Ana Carolina Cellular, Luciana L. da Silva Monteiro, and Marcelo A. Savi*

Numerical investigation of nonlinear mechanical and constitutive effects on piezoelectric vibration-based energy harvesting

Numerische Untersuchung von nichtlinearen mechanischen und konstitutiven Effekten auf piezoelektrisches schwingungsbasiertes Energy Harvesting

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Abstract: Vibration-based energy harvesting has the main objective to convert available environmental mechanical energy into electrical energy. Piezoelectric materials are usually employed to promote the mechanical-electrical conversion. This work deals with a numerical investigation that analyzes the influence of nonlinear effects in piezoelectric vibration-based energy harvesting. Duffing-type oscillator that can be either monostable or bistable represents mechanical nonlinearities. A quadratic constitutive electro-mechanical coupling model represents piezoelectric nonlinearities. The system performance is evaluated for different system characteristics being monitored by the input and the generated power. Numerical simulations are carried out exploring dynamical behavior of energy harvesting system evaluating different kinds of responses, including periodic and chaotic regimes.

Keywords: Nonlinear dynamics, chaos, energy harvesting, piezoelectricity, Duffing.

Zusammenfassung: Schwingungsbasiertes Energy Harvesting hat das Hauptziel, die verfügbare mechanische Energie der Umgebung in elektrische Energie umzuwandeln. Piezoelektrische Materialien werden häufig verwendet, um die mechanische und elektrische Energieumwandlung zu fördern. Diese Arbeit beschäftigt sich mit einer numerischen Untersuchung, die den Einfluss von nichtlinearen Effekten in der piezoelektrischen schwingungsbasierten Energy Harvesting untersucht. Duffing-Oszillator, der entweder monostabil oder bistabil sein kann, stellt die mechanische Nichtlinearitäten dar. Ein quadratisches und konstitutives elektromechanisches Kopplungsmodell stellt die piezoelektrische Nichtlinearitäten dar. Die Systemleistung wird für verschiedene Systemeigenschaften eingeschätzt, die durch die Eingabe und die erzeugte Leistung beaufsichtigt werden. Numerische Simulationen werden durchgeführt, umdas dynamische Verhalten des Energy-Harvesting-Systems zu untersuchen, wobei verschiedene Arten von Reaktionen ausgewertet werden, einschließlich periodischer und chaotischer Verhalten.

Schlagwörter: Nichtlineare Dynamik, Chaos, Energy Harvesting, Piezoelektrizität, Duffing-Oszillator.

1 Introduction

Vibration-based energy harvesting has the main objective to convert available environmental mechanical energy into electrical energy. Piezoelectric materials are usually employed to promote mechanical-electrical conversion. This idea has been encouraged by the growing use of mobile devices where converted energy can either supply the energy or charge batteries. The major challenge is to enhance the energy harvesting performance in order to enlarge the purposes of the potential applications.

Nonlinear energy harvesting systems have been developed to obtain better performances over a broad frequency range providing more power than linear systems. In this regard, nonlinear and random effects are of special interest. Different nonlinearities can be imagined and it is important to highlight mechanical, electrical and constitutive.

Several research efforts have been dealing with nonlinear mechanical system. Ramlan et al. [16] showed the benefits of using an oscillator with nonlinear stiffness in an energy harvesting device. The use of Duffing-type oscillators with monostable and bistable harvesters has been studied by many authors [8, 9, 11, 14, 17]. A Duffing-type oscillator can be imagined as a piezomagnetoelas-

***Corresponding author: Marcelo A. Savi,** Universidade Federal do Rio de Janeiro, COPPE – Department of Mechanical Engineering, Center for Nonlinear Mechanics, 21.941.972, Rio de Janeiro, Brazil; P.O. Box 68.503, e-mail: savi@mecanica.ufrj.br
Ana Carolina Cellular, Universidade Federal do Rio de Janeiro, COPPE – Department of Mechanical Engineering, Center for Nonlinear Mechanics, 21.941.972, Rio de Janeiro, Brazil; P.O. Box 68.503
Luciana L. da Silva Monteiro, CEFET/RJ, Department of Mechanical Engineering, 20.271.110, Rio de Janeiro, Brazil

tic structure composed by a cantilever beam with permanent magnets strategically used in order to produce equilibrium points associated with a two-well potential energy [7, 19, 20]. Stanton et al. [19] analyzed this type of an energy harvester considering either softening or hardening responses. Stanton [20] and Cottone [5] investigated bistable configurations using an experimental approach.

De Paula et al. [6] investigated random aspects on vibration-based energy harvesting of a piezomagnetoelastic structure and main results show details about harvested power. Betts et al. [3] presented a nonlinear device through an arrangement of bistable composites combined with piezoelectric elements for broadband energy harvesting of ambient vibrations. Results showed that it is possible to improve the power harvested over conventional devices. Bai et al. [2] showed that an asymmetric tip mass can induce nonlinear and hysteretic behavior on the energy harvester with a free-standing thick-film bimorph structure.

Regarding nonlinear constitutive effects, Crawley and Anderson [4] discussed nonlinear aspects related to the piezoelectric coupling showing that there is a significant dependence of strains. Triplett and Quinn [24] investigated the nonlinear piezoelectric coupling behavior and also some aspects related to the mechanical nonlinearities. Stanton et al. [20] proposed a quadratic dependence of piezoelectric coupling coefficient on the induced strain. Experimental tests were performed showing a good agreement between numerical and experimental data. Silva et al. [21] investigated the influence of hysteretic behavior of piezoelectric coupling comparing results with linear models. Results suggested that there is an optimum hysteretic behavior that can increase the harvested power output of the energy harvesting systems. Silva et al. [22] showed a comparison among experimental data and numerical simulations performed with distinct nonlinear piezoelectric coupling models. The main conclusions pointed out that the inclusion of nonlinear terms reduces discrepancies predicted by linear models. Moreover, nonlinear aspects as dynamical jumps are associated with dramatic changes of system responses.

Considerable efforts have been made to improve the energy harvesting system using nonlinear electrical circuits [13, 15]. Nonlinear switching techniques have been developed, such as parallel SSHI (synchronized switching harvesting on an inductor) and series SSHI [1]. Lefeuvre et al. [12] showed that the Synchronous Electric Charge Extraction (SECE) could enhance the electromechanical conversion when compared with the classical extraction technique [10, 18]. Recently, synergistic use of smart materials has been considered to enhance the energy harvesting performance. Silva et al. [23] employed shape memory alloys together with piezoelectric elements with this aim.

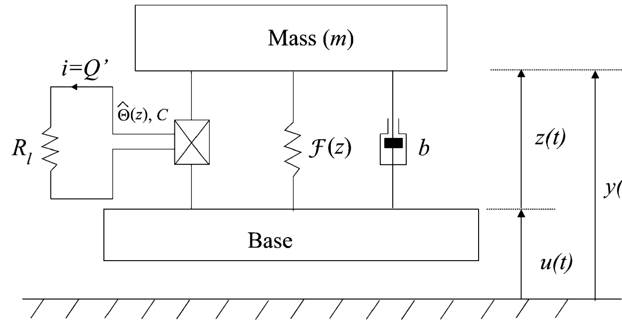

Figure 1: Archetypal model of the vibration-based energy harvesting system.

This article deals with a numerical investigation of nonlinear effects considering both mechanical and piezoelectric aspects. An archetypal model for vibration-based energy harvesting system is considered by assuming a mechanical system coupled to an electric circuit by a piezoelectric element. A Duffing-type oscillator that can be monostable and bistable represents mechanical nonlinearity. Constitutive nonlinearity is investigated considering a piezoelectric element described by an electromechanical quadratic relation. Input and output powers are monitored evaluating the energy harvesting system performance. Special attention is dedicated to the energy harvesting dynamical analysis, exploring different kinds of responses, including periodic and chaotic behaviors.

2 Energy harvesting system

A vibration-based energy harvesting system consists of a mechanical system connected to an electrical circuit by a piezoelectric element (Figure 1). The mechanical system is an oscillator with a mass m displaced by y; the base excitation is represented by $u = u(t)$ while z represents the mass displacement relative to the base. In addition, the oscillator has a linear viscous damping with coefficient b, and a restitution element that provides a force $\mathcal{F}(z)$, which describes a spring with nonlinear force–displacement relationship. Electro-mechanical coupling is provided by a piezoelectric element with coupling coefficient $\hat{\Theta}$. This element is connected to an electric circuit represented by an electrical resistance R_l and a capacitance C; V is the voltage across the piezoelectric element.

