermomechanical loadings

Thermomechanical loadings are now addressed in order to simulate some operational conditions of the origami-stent. Basically, opened configuration is of concern considering \( T_1 = T_0 \) and \( T_2 = 310 \text{ K} \). Under this condition, only \( 90^\circ \) is an equilibrium point and an external force is applied. In order to establish a physical interpretation, the origami-stent system is assumed to represent human body stent. Under this assumption, the magnitude of the external force is compatible to the human body blood pressure. Hence, the dimensionless amplitude \( \mu \) of the external force \( D_{\text{ext}} \) is in a range from 0.01 to 0.85, which is related to force of magnitude \( 10^{-3} \text{ N} \).

Initially, a global analysis is performed considering bifurcation diagrams that present a stroboscopic view of system dynamics under the slow quasi-static variation of parameter \( \mu \) for \( \Omega = 0.30 \) and different number of elements: \( n = 6; n = 8; n = 10 \) (Fig. 25). Fig. 25-a considers \( n = 6 \). Note that, for small force amplitudes, the system presents period-1 response until \( \mu = 0.143 \). After that, a cloud of points suggests a chaotic response. By increasing the value of \( \mu \), the diagram shows other regions where the system response has a periodicity greater than 1. When \( \mu = 0.156 \), bifurcations start from chaotic-like response to period-1 response. A period doubling can be noticed at \( \mu = 0.260 \), nevertheless a period-1 response is predominant until \( \mu = 0.628 \), where the system response becomes chaotic. Inside this chaotic region, there are some periodic windows, especially between \( \mu = 0.740 \) and \( \mu = 0.781 \).

By considering different number of elements, \( n = 8 \) (Fig. 25-b) and \( n = 10 \) (Fig. 25-c), it is noticeable that the increase of the number of elements tends to stretch the diagram and spread the kind of behavior over the values of \( \mu \). Note, for instance, that period-1 response for small
Fig. 24. Comparison of the opening process with thermal oscillation considering different values of the number of elements, n. (a) Thermal load; time history of the deployment angle $\theta$ for: (b) $n = 6$; (c) $n = 8$; (d) $n = 10$.

Fig. 25. Bifurcation diagrams varying excitation amplitude $\mu$ for different number of elements and a fixed frequency ($\Omega = 0.30$): (a) $n = 6$; (b) $n = 8$; (c) $n = 10$. 
amplitudes of the force finishes at $\mu = 0.143$ when $n = 6$ but at $\mu = 0.157$ when $n = 8$ and at $\mu = 0.308$ when $n = 10$. After the region of period-1 response, different bifurcation evolutions occur for each cell number. When $n = 6$, the cloud of points occurred immediately after the period-1 response. When $n = 8$, a period doubling takes place until the system response is quasi-periodic and then, the cloud of points takes place. When $n = 10$, a period doubling occurs then the cloud of points takes place. Subsequently, a periodic response window is noticed until there is a response sudden change suggesting a chaotic response. Afterward, a bifurcation from chaos to period-1 occurs.

Phase space and its respective Poincaré section are presented in Fig. 26 for different amplitudes of the external force taken from the bifurcation diagram of $n = 6$. In all these cases, the external force is great enough to push the origami to its physical limits, which is indicated by the straight lines at $30^\circ$ and $105^\circ$. For $\mu = 0.145$, a point inside the cloud of points region, a chaotic response occurs. By increasing values of $\mu$,
period-1 and period-2 responses are achieved for $\mu = 0.20$ and $\mu = 0.36$, respectively. A chaotic response is again obtained for $\mu = 0.70$.

The same results of Fig. 26, that considers $n = 6$, is now presented for $n = 8$, Fig. 27, and $n = 10$, Fig. 28. When $\mu = 0.145$ (Figs. 27-a and 28-a), the system oscillates around 90° with a period-1 response, without reaching the physical limits of the structure. When $\mu = 0.20$, there is a quasi-periodic response for $n = 8$ and, even if the system still oscillates around 90°, the upper physical limit is reached. In contrast, for $n = 10$, this increase of $\mu$ does not affect the system response substantially. On the other hand, when $\mu = 0.36$, the system response is periodic for $n = 8$, but chaotic for $n = 10$. When $\mu = 0.70$, a period-1 response is noticed.
Resonant condition is usually critical to system response. Concerning chaotic behavior, they can be induced more effectively close to this condition. In this regard, Fig. 29 presents the maximum amplitude of displacement under the variation of frequency $\Omega$. For $n=6$ (Fig. 29a), it presents a typical resonant curve with maximum amplitude at $\Omega = 0.159$. It is also noticeable a small peak at $\Omega = 0.085$, related to constitutive nonlinearities. For $n=8$ and $n=10$, the peaks of maximum angle is shifted to the left and the flat region related to the maximum values is due to the maximum displacement constraints imposed by the origami physical limits. Besides, the smaller peak is more pronounceable. Furthermore, Fig. 29-d presents a zoom in Fig. 29-c where dynamical jumps for different frequencies are observed.

