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Chaos control applied to heart rhythm dynamics

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ABSTRACT

The dynamics of cardiovascular rhythms have been widely studied due to the key aspects of the heart in the physiology of living beings. Cardiac rhythms can be either periodic or chaotic, being respectively related to normal and pathological physiological functioning. In this regard, chaos control methods may be useful to promote the stabilization of unstable periodic orbits using small perturbations. In this article, the extended time-delayed feedback control method is applied to a natural cardiac pacemaker described by a mathematical model. The model consists of a modified Van der Pol equation that reproduces the behavior of this pacemaker. Results show the ability of the chaos control strategy to control the system response performing either the stabilization of unstable periodic orbits or the suppression of chaotic response, avoiding behaviors associated with critical cardiac pathologies.

1. Introduction

Natural phenomena have rhythms that can be both regular and irregular over time and space. Specifically in biomedical systems, these rhythms can be associated with either normal or pathological physiological functioning [15,26]. The cardiovascular rhythm dynamics have been widely studied due to the key aspects of the heart in the physiology of living beings. The heart is a hollow and muscular organ divided into four chambers, two atrium and two ventricles, as illustrated in Fig. 1. The cardiac conduction system can be treated as a network of self-excitatory elements formed by the sino-atrial node (SA), atrio-ventricular node (AV) and His–Purkinje system (HP) [15,16]. The physiological functioning of the heart electrical system initiates at the SA node and the electrical impulse spreads in the form of wave, stimulating the atrium. The impulse reaches the AV node, initiating an electrical impulse that goes down to the His–Purkinje system and myocardial cells. In normal state, the SA node determines the frequency of the heartbeat, being called a normal heart pacemaker.

Heartbeat dynamics have been analyzed by either mathematical models or time series analysis. Wessel et al. [31] presented a general overview about cardiovascular physics pointing that its challenge is to develop methods that are able to improve the medical diagnostics decreasing the patient's risk. In this regard, cardiovascular physics interconnects medicine, physics, biology, engineering and mathematics representing an interdisciplinary collaboration of several specialists. The cardiovascular dynamics analysis introduces the idea that physiological rhythms constitute a central characteristic of life. Christini and Glass [4] presented a general overview of complex cardiac arrhythmias mapping and control.

Concerning mathematical modeling of the cardiac dynamics, the first study of the heartbeat described by nonlinear oscillators was carried out by Van der Pol and Van der Mark [29]. After this work, many studies have been developed to mathematically model the cardiac rhythms. Grudzinski and Zebrowski [16] proposed a variation of the original Van der Pol oscillator in order to reproduce the action potential generated by a natural cardiac pacemaker. Santos et al. [25] presented a model of the cardiac dynamics composed by two coupled modified Van der

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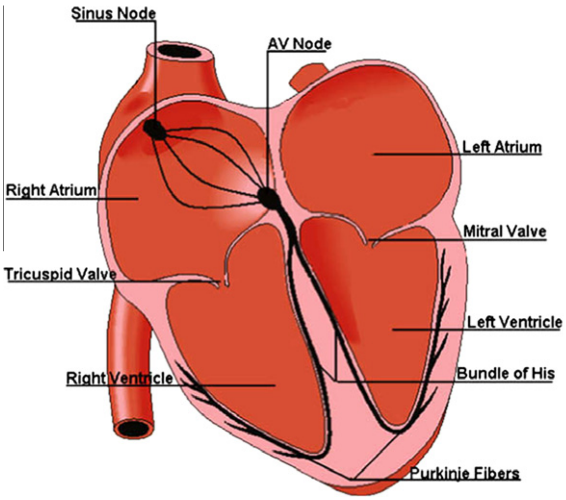

Fig. 1. Schematic picture of the heart [15].

Pol oscillators, representing the behavior of two pacemakers. Gois and Savi [15] reproduced the heart functioning through a mathematical model composed by three modified Van der Pol oscillators with delayed coupling. This model is able to describe the electrocardiograms (ECGs), representing either normal or pathological behaviors.

Several researches are pointing to the fact that some cardiac arrhythmias are associated with chaotic responses. Glass et al. [14] analyzed cardiac arrhythmia by considering periodic stimulation of an aggregate of spontaneously beating cultured cardiac cells. Results pointed to evidences of nonlinear characteristics as period-doubling bifurcations and chaotic dynamics at different values of the stimulation parameters.

Concerning time series analysis, ECG signals are usually employed to represent the cardiac dynamics. This kind of analysis is important because it is not necessary a previous knowledge about the system behavior. Kaplan and Cohen [19] investigated the relation between chaos and fibrillation. The article analyzed data from three-lead surface ECGs from dogs in which fibrillation had been either electrically induced or spontaneously occurred. Results indicated that the system has nonlinear evidences but also random characteristics. Witkowski et al. [30] investigated electrically induced ventricular fibrillation in dogs. Results pointed to deterministic characteristics of the time series. Babloyantz and Destexhe [2] investigated the chaoticity of normal human ECGs. The authors used several tools as Poincaré section, power spectrum, autocorrelation function, correlation dimension, Kolmogorov entropy and concluded that the normal ECG has chaotic characteristics. Kantz and Schreiber [18] investigated human ECG time series establishing a comparison between nonlinear deterministic and stochastic aspects. At the end, the authors explained that "stochastic fluctuations, intrinsic instability, and a changing environment act together to produce the intriguing patterns we observe". Moreover, they pointed to the importance of the use of nonlinear tools in the investigation of heart dynamics.

The idea that heart dynamics may present chaotic behavior suggests therapeutic strategies different from the classic approaches and the control of chaotic heartbeat is a key issue in this subject [15,26]. Garfinkel et al. [12,13] discussed the application of chaos control techniques to prevent arrhythmic cardiac responses, explaining that this approach can be incorporated into pacemakers to prevent ventricular fibrillation, for example. These authors presented an in vitro excitable biological tissue, suggesting the potential application of the chaos control approach.

Christini et al. [3] showed that the chaos control approach can modulate human cardiac electrophysiological dynamics. The procedure successfully achieved control in humans. Therefore, this article presented a proof-of-concept demonstration of the clinically feasibility of this approach in humans.

The use of other control procedures is also a possibility to avoid undesirable responses of the heart. Hall et al. [17] applied control procedures to a piece of dissected rabbit heart using electrical excitation in order to suppress alternans arrhythmia. This arrhythmia is related to a beat-to-beat variation in the electric wave propagation morphology in myocardium. The authors proposed a map to model the cardiac rhythm and the control of alternans was achieved by directing the system towards to the unstable fixed point. Dubljevic et al. [10] discussed the use of feedback control to suppress alternans behavior in an extracted rabbit heart and in a cable of cardiac cells. The idea was to perform real-time control of cardiac alternans. Lópes et al. [20] employed a controller to avoid pathological cardiac rhythms described by the model proposed by Gois and Savi [15].

Chaos control is based on the richness of unstable periodic patterns that exist in chaotic behavior and can be understood as the use of small perturbations to stabilize unstable periodic orbits (UPOs) embedded in chaotic attractors [7–9]. The ability to stabilize UPOs confers a great flexibility to the system since one of these UPOs can provide better performance than others in a particular situation. Therefore, the use of chaos control can make chaotic behavior to be desirable in a variety of applications.