The equations of motion of the energy harvesting system can be written as follows:

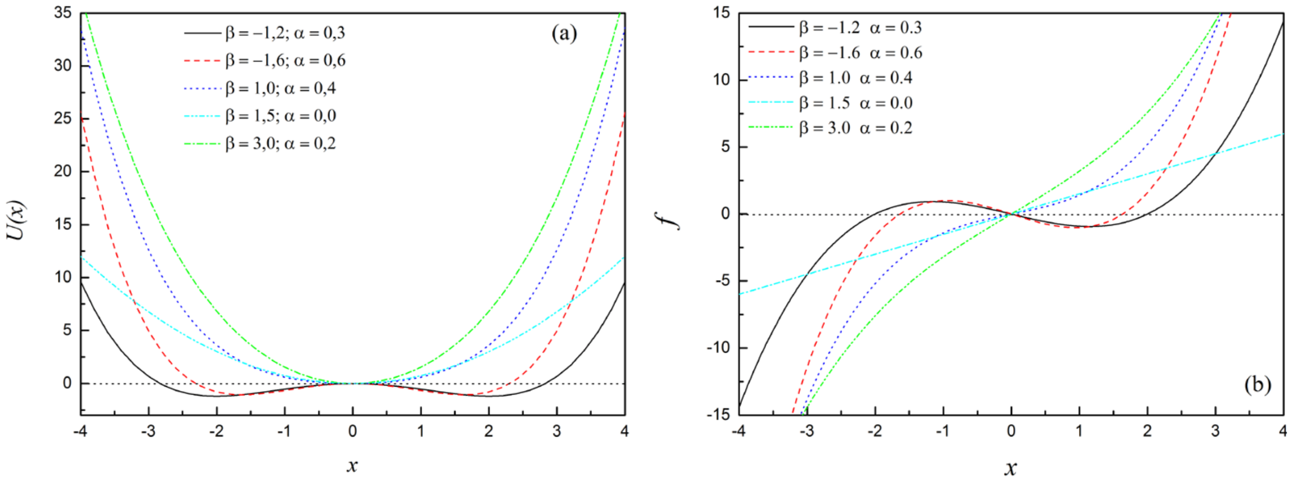

Figure 2: Duffing-type system: potential energy and corresponding force for different values of α and β.

$$m\ddot{z} + b\dot{z} + \mathcal{F}(z) - \hat{\Theta}V = -m\ddot{u} \quad (1)$$

$$\hat{\Theta}\dot{z} + C\dot{V} + \frac{1}{R_l}V = 0 \quad (2)$$

Mechanical nonlinearity is considered assuming a Duffing-type restitution force:

$$\mathcal{F}(z) = \hat{\beta}z + \hat{\alpha}z^3 \quad (3)$$

where $\hat{\alpha}$ and $\hat{\beta}$ characterize different behaviors. The piezoelectric electro-mechanical coupling is described by considering the following equation:

$$\hat{\Theta}(z) = \mu(1 + \delta_1 |z| + \delta_2 z^2) \quad (4)$$

where δ_1 and δ_2 are linear and nonlinear piezoelectric coupling coefficients, respectively. The proper description of the electro-mechanical coupling is a crucial point in the energy harvesting analysis. Nonlinear effects have a significant influence on the energy harvesting performance, especially under resonant conditions. The electro-mechanical quadratic nonlinear model captures the general behavior of the energy harvester, being in good agreement with experimental data close to resonant conditions [23].

2.1 Nondimensional equations of motion

A dimensionless analysis of the mathematical model is now in focus. Hence, consider spatial and electrical new coordinates as $x = z/l$ and $v = V/\hat{V}$ where l is a reference length and $\hat{V}$ is a reference voltage; $\omega_0 = \sqrt{|\hat{\beta}|/m}$ is a reference frequency and a harmonic excitation $-m\ddot{u} = A\sin(\omega t)$ is assumed. Using $2\zeta = b/m\omega_0$, $\epsilon = \hat{V}^2 C/m\omega_0^2 l^2$, $\phi = (l/C\hat{V})\hat{\Theta}$, $\rho = R_l C\omega_0$, $\overline{\omega} = \omega/\omega_0$, $\beta = \hat{\beta}/m\omega_0^2$, $\alpha = \hat{\alpha}l^2/m\omega_0^2$, $\theta = (l/C\hat{V})\mu$ and $\gamma = \frac{A}{m\omega_0^2 l}$, the equations of motion (1–2) are rewritten as follows:

$$x'' + 2\zeta x' + f(x) - \epsilon\phi v = \gamma\sin(\overline{\omega}\tau) \quad (5)$$

$$\phi x' + v' + v/\rho = 0 \quad (6)$$

where $(\blacksquare') \equiv d(\blacksquare)/d\tau$, and $\tau = \omega_0 t$ is the nondimensional time.

Mechanical nonlinearity is represented by a Duffing-type restitution force that describes the general behavior expressed by Eq. (3) is given by:

$$f(x) = \beta x + \alpha x^3 \quad (7)$$

This restitution force is related to the following expression of potential energy:

$$U(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4 \quad (8)$$

Different values of β and α characterize distinct mechanical system behaviors. Note that when $\beta < 0$, the system has a double-well potential with bistable aspects. On the other hand, when $\beta > 0$, the system is nonlinear monostable. A linear system is characterized by $\beta > 0$ and $\alpha = 0$. Figure 2 shows the potential energy and the corresponding restitution force for different values of these parameters.

On the other hand, the dimensionless piezoelectric coupling nonlinearity given by Eq. (4) is rewritten in dimensionless form as:

$$\phi = \theta\left(1 + \xi_1 |x| + \xi_2 x^2\right) \quad (9)$$

where $\xi_1 = l\delta_1$ and $\xi_2 = l^2\delta_2$ are the dimensionless piezoelectric constitutive coefficients.

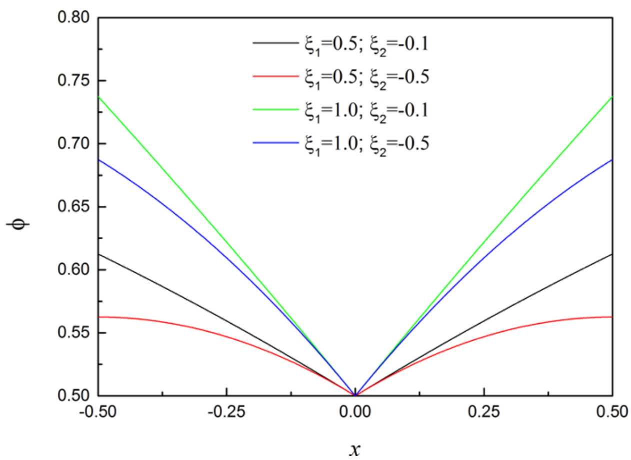

Figure 3: Electro-mechanical behavior as function of induced strain.

The general behavior of the nonlinear electro-mechanical coupling (Eq. 9) can be observed in Figure 3 for different parameters. It should be highlighted a strong strain dependence, as showed in Crawley and Anderson [4]. This nonlinearity has a significant influence on energy harvesting response, especially under resonant conditions where large strains are involved.

The energy harvesting electrical power can be evaluated by considering instantaneous dimensionless power defined as $P = v^2/\rho$. This electrical dimensionless power is associated with system output, where RMS average is given by

$$\overline{P}_{out} = \sqrt{\frac{1}{\tau}\int_0^\tau P^2 d\tau} \tag{10}$$

On the other hand, the instantaneous dimensionless input power given by $P_{in} = x'\gamma \sin(\overline{\omega}\tau)$, mechanical dimensionless power is related to the system excitation, and RMS average is defined as follows,

$$\overline{P}_{in} = \sqrt{\frac{1}{\tau}\int_0^\tau P_{in}^2 d\tau} \tag{11}$$

The system performance can be evaluated by considering the instantaneous conversion efficiency, $\eta = P/P_{in}$, that establishes a relation between instantaneous electrical and mechanical powers, or in other words, output and input powers. Analogous definition can be done considering RMS average given by: $\overline{\eta} = \overline{P}_{out}/\overline{P}_{in}$. From now on, the term power is used associated with dimensionless power. Besides, the different definitions are conveniently employed in order to show distinct behaviors.