Fig. 30 shows the bifurcation diagram for $n=6$ varying $\mu$ under resonant condition ($\Omega = 0.16$). Note that system response is predominantly chaotic indicating the influence of the resonant condition on system dynamics. Phase space and Poincaré section is presented in Fig. 31 for $\Omega = 0.16$ and $\mu = 0.20$. A chaotic-like response is observed presenting a strange attractor in Poincaré section.

5. Conclusions

This paper deals with the nonlinear dynamical analysis of an origami-stent actuated by shape memory alloys. A polynomial constitutive model is employed to describe the thermomechanical behavior of SMA actuators. Geometrical relations allow one to build a single degree of freedom system to analyze the system dynamics. This system has both constitutive and geometrical nonlinearities, presenting non-smooth characteristics. Numerical simulations are carried out showing different thermomechanical loading situations representing operational conditions. Concerning thermal loadings, it is possible to identify that different configurations can be reached by changing the temperature of each actuator. Results are affected by the order of actuation. Moreover, thermal fluctuations can affect the system response depending on conditions. It depends on either the origami configuration or the actuator temperature. The number of cell elements interferes on the response stabilization time as well. An origami with more cell elements takes more time to stabilize, being more affected by thermal fluctuations. Thermomechanical loadings can produce complex responses including chaos. The number of cell elements is critical in order to define the system response. In general, the increase of the number of cell elements reduces chaotic regions. Furthermore, greater values of external force are needed to push the origami to reach its physical limits. Dynamical jumps are also observed on origami-stent dynamics.

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Appendix

Origami-stent equations of motion are related to complex functions that express geometric relations. Kinetic energy definition (Eq. (23)) is described on the global coordinate frame $F$, using transformation matrix. Therefore, consider that kinetic energy, Eq. (22), can be written as follows:

$$E = g(\theta)\dot{\theta}^2$$

Applying this equation in the left hand side of the Lagrange’s equation (Eq. (28)):

$$\frac{\partial}{\partial \theta} \left( \frac{\partial E}{\partial \dot{\theta}} \right) - \frac{\partial E}{\partial \theta} = 2g(\theta)\ddot{\theta} + 2g(\theta)\dot{\theta}^2 = 2g(\theta)\ddot{\theta} + 2g(\theta)\dot{\theta}^2$$

Combining the terms with $\ddot{\theta}$ and $\dot{\theta}^2$, two others functions, $g_1(\theta)$ and $g_2(\theta)$, are defined:

$$\frac{\partial}{\partial \theta} \left( \frac{\partial E}{\partial \dot{\theta}} \right) - \frac{\partial E}{\partial \theta} = 2g_1(\theta)\ddot{\theta} + 2g_2(\theta)\dot{\theta}^2$$

Therefore, $g_1(\theta) = 2g(\theta)$ and $g_2(\theta) = g(\theta)/\dot{\theta}$. The equation $g(\theta)$ is presented in the sequence, where $(\gamma = \dot{\theta})/\dot{\theta}$, reminding that $\varphi$ and

![Fig. 30. Bifurcation diagram varying the excitation amplitude $\mu$ for $\Omega = 0.16$ and for $n=6$.](image1)

![Fig. 31. Chaotic response for $\mu = 0.20$, $\Omega = 0.16$ and $n=6$. (a) Phase space. (b) Poincaré section.](image2)
\[ g(\theta) = \frac{1}{2} m_1 L^2 \left\{ \frac{1}{2} \beta^2 \cos^2(\beta) \right\}
\]

\[ \left[ -\frac{1}{2} \beta \sin(\beta) \cos(\beta) - \frac{1}{2} \beta \cos(\beta) \sin(\beta) + \frac{1}{2} \sin(\beta) \cos(\beta) \right]^2
\]

\[ + \frac{1}{2} \beta \sin(\beta) \cos(\beta) + \frac{1}{2} \beta \cos(\beta) \sin(\beta) - \frac{1}{2} \sin(\beta) \cos(\beta) + R_1 \right)^2
\]

\[ + \frac{1}{4} \beta^2 \left( \frac{1}{2} + \frac{1}{2} \sin(\beta) \cos(\beta) + \frac{1}{2} \sin(\beta) \cos(\beta) \right) + 2 R_1 \left( -\frac{1}{2} \sin(\beta) \cos(\beta) \right) \]

\[ + R_1^2 \left( \frac{1}{2} \sin(\beta) \cos(\beta) + \frac{1}{2} \sin(\beta) \cos(\beta) \right) \]

\[ + \beta \sin(\beta) \cos(\beta) \left( \frac{1}{2} \beta \sin(\beta) \cos(\beta) + \frac{1}{2} \beta \cos(\beta) \sin(\beta) \right)
\]

References