Pyragas [22] proposed a continuous-time control method, called time-delayed feedback (TDF), to stabilize UPOs embedded in the chaotic behavior. The control law of this method is based on the difference between the present and a delayed state of the system. Several methods were proposed based on this continuous method in order to overcome some limitations of the original technique, as described by Pyragas [24]. Pyragas [24] also states that the extended time-delayed feedback control (ETDF), proposed by Socolar et al. [27], is presumably the most important modification of the TDF. The ETDF control method considers not only one but several delayed states of the system on its control law.

This contribution employs the ETDF control approach in order to avoid critical pathological responses of natural cardiac pacemakers. Initially, the behavior of the natural cardiac pacemaker is modeled by a modified Van der Pol oscillator proposed by Grudzinski and Zebrowski [16]. Using this model, periodic and chaotic responses are treated respectively representing normal and pathological functioning of the heart rhythm. Afterward, the close-return method [1] is employed to identify UPOs embedded in the chaotic attractor. The controller parameters are then determined for each desired UPO evaluating the maximum Lyapunov exponent, calculated using the algorithm due to Wolf et al. [32]. Finally, stabilization of some UPOs is performed. Results show the possibility of using this chaos control strategy in order to control and suppress some critical cardiac pathologies.

This paper is organized as follows. After this introduction, a brief discussion about the extended time-delayed feedback control method is presented. The calculation of UPO Lyapunov exponent is then discussed establishing a procedure to estimate controller parameters. The mathematical modeling of the heart pacemaker is presented by assuming a modified Van der Pol equation and the calculation of Lyapunov exponent is explained for this specific system. Numerical simulations is then presented split in three parts: uncontrolled system, where periodic and chaotic behaviors are presented; the chaos control of the heart rhythms, where ETDF is employed to stabilize UPOs; chaos suppression where chaotic behavior is eliminated. Finally, conclusions are discussed.

2. Extended time-delayed feedback control method

Chaos control methods may be understood as a two stage approach. The first stage is related to the identification of UPOs embedded in chaotic attractors and the evaluation of the controller parameters. The second stage is related to the stabilization of some desired UPOs. Basically, chaos control methods can be classified into discrete [21,6,7] and continuous approaches [22,8].

The time-delayed feedback control method (TDF) was the first continuous chaos control method proposed in the literature by Pyragas [22]. This approach states that chaotic systems can be stabilized by feedback perturbations proportional to the difference between the present and a delayed state of the system. In order to overcome some limitations of the TDF method, Socolar et al. [27] proposed a control strategy that considers information from several delayed states. This control technique can be modeled by a set of nonlinear ordinary differential equations as follows:

$$\begin{aligned} \dot{x} &= Q(x,y), \\ \dot{y} &= P(x,y) + C(t,y), \end{aligned} \tag{1}$$

where x and y are the state variables, $Q(x,y)$ and $P(x,y)$ define the system dynamics, while $C(t,y)$ is associated with the control action. The perturbation action is given by:

$$\begin{aligned} C(t,y) &= K[(1-R)S_\tau - y], \\ S_\tau &= \sum_{m=1}^{N_\tau} R^{m-1} y_{m\tau}, \end{aligned} \tag{2}$$

where $y = y(t)$, $y_{m\tau} = y(t - m\tau)$, $0 \leqslant R < 1$ and K are controller parameters. In general, N_τ is infinity but it can be properly defined depending on the dynamical system. For any value of R, the perturbation of Eq. (2) is zero when the trajectory of the system is on an UPO since $y(t - m\tau) = y(t)$ for all m if $\tau = T_i$, where T_i is the periodicity of the ith UPO. The stabilization of some UPO depends on the proper choice of R and K. It is important to mention that the ETDF method is equivalent to the original TDF method when $R = 0$.

The controlled dynamical system, Eq. (2), is described by a delay differential equation (DDE) and its solution imposes to establish an initial function $y_0 = y_0(t)$ over the interval $[-N_\tau\tau, 0]$. In this paper, this function is estimated by a Taylor series expansion as proposed by Cunningham [5]:

$$y_{m\tau} = y - m\tau\dot{y}. \tag{3}$$

Under this assumption, the following system is obtained:

$$\begin{aligned} \dot{x} &= Q(x,y), \\ \dot{y} &= P(x,y) + K[(1-R)S_\tau - y], \end{aligned} \tag{4}$$

$$\text{where} \quad \begin{cases} S_\tau = \sum_{m=1}^{N_\tau} R^{m-1}[y - m\tau\dot{y}], & \text{for } (t - N_\tau\tau) < 0, \\ S_\tau = \sum_{m=1}^{N_\tau} R^{m-1} y_{m\tau}, & \text{for } (t - N_\tau\tau) \geqslant 0. \end{cases}$$

Note that DDEs contain derivatives that depend on the solution at delayed time instants. Therefore, besides the special treatment that must be given for $(t - N_\tau\tau) < 0$, it is necessary to deal with time-delayed states while integrating the system. A fourth-order Runge–Kutta method with linear interpolation on the delayed variables is employed in this work for the numerical integration of the controlled dynamical system [8]. Moreover, it is worth mentioning that, since the control law depends on delayed states of the system, the control action is initiated only when all these states are known. Thus, by assuming three delayed states in Eq. (4), $N_\tau = 3$, the perturbations performed by the controller starts at $t = t_0 + 3\tau$, being t_0 the initial time instant.

During the learning stage it is necessary to identify the UPOs embedded in the chaotic attractor, which can be done by employing the close-return method [1]. Moreover, it is necessary to establish a proper choice of controller parameters, R and K, for each desired orbit. This choice can be done by analyzing Lyapunov exponents of the correspondent orbit, as presented in the next section [8]. After this first stage, the control stage is performed, trying to stabilize desired UPOs.

3. UPO Lyapunov exponent

The idea behind the time-delayed feedback control is the construction of a continuous-time perturbation, as presented in Eqs. (2) and (3), in such a way that it does not change the desired UPO of the system, but only its characteristics. This is achieved by changing the controller parameters in order to force Lyapunov exponents related to an UPO to become all negatives, which means that the UPO becomes stable. In this regard, it is enough to determine only the largest Lyapunov exponent, evaluating values of R and K that change the sign of the exponents. In other words, it is necessary to look for a situation where the maximum exponent is negative, $\lambda(R,K) < 0$, situation where the orbit becomes stable [8].

The calculation of the Lyapunov exponent from DDEs is more complicated than ODEs. This is because the terms associated with the control law of ETDF, Eq. (3), involve the knowledge of system states delayed in time. Considering three delayed states ($N_\tau = 3$), the last equation of the system presented in Eq. (4) consists in the DDE as follows:

$$\begin{aligned} \dot{x} &= Q(x,y), \\ \dot{y} &= P(x,y) + C(t,y,y_\tau,y_{2\tau},y_{3\tau}). \end{aligned} \tag{5}$$

Therefore, the calculation of $y = y(t)$ for time instants greater than t implies in the previous knowledge of the function $y(t)$ in the interval $(t - 3\tau, t)$. Equations of this type consist of infinite-dimensional system that presents an infinite number of Lyapunov exponents, from which only a finite number can be determined from a numerical analysis. However, for the stability analysis of UPOs is sufficient to determine only the maximum Lyapunov exponent [23].