3 Numerical simulations

Numerical simulations are carried out exploring the energy harvesting system dynamics. System parameters are experimentally identified from a piezomagnetoelastic structure [6]. In this regard, the following parameters are adopted for all simulations: $\zeta = 0.01$, $\epsilon = 0.1$, $\theta = 0.5$, $1/\rho = 0.05$ and $\gamma = 0.1$. A parametric analysis is performed for the other parameters. Different nonlinearities and excitation aspects are investigated. Concerning mechanical nonlinearities, different Duffing parameters, β and α, are evaluated considering linear, monostable and bistable characteristics. Constitutive nonlinearities of the piezoelectric element are also investigated assuming coupling coefficients (ξ_1 and ξ_2) and establishing a comparison with linear constitutive model ($\xi_1 = \xi_2 = 0$). Moreover, excitation amplitudes and frequencies are varied evaluating distinct situations that change system response. Efficiency is analyzed through the main aspects of energy harvesting power. The fourth order Runge-Kutta method is employed for numerical simulations. Time steps less than 10^{-3} are employed, being defined after a convergence analysis.

3.1 Influence of the excitation frequency

The most desirable responses for energy harvesting purposes are usually related to large amplitudes. Therefore, resonant conditions are of special interest. Nevertheless, nonlinearities are associated with complex responses that can change this condition and, therefore, need to be properly investigated. Dynamical jumps and chaos are some nonlinear characteristics that need to be treated in order to evaluate the system performance.

Initially, the system efficiency is evaluated changing the excitation frequency and adopting different mechanical parameters with a linear constitutive model ($\xi_1 = \xi_2 = 0$). Figure 4 shows the efficiency parameter under the slow quasi-static variation of excitation frequency for different characteristics of the mechanical system, discarding the transient response. The response is characterized by a two-peak curve due to nonlinear effects. The first one is related to high values of output/input power ratio, but associated with small values of each one of them, whereas the second one is a typical nonlinear resonance, presenting dynamical jumps.

In general, for these sets of parameters, monostable systems (associated with positive β) have better efficiency than the bistable ones (associated with negative β). Note that the maximum efficiency occurs at different frequencies, and this conclusion is more evident for bistable sys-

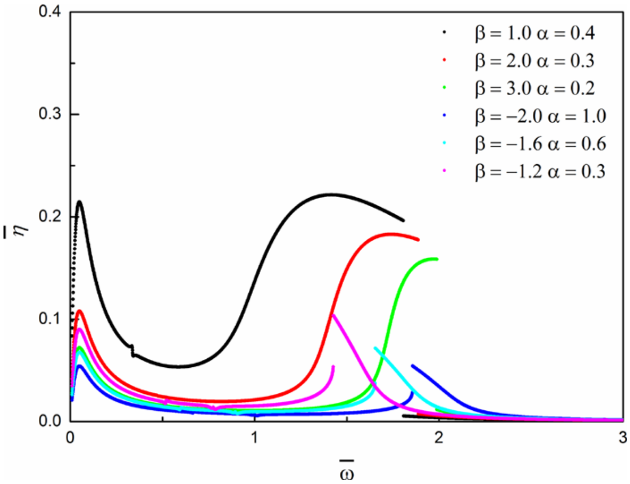

Figure 4: Efficiency versus excitation frequency for different mechanical properties and linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$).

tems. Details of this kind of behavior are presented in Figure 5 for monostable and bistable systems, showing both up-sweep and down-sweep responses, highlighting dynamical jumps. It should be mentioned that monostable system can reach higher efficiency values during up-sweep. On the other hand, bistable systems can reach higher efficiency values during down-sweep test.

Figure 6 presents a comparison between input, P_{in}, and output, P_{out}, powers for monostable and bistable systems. Note that monostable systems have better efficiency than the bistable ones for these sets of parameters. Moreover, bistable systems present two peaks while monostable systems have only one. Besides, monostable systems show hardening trend while the bistable ones present softening behavior.

Energy harvesting system response is now investigated assuming nonlinear piezoelectric coupling ($\xi_1 = 0.5$ and $\xi_2 = -0.1$), Figure 7. Comparing results of the linear and nonlinear constitutive models (Figures 4 and 7, respectively), it is noticeable that the nonlinear piezoelectric coupling increases the power output for all analyzed cases. These curves suggest that nonlinear effects have considerable influence on the results. More details can be observed in Figure 8, where monostable and bistable results are shown together, comparing the effects of linear or nonlinear piezoelectric couplings. Note that the efficiency increases when compared with the linear constitutive model, being an essential advantage for energy harvesting purposes. In addition, the maximum value tends to shift to higher frequencies for bistable systems (Figure 8b), but this effect is barely visible in the monostable system (Figure 8a). Figure 9 presents a comparison between the input and output powers for monostable and bistable systems showing the effect piezoelectric couplings.

Different values of nonlinear piezoelectric couplings are now analyzed by considering the monostable case ($\beta = 1.0$, $\alpha = 0.4$). Figure 10a shows the efficiency versus excitation frequency varying ξ_2 and keeping ξ_1 constant and Figure 10b varying ξ_1 and keeping ξ_2 constant. By increasing the modulus of ξ_2, the response amplitude is reduced and the maximum occurs at the lower frequency (Figure 9a). The increase of ξ_1 induces the increase of efficiency in the same way (Figure 10b). Figure 11 shows the same analyses for a bistable system ($\beta = -1.2$, $\alpha = 0.3$). As ξ_2 increases, the maximum efficiency initially decreases and then increases, but when ξ_2 is equal 0.5, the efficiency almost vanishes (Figure 11a). The same behavior showed before have been found by increasing ξ_1 (Figure 11b).

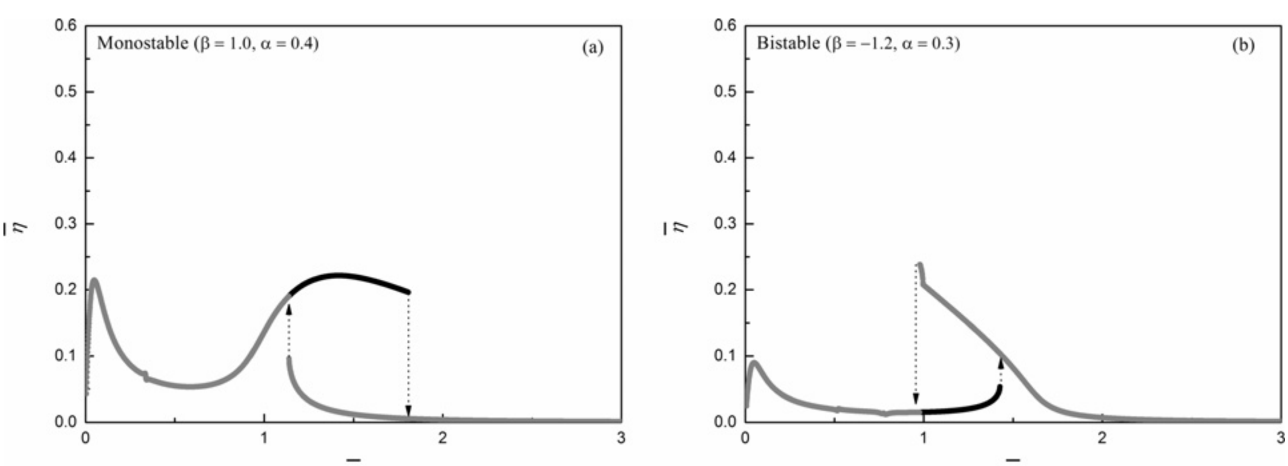

Figure 5: Dynamical jumps: (a) monostable system ($\beta = 1.0$, $\alpha = 0.4$); (b) bistable system ($\beta = -1.2$, $\alpha = 0.3$).

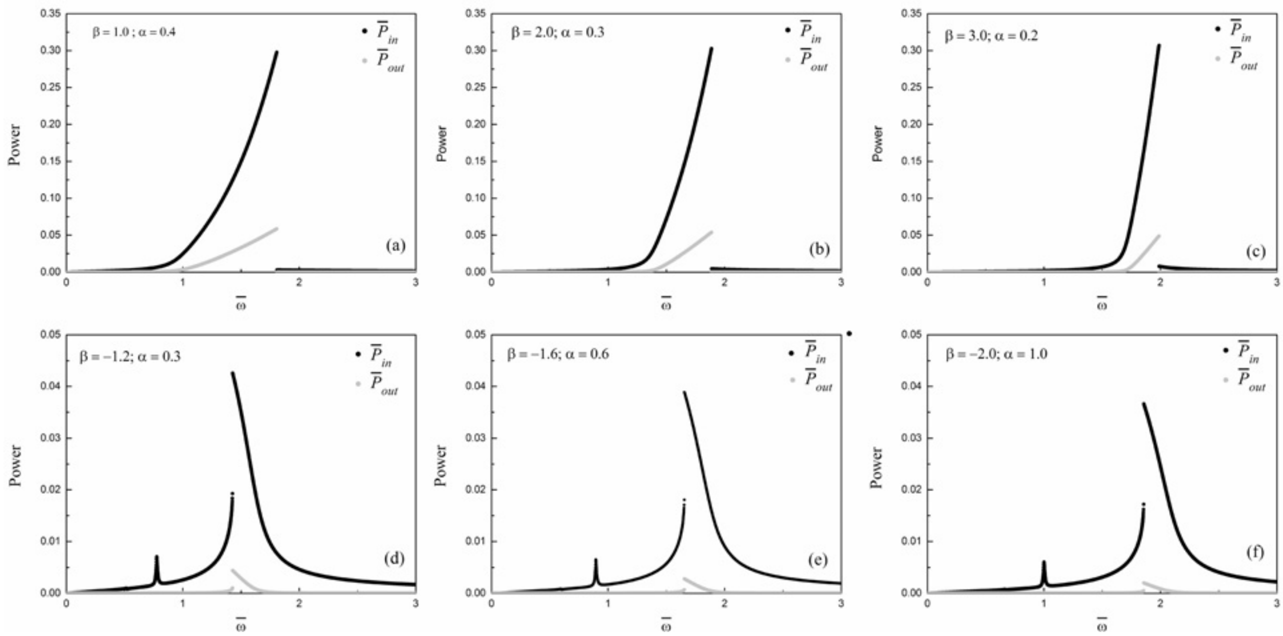

Figure 6: Input and output powers for different mechanical parameters: (a) $\beta = 1.0$, $\alpha = 0.4$; (b) $\beta = 2.0$, $\alpha = 0.3$; (c) $\beta = 3.0$, $\alpha = 0.2$; (d) $\beta = -1.2$, $\alpha = 0.3$; (e) $\beta = -1.6$, $\alpha = 0.6$ and (f) $\beta = -2.0$, $\alpha = 1.0$.

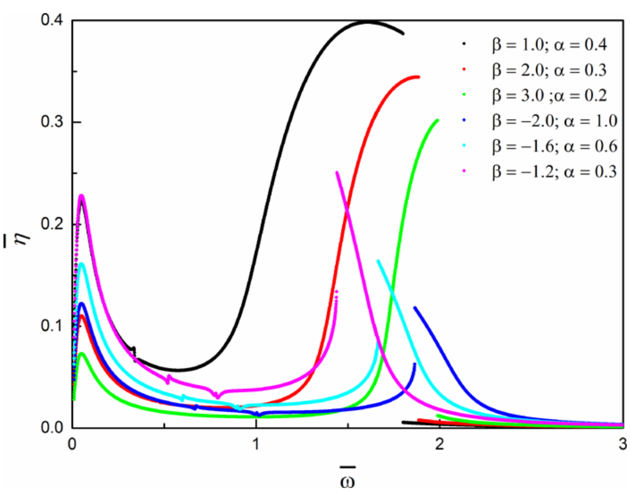

Figure 7: Efficiency versus excitation frequency for different mechanical properties and with nonlinear piezoelectric coupling ($\xi_1 = 0.5$ and $\xi_2 = -0.1$).

3.2 Influence of the excitation amplitude

Different excitation conditions are now investigated exploring the influence of the excitation amplitude. Basically, the system response is considered under a constant excitation frequency, but varying the excitation amplitude. Initially, a monostable system ($\beta = 1.0$, $\alpha = 0.4$) is treated considering linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$) with $\overline{\omega} = 1.5$, a frequency close to the resonance frequency. Figure 12 and Figure 13 present bifurcation diagrams considering instant, P, and average, P_{out}, powers for different values of the excitation amplitude. Basically, the bifurcation diagram is built considering a slow quasi-static variation of the parameter, discarding the transient response. Instant power is plotted using Poincaré section. Different procedures are employed to build the bifurcation diagrams. The first one, Figure 12, uses the same initial conditions for all parameters; on the other hand, the second procedure, Figure 13, adopts the previous parameter response as initial conditions. The difference between them suggests the existence of multiple stable solutions for the same set of parameters. Details of the system dynamics for some set of parameters, but with different initial conditions, are depicted in both Figures. They are essentially different, being associated with distinct levels of harvested power. Note that for $\gamma = 0.5$, it is possible to collect a maximum power of 0.062 for the first case and 0.0023 for the second. In terms of average power the difference goes from 0.029 to 0.001. Similar situation can be observed for $\gamma = 4.23$: 0.169 to 0.118 in terms of maximum power; or 0.053 to 0.052 in terms of average power. Figure 14 presents a comparison between the efficiency of the monostable system for both of the previous cases. Note that the variation on initial conditions can be related to different levels of efficiency, especially in regions related to dynamical jumps.

In the following, the influence of the piezoelectric coupling is considered. The system response is investigated

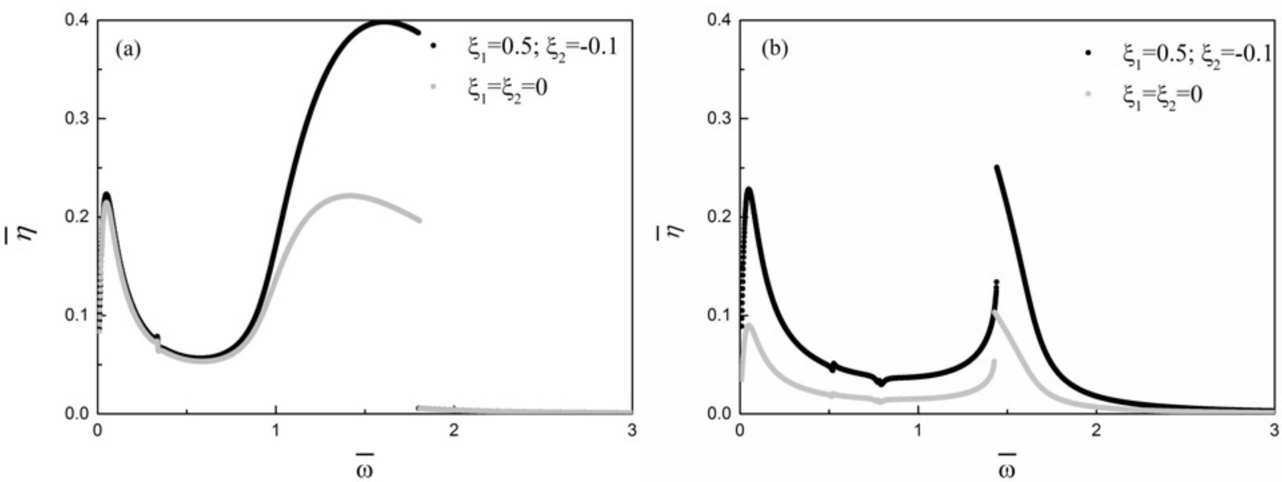

Figure 8: Efficiency versus excitation frequency comparing linear with nonlinear piezoelectric couplings: (a) monostable system ($\beta = 1.0$, $\alpha = 0.4$); (b) bistable system ($\beta = -1.2$, $\alpha = 0.3$).