In this paper, the calculation of Lyapunov exponent is conducted by approximating the continuous evolution of the infinite-dimensional system by a finite number of elements where values change at discrete time steps [11]. In this regard, a function $y(t)$ in the interval $(t - 3\tau, t)$ can be approximated by N samples taken at intervals $\Delta t = 3\tau/(N - 1)$. Thus, instead of the two variables shown in Eq. (5), now considered $N + 1$ variables represented by the vector $\boldsymbol{z}$, where components $z_3, \ldots, z_{N+1}$ are related to the delayed states of $y(t)$:

$$\begin{aligned} \boldsymbol{z} &= (z_1, z_2, z_3, \ldots, z_{N+1}) \\ &= (x(t), y(t), y(t - \delta t), \ldots, y(t - 3\tau)). \end{aligned} \tag{6}$$

There are several forms to accomplish this kind of approach. In this work, based on the procedure proposed by Sprott [28], the DDE is replaced by a set of ODEs. Under this assumption, the infinite-dimensional continuous system shown in Eq. (5) is represented by $N + 1$ finite-dimensional ODEs, as follows:

$$\begin{aligned} \dot{z}_1 &= Q(z_1, z_2), \\ \dot{z}_2 &= P(z_1, z_2) + C(z_2, z_{(N-1)/3+2}, z_{2(N-1)/3+2}, z_{N+1}), \\ \dot{z}_i &= N(z_{i-1} - z_{i+1})/2\tau, \quad \text{for } 2 < i < N+1, \\ \dot{z}_{N+1} &= N(z_N - z_{N+1})/\tau, \end{aligned} \tag{7}$$

where $N = 3\tau/\Delta t + 1$. This system can be solved by any standard integration method such as the fourth-order Runge–Kutta method. Besides, Lyapunov exponents can be calculated using the algorithm proposed by Wolf et al. [32]. Moreover, in order to calculate the exponent of a specific UPO, the system is integrated along the orbit of interest [8].

4. Mathematical model

The mathematical modeling of the heartbeat dynamics was first established by the coupling of nonlinear oscillators by Van der Pol and Van der Mark [29]. Thereafter, the Van der Pol equation has been frequently used in theoretical models of cardiac rhythms due to the similarity between its characteristics and biological system behaviors, such as limit cycle, synchronization and chaos [16,26]. Moreover, the Van der Pol equation adapts its intrinsic frequency to the frequency of the external driving signal, without changing its amplitude, which is an important feature related to cardiac pacemaker. Santos et al. [25] discussed other criteria that justify the use of Van der Pol equation as a phenomenological model of the heartbeat dynamics.

Grudzinski and Zebrowski [16] proposed a modification of the classic Van der Pol equation by adding two fixed points, a saddle and a node, and an asymmetric damping term related to the voltage. This new model allows one to simulate important physiological characteristics of a natural cardiac pacemaker. The proposed model is represented by the following equation:

$$\ddot{x} + \alpha(x - v_1)(x - v_2)\dot{x} + \frac{x(x+d)(x+e)}{ed} = F(t), \tag{8}$$

where α modifies the pulse shape, which changes the time that the heart receives the stimulus, v_1 and v_2 compose an asymmetric term that replaces the damping term existing in the classic Van der Pol equation, e controls the atrial or ventricular contraction period, d is a parameter that arises when the harmonic forcing of classic equation is replaced by a cubic term and $F(t)$ is an external forcing. Therefore, the pacemaker is described by the following set of first order ordinary differential equations:

$$\begin{aligned} \dot{x} &= x_2, \\ \dot{x} &= F(t) - \alpha(x_1 - v_1)(x_1 - v_2)x_2 - \frac{x_1(x_1+d)(x_1+e)}{ed} + C(t, x_2), \end{aligned} \tag{9}$$

where $C(t,x_2)$ represents the control perturbation.

4.1. Calculation of Lyapunov Exponents

The calculation of the Lyapunov exponents is performed by an alternative representation of the system. By assuming $z_1 = x_1$, $z_2 = x_2$, and considering that the variables $z_3, \ldots, z_{N+1}$ represent the delayed states of x_2 over the interval $(t - 3\tau, t - h)$, the set of equations that governs the system dynamics is given by:

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= \left[-2\alpha z_1 z_2 + \alpha v_1 z_2 + \alpha v_2 z_2 \left(-3z_1^2 - 2ez_1 - 2dz_1 - ed\right)/ed\right] z_1 \\ &\quad + (-\alpha z_1^2 + \alpha v_1 z_1 + \alpha v_2 z_1 - \alpha v_1 v_2) z_2 \\ &\quad + K[(1-R)(z_{(N-1)/3+2} + R z_{2(N-1)/3+2} + R^2 z_{N+1}) - z_2], \\ \dot{z}_i &= N(z_{i-1} - z_{i+1})/2\tau, \quad \text{for } 2 < i < N+1, \\ \dot{z}_{N+1} &= N(z_N - z_{N+1})/\tau. \end{aligned} \tag{10}$$

where $N = 3\tau/h + 1$ and h is the integration time step. The set of equations given by Eq. (10) can be numerically integrated using the fourth order Runge–Kutta method and the maximum Lyapunov exponent is calculated using the algorithm proposed by Wolf et al. [32]. Besides this, it is important to be pointed out that the fiducial trajectory associated with the original system (z_1, z_2) is replaced by a time series that represents the orbit, obtained in UPO identification stage [8].

In order to verify the capability of the ETDF control method to stabilize UPOs, the maximum Lyapunov exponent of the desired UPO is calculated for different values of controller parameters, R and K. In principle, the stabilization of the desired orbit can be achieved for parameters related to negative values of the exponent. Moreover, the choice of parameter values should be done in such a way that the maximum Lyapunov exponent is close to its minimum value [23].

5. Numerical simulations

This section presents numerical simulations related to cardiac rhythms. The analysis starts with uncontrolled system, presenting periodic and chaotic responses, respectively representing normal and pathological functioning of the heart rhythms. Afterward, the controlled system is investigated. Initially, the stabilization of UPOs that belong to the system dynamics is of concern. After that, chaos control method is employed to suppress chaos, avoiding chaotic behavior of the heart.

5.1. Uncontrolled Behavior

Numerical simulations of the cardiac pacemaker are carried out showing some aspects of the uncontrolled system behavior which means that $C(t,x_2) = 0$. Initially, the parameters proposed by Grudzinski and Zebrowski [16] are used, representing the normal activity of the natural pacemaker. Basically, the following parameters are considered: $\alpha = 3$, $\upsilon_1 = 0.83$, $\upsilon_2 = -0.83$, $d = 3$, $e = 6$, $F(t) = 0$. Moreover, initial conditions are defined by $[x_1(0)\ x_2(0)] = [-0.1\ 0.025]$. Fig. 2 shows the typical system response presenting the steady-state phase space and time history.