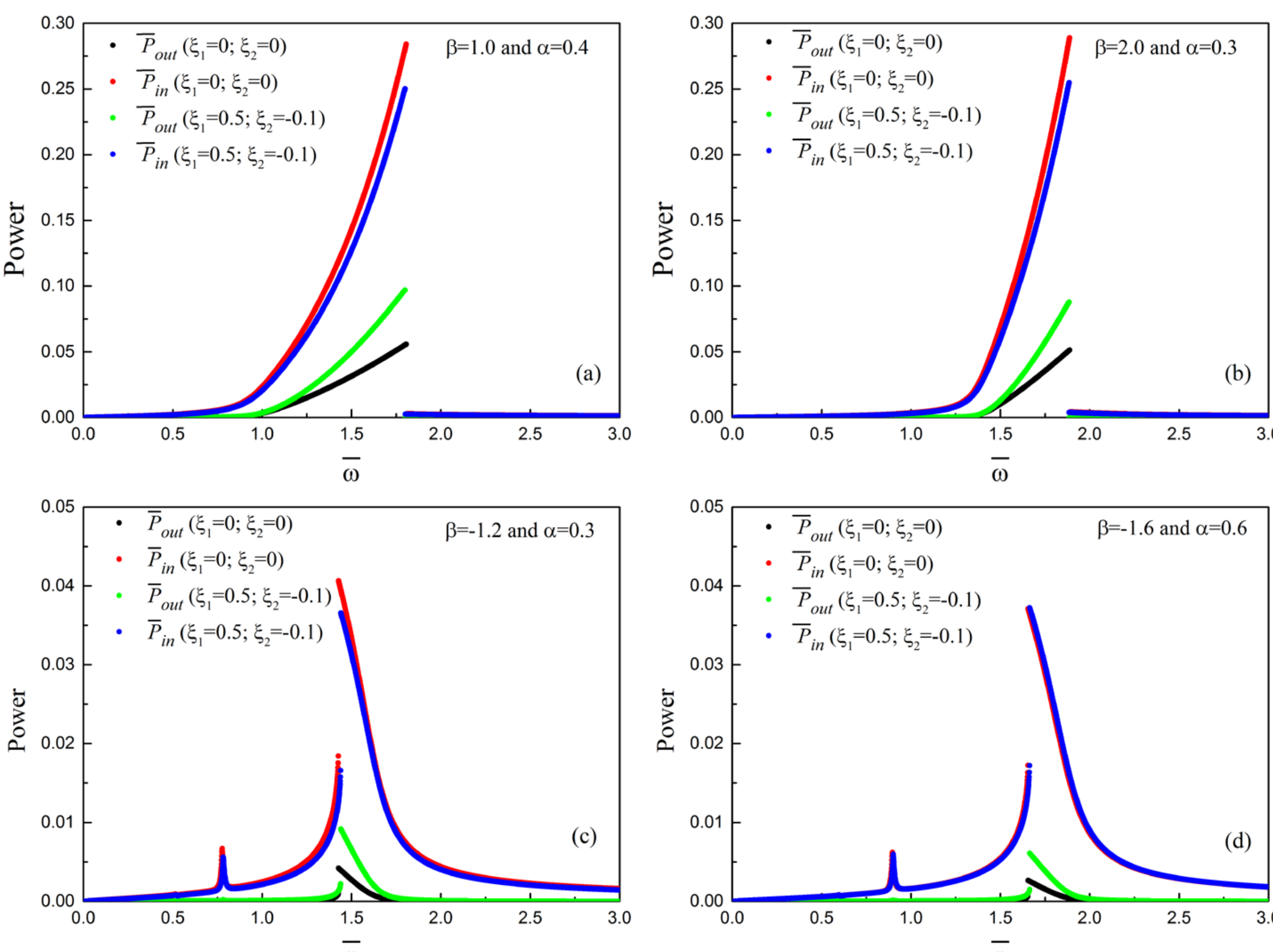

Figure 9: Input and output powers comparing linear and nonlinear piezoelectric couplings for monostable systems: (a) $\beta = 1.0$, $\alpha = 0.4$, (b) $\beta = 2.0$, $\alpha = 0.3$; and bistable systems: (c) $\beta = -1.2$, $\alpha = 0.3$ and (d) $\beta = -1.6$, $\alpha = 0.6$.

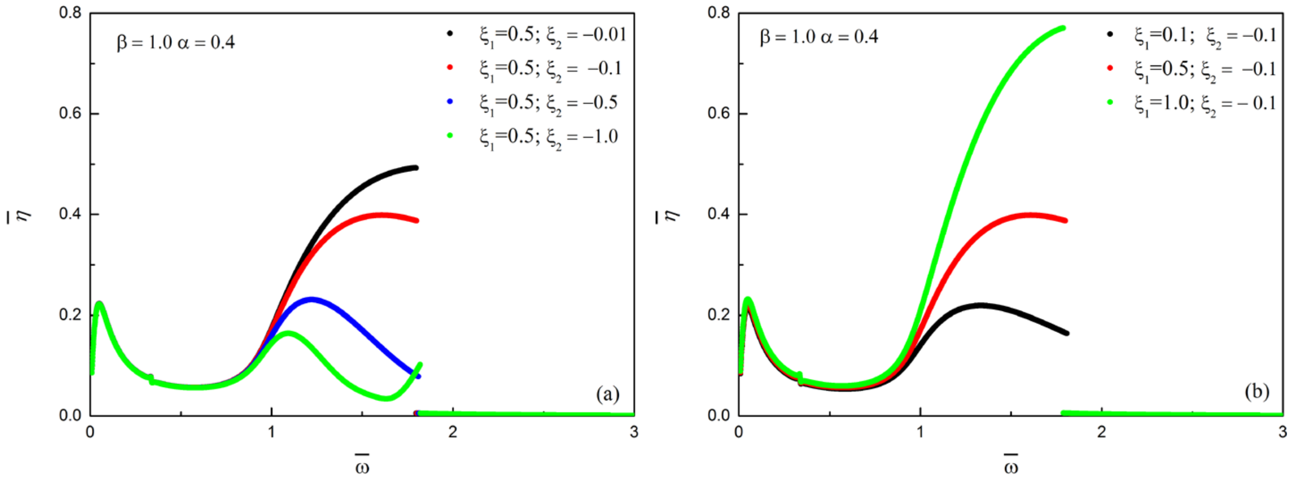

Figure 10: Efficiency versus excitation frequency for a monostable system ($\beta = 1.0$, $\alpha = 0.4$) and different types of nonlinear piezoelectric coupling: (a) varying ξ_2; (b) varying ξ_1.

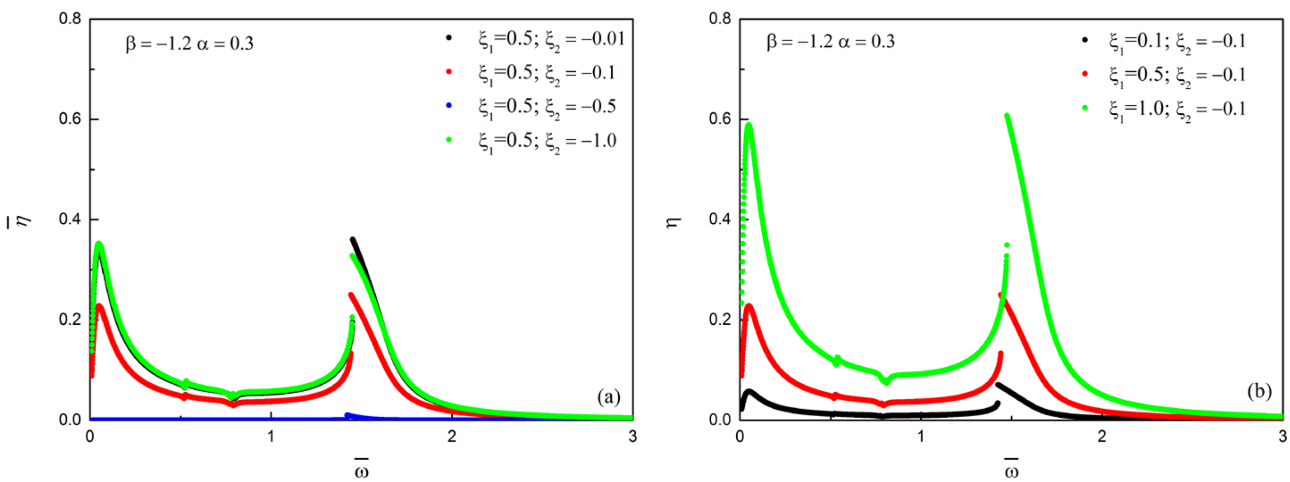

Figure 11: Efficiency versus excitation frequency for a bistable system ($\beta = -1.2$, $\alpha = 0.3$) and different types of nonlinear piezoelectric coupling: (a) varying ξ_2 and (b) varying ξ_1.

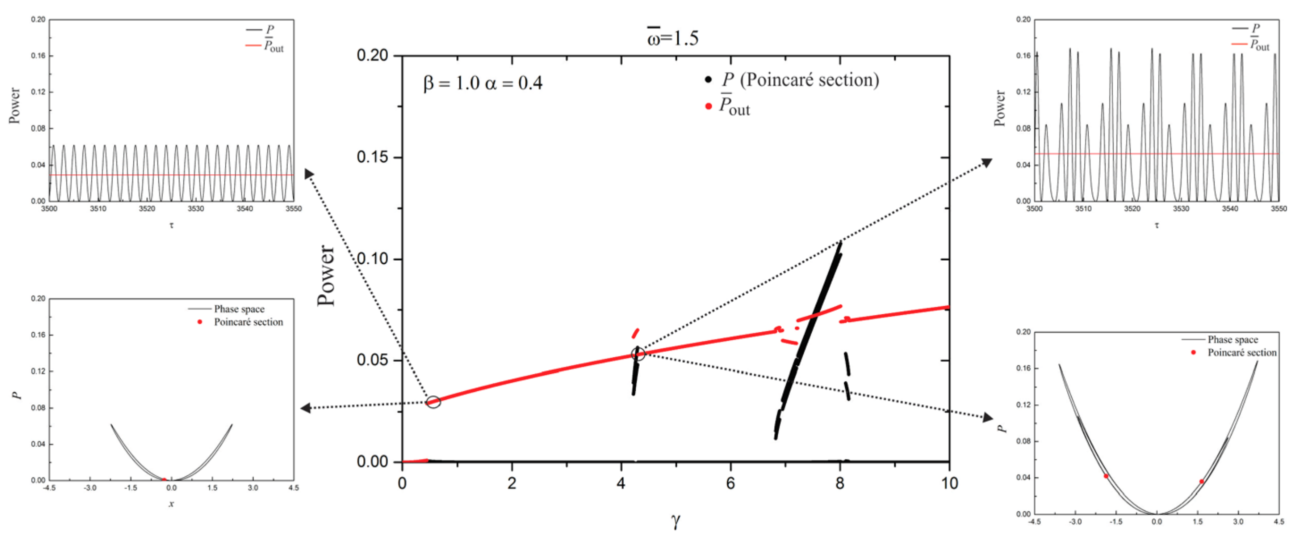

Figure 12: Power and average power considering different values of the excitation amplitude for a monostable system ($\beta = 1.0$, $\alpha = 0.4$) and linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$) using the same initial conditions for each parameter.

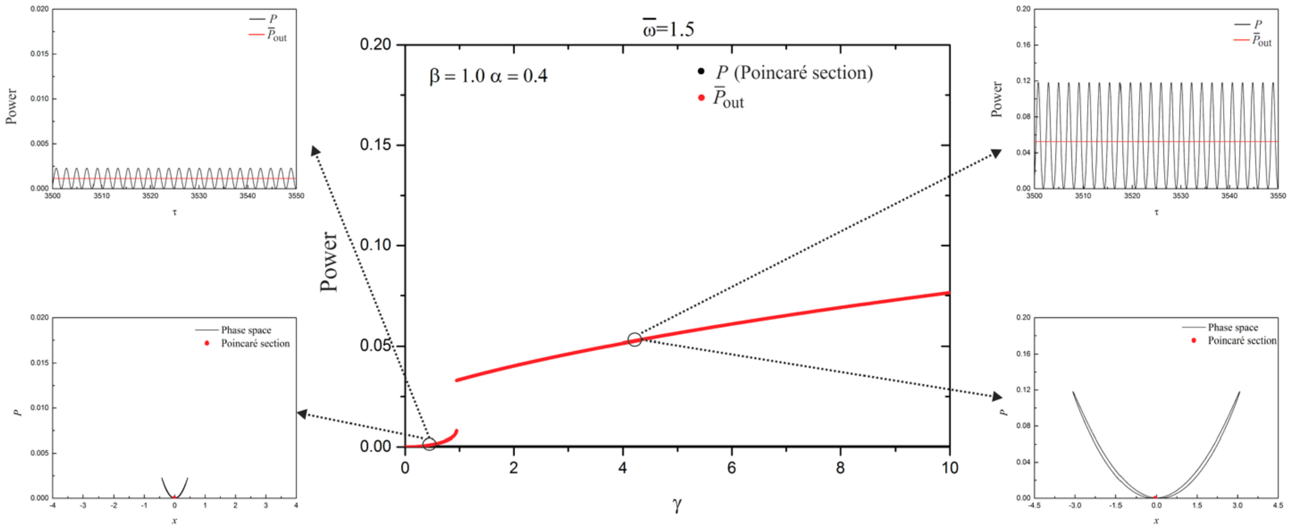

Figure 13: Power and average power considering different values of the excitation amplitude for a monostable system ($\beta = 1.0$, $\alpha = 0.4$) and linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$) using different initial conditions for each parameter.

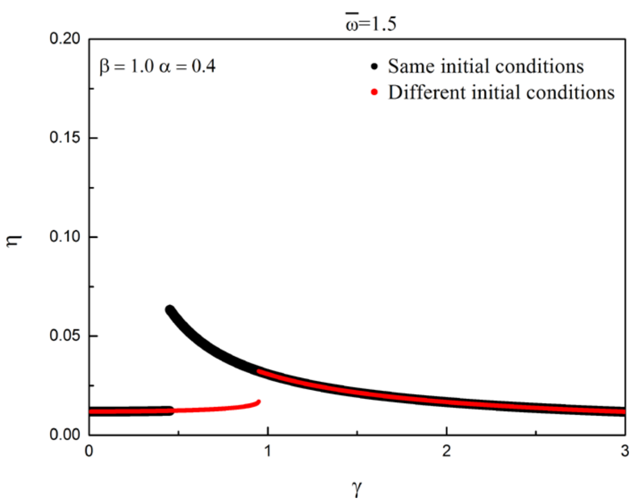

Figure 14: Efficiency versus excitation amplitude for a monostable system ($\beta = 1.0$, $\alpha = 0.4$) and linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$) using different initial conditions for each parameter (red line) and the same initial conditions for each parameter (black line).

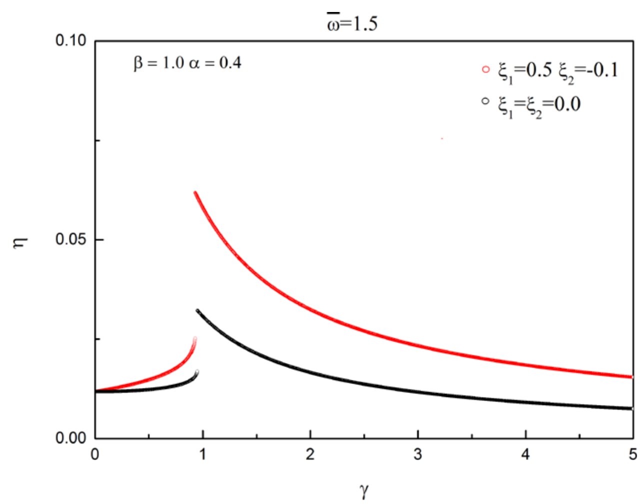

Figure 15: Efficiency versus excitation amplitude for a monostable system ($\beta = 1.0$, $\alpha = 0.4$) with linear ($\xi_1 = \xi_2 = 0$) and nonlinear ($\xi_1 \neq 0$ and $\xi_2 \neq 0$) piezoelectric coupling.

assuming linear or nonlinear piezoelectric coupling using $\overline{\omega} = 1.5$, which means for the same conditions as in the previous cases. Figure 15 presents bifurcation efficiency diagrams. Once again, nonlinear piezoelectric coupling tends to increase the efficiency. It is noticeable a generally low efficiency for the considered parameters and a jump close to $\gamma = 1.0$.

A bistable system ($\beta = -1.2$, $\alpha = 0.3$) is now investigated considering linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$) with $\overline{\omega} = 1.5$. Figure 16 and Figure 17 present bifurcation efficiency diagrams considering different procedures. The first one, Figure 16, uses the same initial conditions for all parameters; the second procedure, Figure 17, adopts the previous parameter response as initial condition. Once again, multistability of solutions is observed by the differences between both diagrams. Chaotic regions are observed in regions associated with cloud of points, being related to good performance in terms of generated power and, therefore, are of special interest in terms of energy harvesting. The average power values are more or less the same but the maximum power value varies as follows: 0.221 ($\gamma = 0.74$), 0.322 ($\gamma = 1.944$) and 0.503 ($\gamma = 4.966$) for