Chaotic responses of cardiac systems may be associated with pathological functioning such as ventricular fibrillation, which is one of the most dangerous cardiac arrhythmias [12,13,15]. In order to represent this kind of pathology, different parameters are assumed: $\alpha = 0.5$, $\upsilon_1 = 0.97$, $\upsilon_2 = -1$, $d = 3$, $e = 6$ and $F(t) = A\sin(\omega t)$, where $A = 2.5$ and $\omega = 1.9$. Under this condition, the system has a coexistence of period-1 and chaotic attractors. Fig. 3 shows the basin of attraction of the system for this set of parameters and different initial conditions. It is noticeable the coexistence of a period-1attractor (black points) and a chaotic attractor (pink points).

Both kinds of responses can be achieved by assuming different initial conditions. The chaotic response can be seen in Fig. 4 that shows phase space, time history and Poincaré section for $x_1(0)\ x_2(0) = [-0.1\ 0.025]$. By changing the initial condition to $[x_1(0)\ x_2(0)] = [-6\ 0]$, the period-1 attractor is achieved, as shown in Fig. 5. This behavior is illustrated by steady-state phase space, Poincaré section and time history.

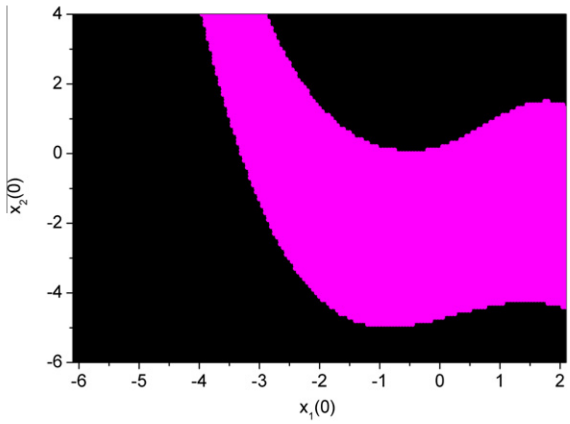

Fig. 3. Basin of attraction showing the coexistence of a period-1 attractor (black) and a chaotic attractor (pink). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

5.2. Controlling the heart rhythms

This section presents the control of the heart rhythms using the continuous chaos control approach (ETDF). Basically, we are interested to avoid the chaotic pathological, functioning of the heart. The most interesting situation is to stabilize unstable periodic orbits because of the low energy consumption related to this procedure. Nevertheless, in terms of clinical point of view, the chaos suppression is also a good alternative. We called chaos suppression a situation where the resulting controlled orbit is not an UPO that belongs to the system dynamics, but a generic orbit. Under this condition, the controller has great effort being related to high values of the control perturbations. This procedure evades the central idea of chaos control but is useful to avoid chaotic behavior that can be critical for life. Chaos suppression will be treated in the next section.

The first stage of the control procedure consists in the identification of UPOs embedded in the chaotic attractor.

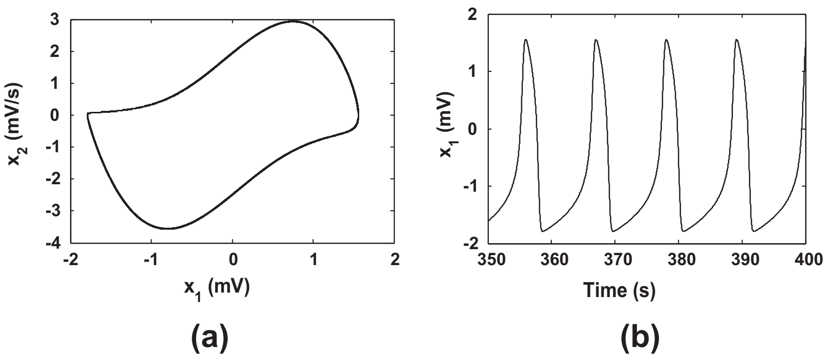

Fig. 2. Normal activity of the cardiac pacemaker in steady-state: (a) Phase space; (b) Time history.

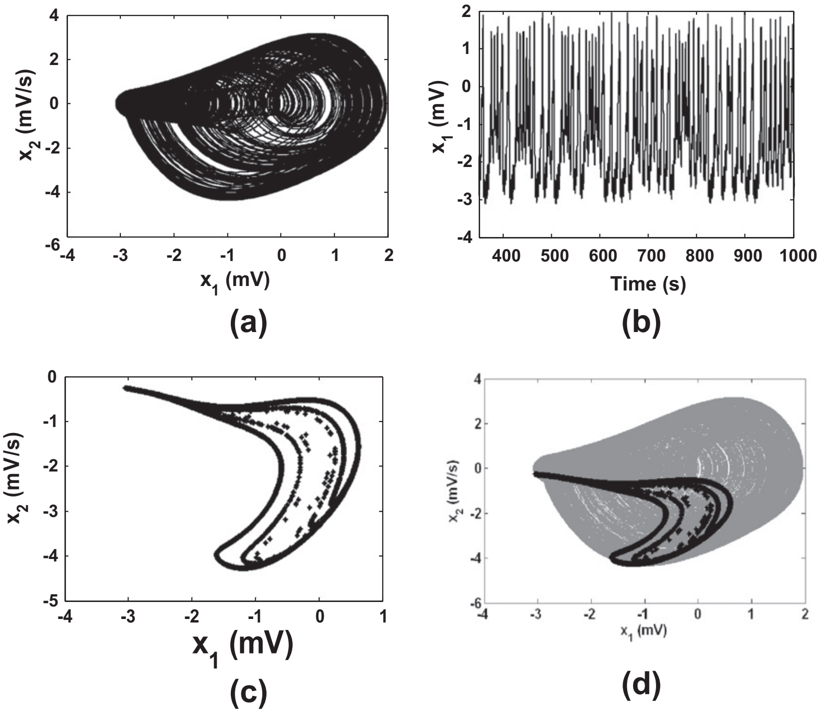

Fig. 4. Chaotic activity of the cardiac pacemaker: (a) Phase space; (b) Time history; (c) Poincare section; (d) Phase space and Poincaré section.

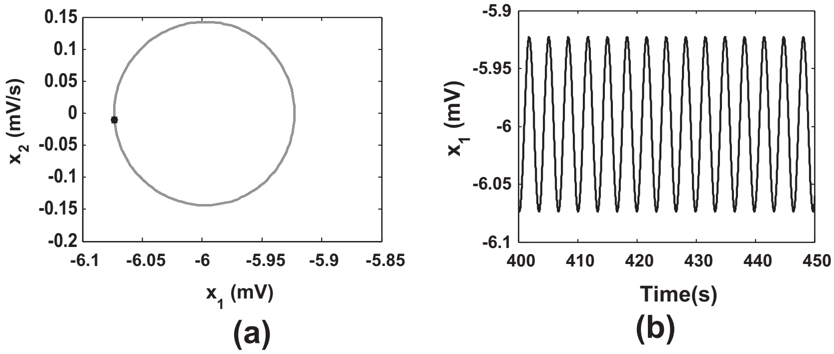

Fig. 5. Periodic activity of the cardiac pacemaker: (a) Phase space and Poincaré section; (b) Steady-state time history.