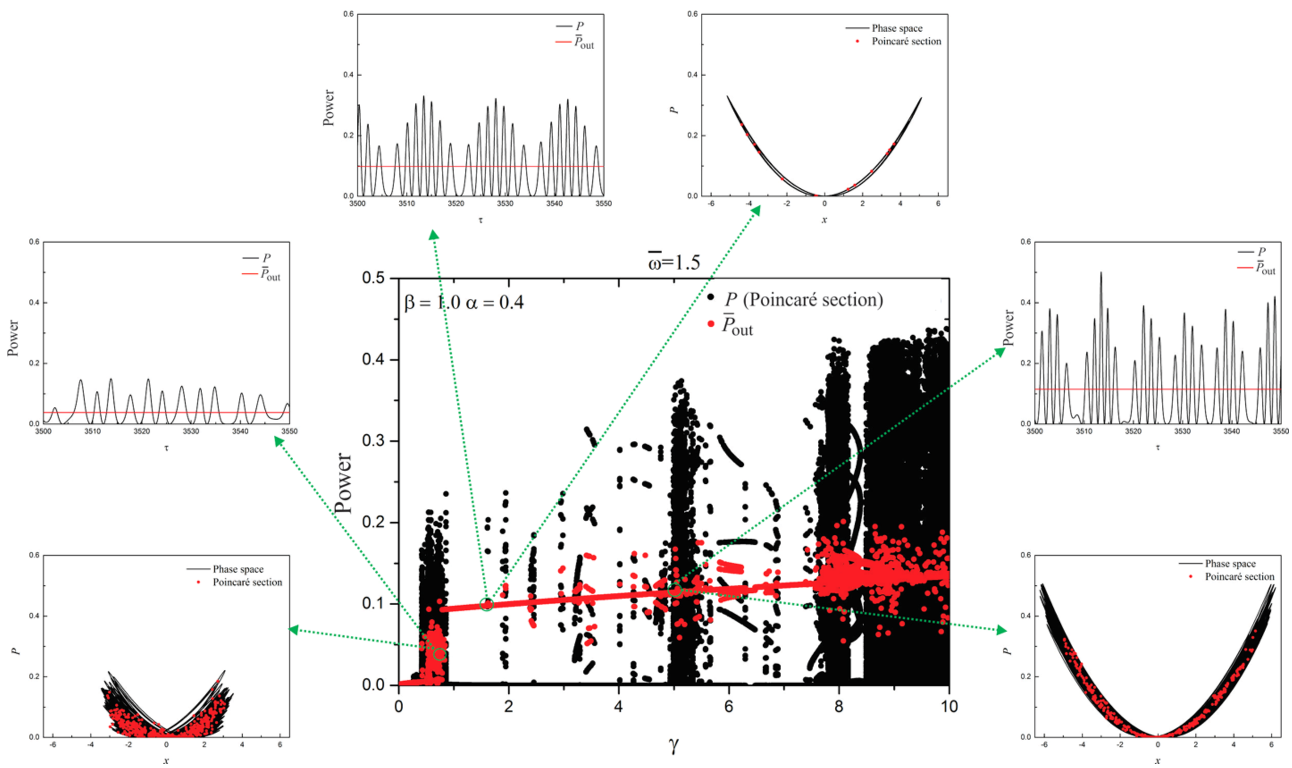

Figure 16: Power and average power considering different values of excitation amplitude for a bistable system ($\beta = -1.2$, $\alpha = 0.3$) with linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$) using the same initial conditions for each parameter.

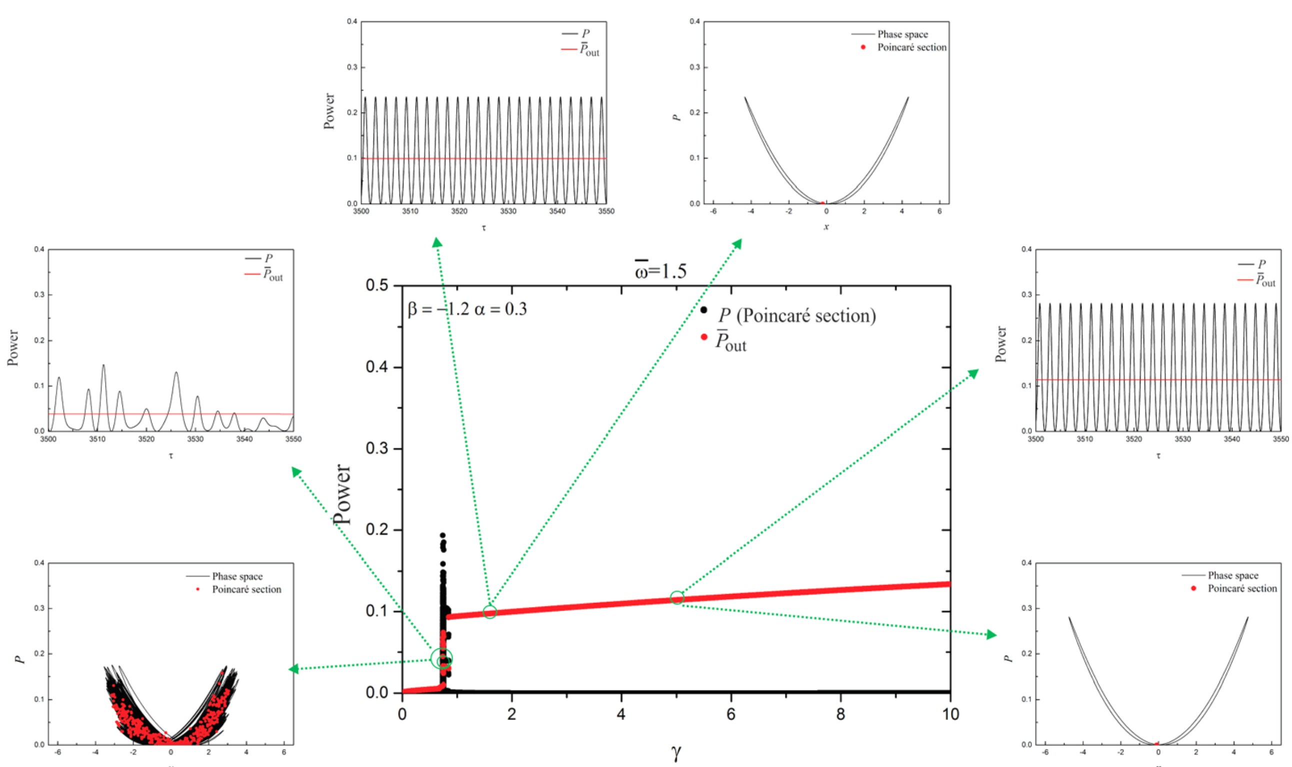

Figure 17: Power and average power considering different values of excitation amplitude for a bistable system ($\beta = -1.2$, $\alpha = 0.3$) with linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$) using different initial conditions for each parameter.

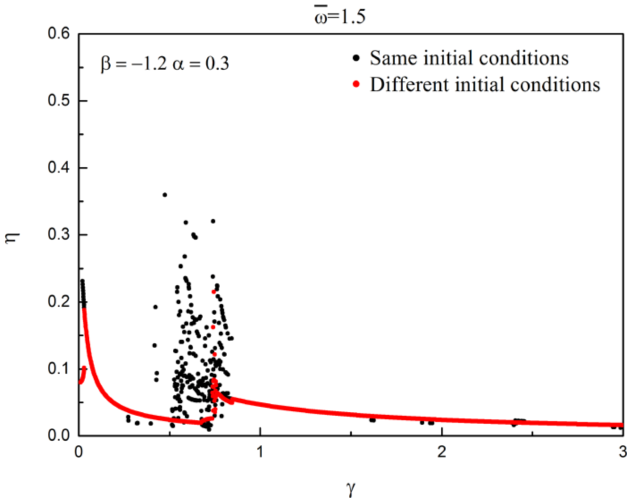

Figure 18: Efficiency versus excitation amplitude for a bistable system ($\beta = -1.2$, $\alpha = 0.3$) with linear piezoelectric coupling using different initial conditions for each parameter (red line) and the same initial conditions for each parameter (black line).

the first procedure to 0.173 ($\gamma = 0.74$), 0.235 ($\gamma = 1.944$), and 0.281 ($\gamma = 4.966$) for the second one.

Figure 18 establishes a comparison in terms of the efficiency of both cases showed in Figures 16 and 17. Due to multistable solutions, the set of initial conditions can dramatically change system dynamics. Figure 19 presents details of the system dynamics for some set of parameters with $\overline{\omega} = 1.5$ and different kinds of solutions. Note that periodic solutions that oscillate around one equilibrium point, around three equilibrium points (around the potential energy two-well), and chaos are highlighted.

Figure 20 presents bifurcation efficiency diagrams for comparing linear ($\xi_1 = \xi_2 = 0$) and nonlinear ($\xi_1 \neq 0$ and $\xi_2 \neq 0$) piezoelectric couplings. Three different values of excitation frequency are analyzed ($\overline{\omega} = 1.0, 1.5$ and 2.0). Under these conditions, it is possible to observe chaotic regions close to dynamical jumps. These regions are associated with good efficiency when compared with others.