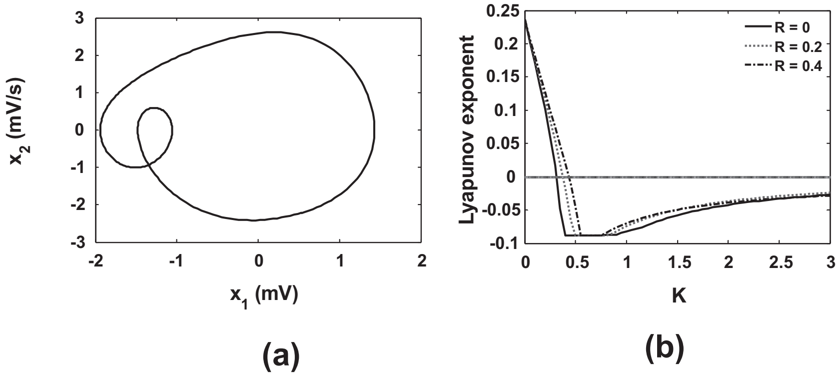

Fig. 6. Identified period-2 UPO and its maximum Lyapunov exponent.

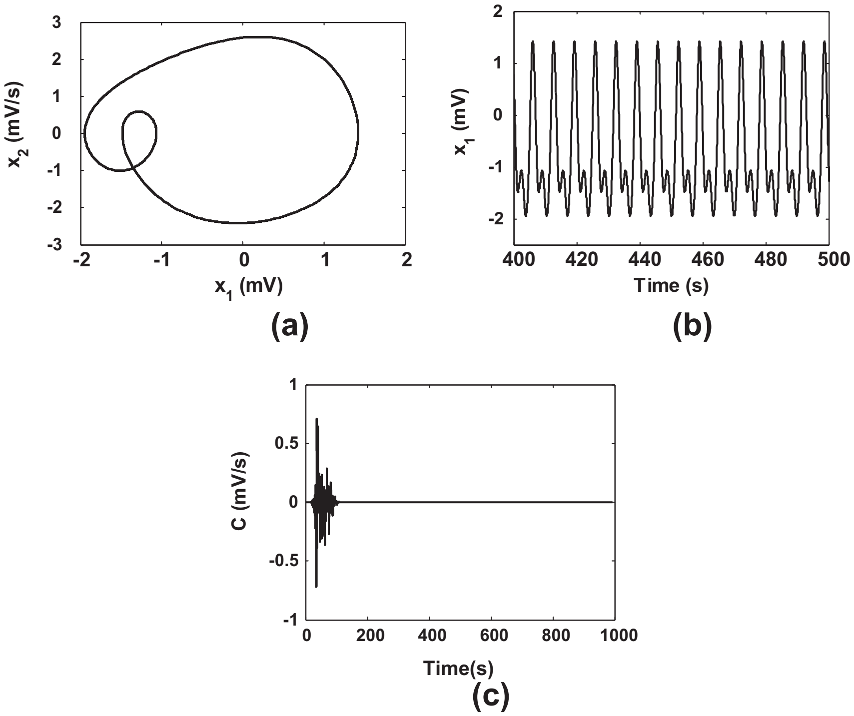

Fig. 7. Stabilization of period-2 UPO: (a) Phase space; (b) Time history; (c) Control perturbation.

Here, it is done by employing the close-return method [1]. After this identification, the maximum Lyapunov exponent needs to be calculated by considering different values of the controller parameters for each desired UPO, defining regions related to negative exponents. This is done by the procedure presented in the preceding sections [8]. The identification of the controller parameters finishes the first stage and then we go to the second stage, where control perturbation is applied to the system in order to achieve UPO stabilization.

Let us start by analyzing a period-2 UPO embedded in the chaotic attractor. Fig. 6 shows the desired orbit and its maximum Lyapunov exponent calculated for different values of controller parameters, R and K. Regions associated with negative values of the maximum Lyapunov exponent indicate that the system stabilization can be achieved for any value of R, including $R = 0$, using appropriate values of K. Fig. 7 shows the steady-state response (phase space and time history) and control perturbation imposed by the controller using $R = 0$ and $K = 0.4$, considering $\tau = 2\ (2\pi/\omega)$, corresponding to the periodicity 2, and $[x_1(0)\ x_2(0)] = [-0.1\ 0.025]$. It is noticeable that the controller is able to stabilize the UPO and, hence, remove the system from an undesirable chaotic behavior through small perturbations. It is also important to observe the low values of the control perturbation, which is the essential characteristic of chaos control.

Next, it is analyzed a period-4 UPO. Fig. 8 shows the identified UPO and its maximum Lyapunov exponents considering $\tau = 4(2\pi/\omega)$, corresponding to the periodicity 4.

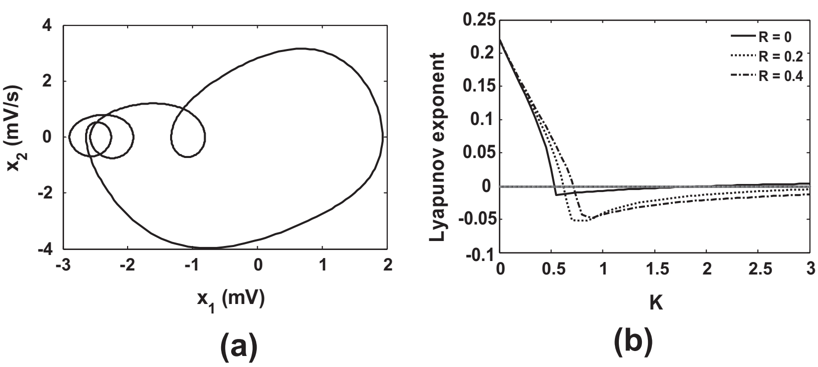

Fig. 8. Identified period-4 UPO and its maximum Lyapunov exponents.

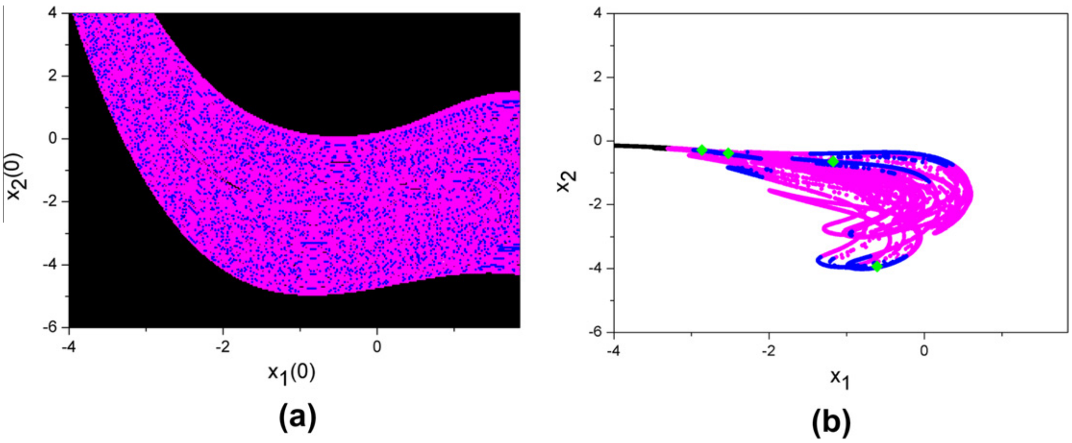

Fig. 9. Attempt of the period-4 UPO stabilization using $R = 0$ and $K = 0.6$: (a) Basin of attraction basin; (b) System state when the control perturbation starts.

These results show that the stabilization of the orbit can be achieved for a range of values of K, when $R = 0$, $R = 0.2$, and $R = 0.4$. The stabilization of the period-4 UPO is performed by assuming $R = 0$ and $K = 0.6$. Under this condition, the system trajectory can be stabilized on the UPO but, actually, this stabilization depends on initial conditions. Fig. 9a shows the basin of attraction of the controlled system. Note that, according to initial conditions, the system can achieve a period-1 attractor (black points), period-4 UPO (blue points) or may exhibit chaotic behavior (pink points). Fig. 9b shows the system state when control perturbations start, at $t = t_0 + 3\tau$, together with the points related to the period-4 UPO (green points). Once again, black, blue and pink points respectively indicate the stabilized orbits: period-1 orbit, period-4 UPO and chaotic orbit. It should be observed that the controller has a better chance to achieve the desired stabilization when the control perturbation starts in the neighborhood of the desired UPO.

Based on this analysis, it is possible to show the system stabilization in one of the three situations described. Initially, let us consider initial conditions that belong to the basin of attraction of the period-4 UPO: $[x_1(0)\ x_2(0)] = [-2.8542\ -0.3339]$. Under this condition, the period-4 UPO is stabilized as shown in Fig. 10 that presents the steady-state behavior (phase space and time history)

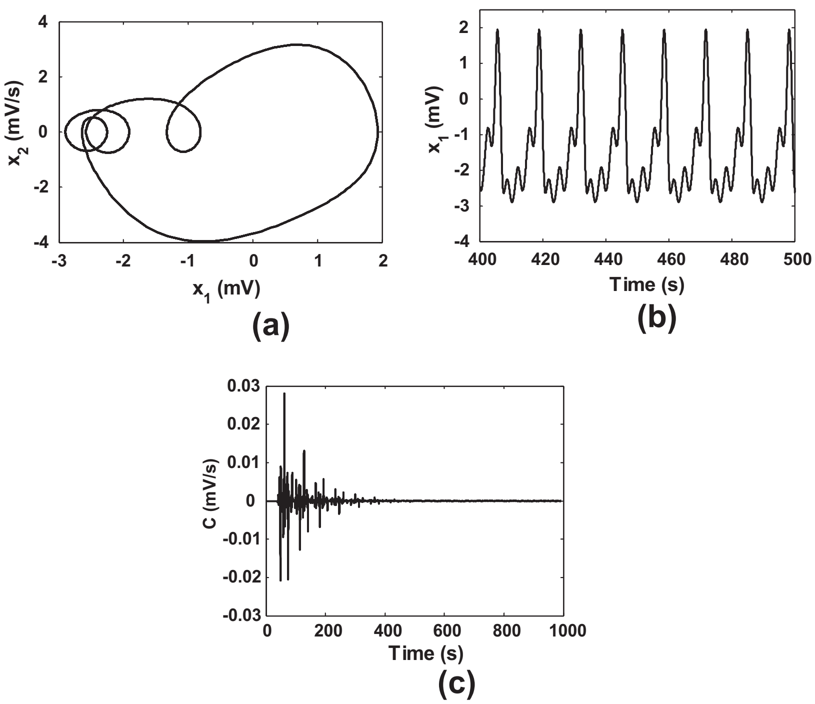

Fig. 10. Stabilization of period-4 UPO using $R = 0$ and $K = 0.6$: (a) Phase space; (b) Time history; (c) Perturbation.

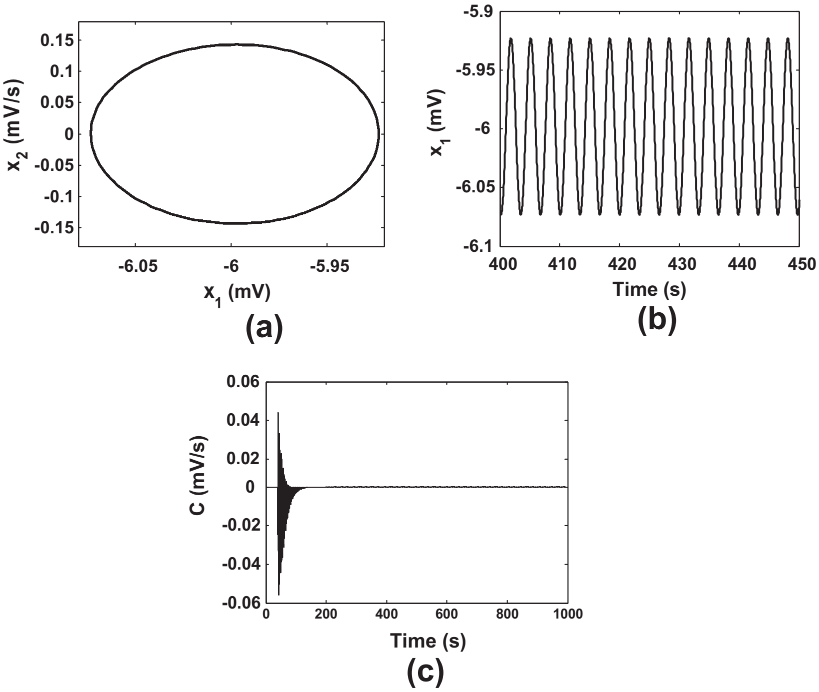

Fig. 11. Stabilization of a period-1 orbit using $R = 0$ and $K = 0.6$: (a) Phase space; (b) Time history; (c) Control perturbation.

together with the control perturbations. It should be highlighted that small perturbations are used to perform the system stabilization.

By assuming different initial conditions, $[x_1(0)\ x_2(0)] = [-2.5\ -3.5]$, the stabilization of the period-1 attractor is achieved. Fig. 11 shows the steady-state period-1 response

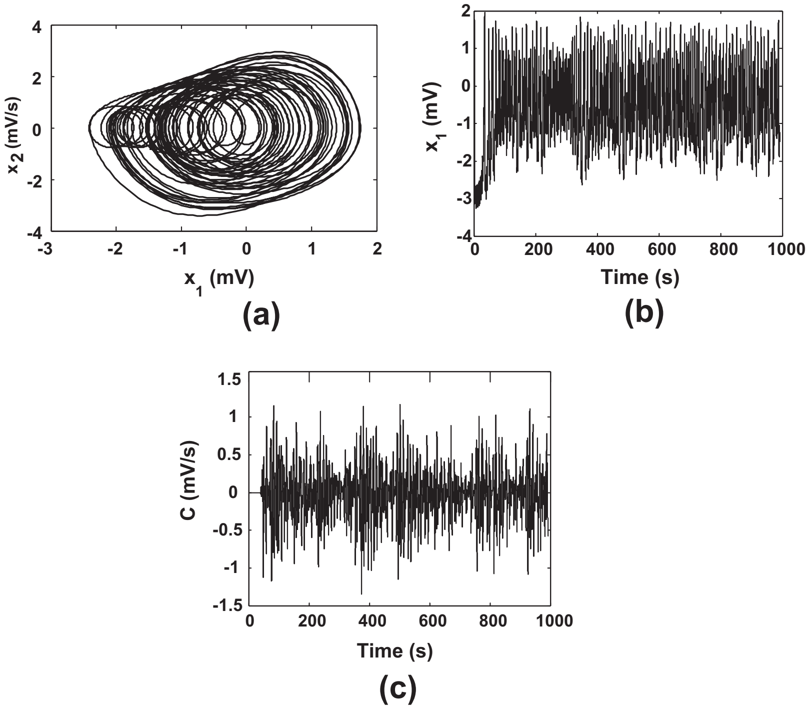

Fig. 12. Chaotic behavior: (a) Phase space; (b) Time history; (c) Control perturbation.

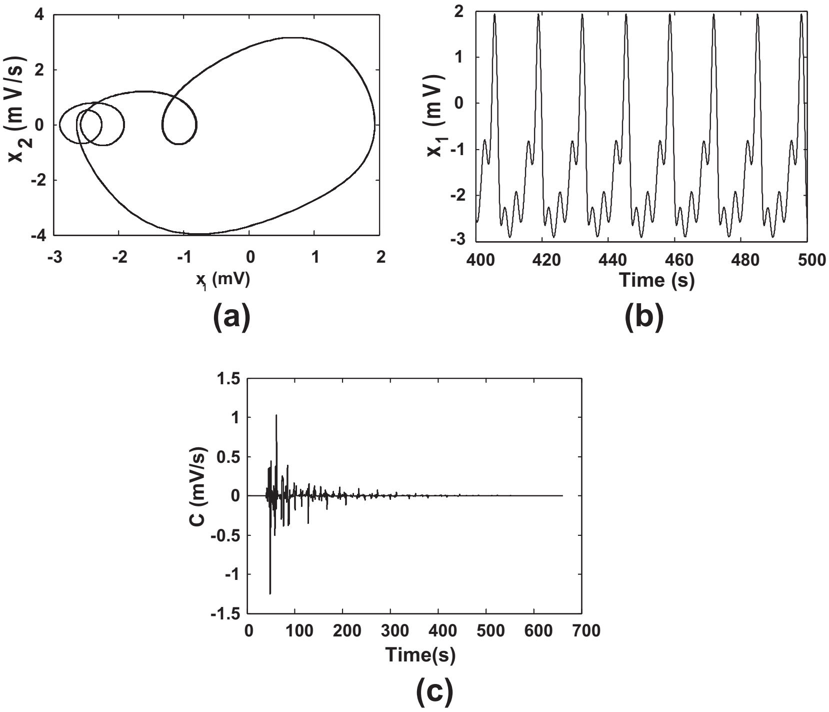

Fig. 13. Stabilization of period-4 UPO using $R = 0$ and $K = 0.8$: (a) Phase space; (b) Time history; (c) Control perturbation.

(phase space and time history) and the perturbation applied to the system. Note that, using these initial conditions, the system converges to the periodic attractor shown in Fig. 5 which does not consist in an UPO of the system. Actually, the control perturbations promote the transfer between two coexisting trajectories: from the chaotic and to the period-1 attractor.

By assuming another initial conditions, $[x_1(0)\ x_2(0)] = [-0.1\ 0.025]$, a different response is achieved. Under this condition, the controller does not succeed to promote system stabilization and chaotic behavior persists. Fig. 12 shows the system response and the control perturbation imposed to the system.

The basin of attraction of the heart system can be altered by changing controller parameters. Hence, keeping the same initial conditions that originally belongs to the chaotic attractor, $[x_1(0)\ x_2(0)] = [-0.1\ 0.025]$, and using different controller parameters, $R = 0$ and $K = 0.8$, the perio-4 UPO is stabilized as presented in Fig. 13.

The ETDF approach usually is not able to stabilize orbits with high periodicity. In order to verify its efficacy to control heart rhythms, a period-10 UPO is investigated. Fig. 14 shows the identified UPO and its maximum Lyapunov exponents calculated by assuming $\tau = 10(2\pi/\omega)$. Note that there are no negative exponents, pointing that the controller is not able to perform system stabilization.

5.3. Chaos suppression

Although the stabilization of some UPOs is not possible to be achieved, there is an alternative approach that can be employed to suppress chaos. This procedure evades the

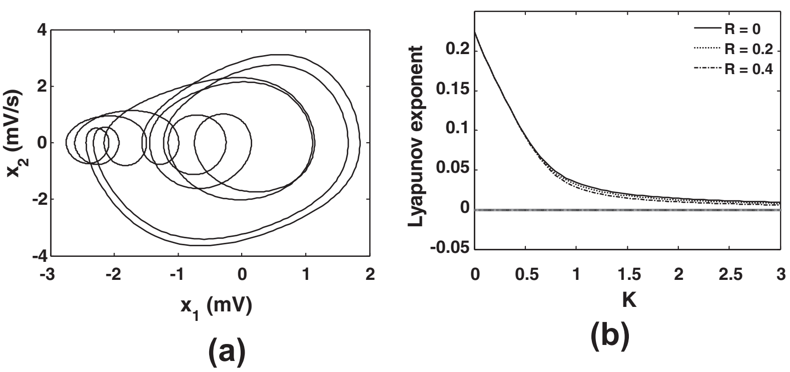

Fig. 14. Period-10 UPO and its maximum Lyapunov exponents.

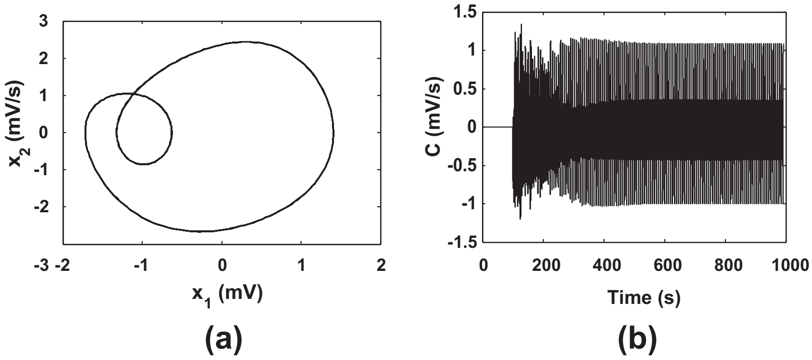

Fig. 15. Chaos suppression using $R = 0.8$ and $K = 0.8$: (a) Stabilized period-2 orbit. (b) Control perturbation.

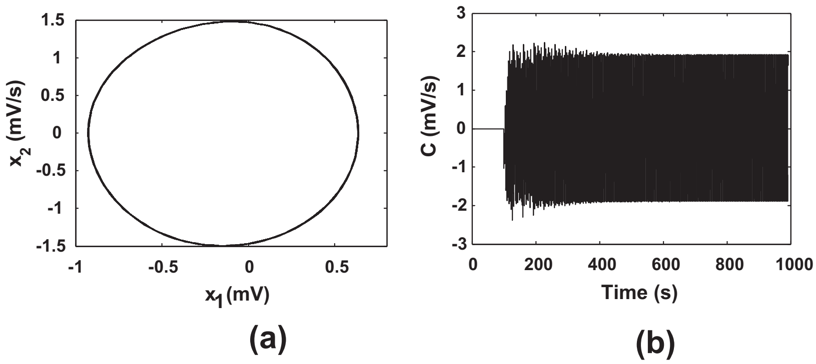

Fig. 16. Chaos suppression using $R = 0.8$ and $K = 2.5$: (a) Stabilized period-1 orbit. (b) Control perturbation.

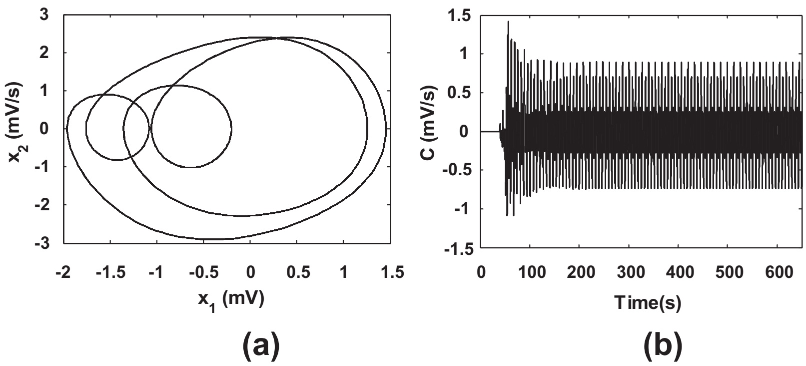

Fig. 17. Chaos suppression using $R = 0.8$ and $K = 0.6$: (a) Stabilized period-4 orbit. (b) Control perturbation.

central idea of chaos control using small perturbations to stabilize UPOs that belong to the system dynamics. However, it can be useful to avoid chaotic behavior that can be critical for life. Basically, the chaos suppression is promoted by increasing the control perturbations, which is achieved by increasing the controller parameters. De Paula & Savi [8] showed that this increase can suppress chaos, stabilizing an orbit that, essentially, is not an UPO embedded in chaotic attractor. This procedure is associated with a great controller effort being related to high values of the control perturbations. Since the clinical point of view is interested to avoid chaotic behavior of the heart dynamics, we employ the chaos suppression approach to eliminate chaos in situations where chaos control does not achieve system stabilization.

In this regard, let us consider the period-10 UPO showed in Fig. 14. By assuming controller parameters $R = 0.8$, $K = 0.8$ and $\tau = 10(2\pi/\omega)$, it is possible to stabilize a period-2 orbit, shown in Fig. 15. The change of the controller parameters to $R = 0.8$ and $K = 2.5$, stabilizes the system in a period-1 orbit, Fig. 16. Once again, it is important to mention that, in principle, these orbits are not UPOs of

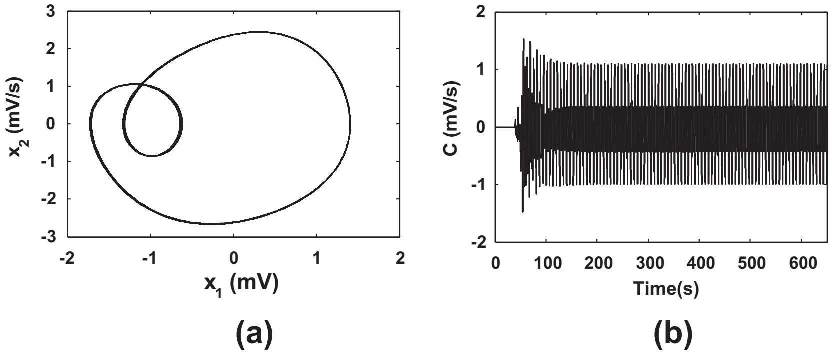

Fig. 18. Chaos suppression using $R = 0.8$and $K = 0.8$: (a) Stabilized period-2 orbit. (b) Control perturbation.

the uncontrolled system and this fact becomes clear by observing the high values of the control perturbation.

Another example where chaos suppression can be employed is related to the chaotic attractor discussed in Fig. 12. There, it is clear that initial condition can define the stabilization of the desired period-4 UPO. Fig. 12 showed that the controller is not able to stabilize system trajectory with $R = 0$, $K = 0.6$ and $[x_1(0)\ x_2(0)] = [-0.1\ 0.025]$ since it belongs to the chaotic basin of attraction. Nevertheless, Fig. 13 showed that the change of controller parameters can change the basin of attraction making the controller succeed to stabilize the desired period-4 UPO. Another alternative to avoid chaotic behavior presented in Fig. 12 is the chaos suppression. Fig. 17 presents the chaos suppression by assuming initial conditions that belongs to the basin of attraction of the chaotic attractor, $[x_1(0), x_2(0)] = [-0.1, 0.025]$, and $R = 0.8$ and $K = 0.6$. Under these conditions, the controller stabilizes a period-4 that, actually, does not belong to the system dynamics. By assuming $R = 0.8$ and $K = 0.8$ a period-2 orbit is stabilized (Fig. 18).

6. Conclusions

Chaos control of heart rhythms is of concern by analyzing the dynamical response of a natural cardiac pacemaker. Periodic and chaotic behaviors are investigated respectively representing normal and pathological behaviors. The extended time-delayed feedback method is employed to control chaotic signals. The central idea of this control strategy is to stabilize unstable periodic orbits (UPOs) embedded in the chaotic attractor through small time-continuous perturbations. Results show that the method is effective for the stabilization of UPOs with low periodicity. However, the multi-stability related to some set of parameters makes the system control dependent of initial conditions. Therefore, according to the initial conditions, the system can be stabilized in different behaviors. Moreover, the controlled system basin of attraction can be altered by changing controller parameters that can avoid problems related to multi-stability issues. The controller also presents some problems to achieve stabilization of UPOs with high periodicity. An alternative to deal with situations where the stabilization of UPOs cannot be achieved is the chaos suppression. Basically, the idea is to promote the increase of the control perturbations defined by the controller parameters. It should be pointed out that, in principle, this suppression is not related to the stabilization of natural orbits that belong to the system dynamics and, therefore, it is associated with great controller effort. However, it is a useful procedure for the clinical point of view, avoiding undesirable behaviors. In terms of clinical application, the controller effort can be imagined as electrical pulses. Based on the presented results, it is possible to conclude that the ETDF is effective to avoid chaotic responses, which are associated with pathological behaviors, eliminating undesirable responses of the heart that is dangerous to life.

Acknowledgements

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