In order to establish a better comparison among different kinds of solutions, a bistable system is treated with linear piezoelectric coupling ($\beta = -1.2$; $\alpha = 0.3$; $\xi_1 = \xi_2 = 0$). Three excitation frequencies are analyzed and efficiency diagrams are plotted together (Figure 21). Note that for $\gamma = 1.609$, the three systems have more or less the same efficiency but present distinct kinds of solutions, also depicted in Figure 21. Two periodic and a chaotic response can be seen. The power harvested associated with the periodic solutions are directly related to the response amplitude. Nevertheless, the power generated by chaotic response is different being not directly associated with amplitude.

A more detailed analysis related to the influence of the qualitative kind of response on energy harvesting

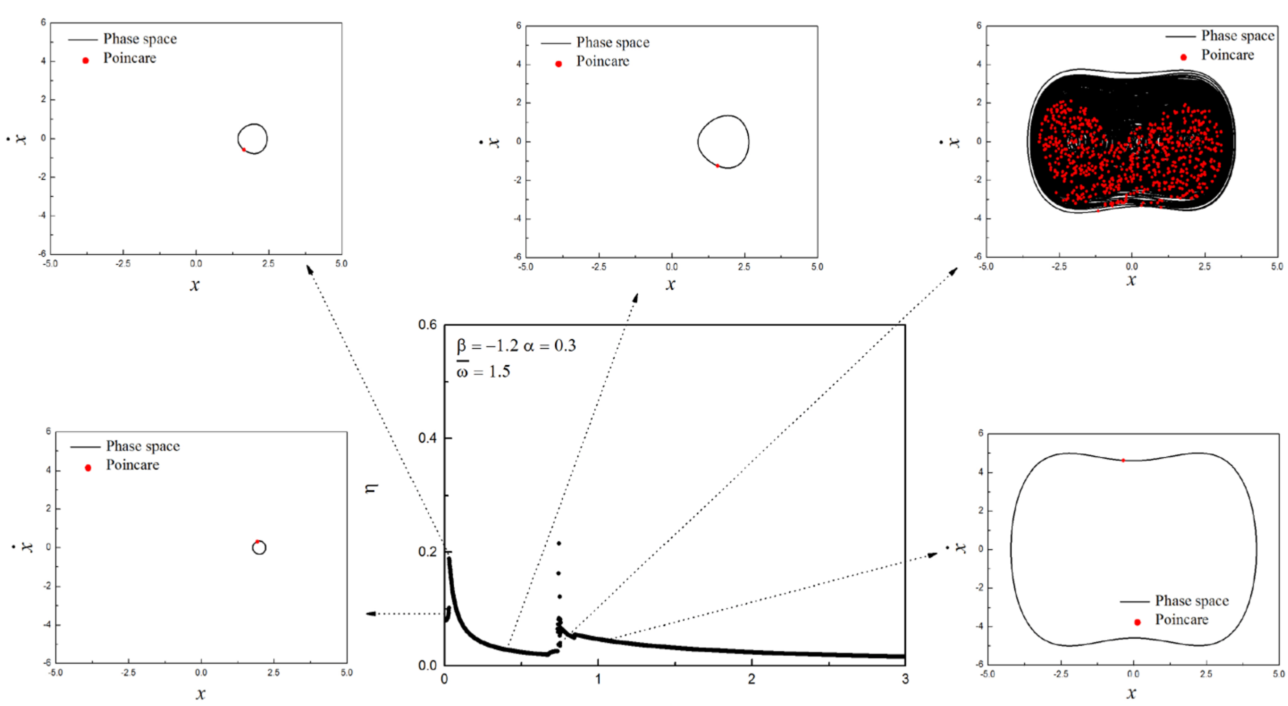

Figure 19: Details of the dynamics of a bistable system ($\beta = -1.2$; $\alpha = 0.3$; $\overline{\omega} = 1.5$; $\xi_1 = \xi_2 = 0$).

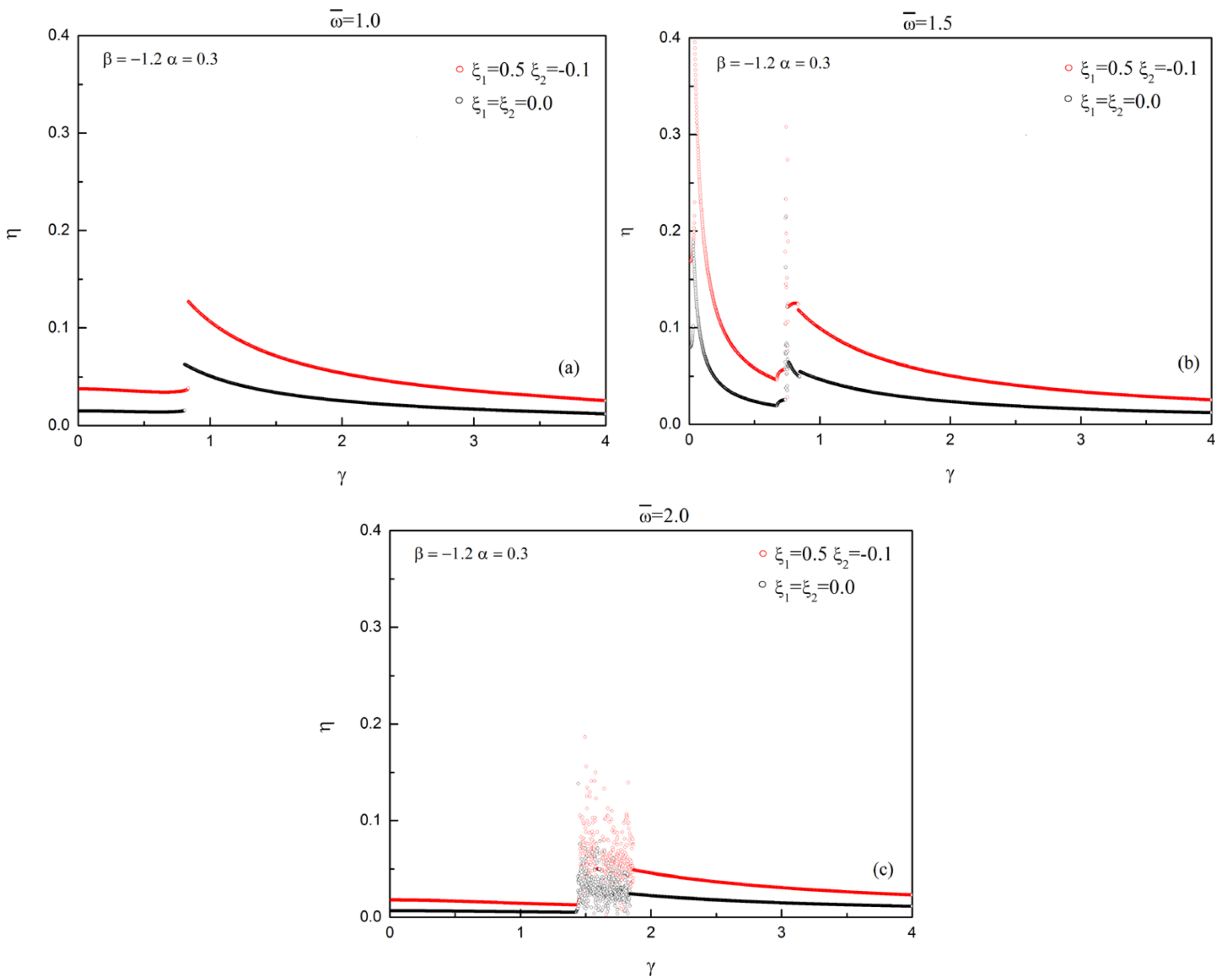

Figure 20: Efficiency for a bistable system ($\beta = -1.2$, $\alpha = 0.3$) with different values of the excitation amplitude considering linear ($\xi_1 = \xi_2 = 0$) and nonlinear ($\xi_1 \neq 0$ and $\xi_2 \neq 0$) piezoelectric couplings. (a) $\overline{\omega} = 1.0$, (b) $\overline{\omega} = 1.5$ and (c) $\overline{\omega} = 2.0$.

is treated comparing the power harvested of responses with similar efficiency, but qualitative distinct kinds of response. Figure 22 establishes a comparison between different qualitative behaviors, periodic and chaotic, and it shows that chaotic response generates more energy around the resonance frequency but the influence of the amplitude is important in this kind of response. Therefore, the conclusion about what kind of response is better in terms of energy harvesting performance is more complex than that.

4 Conclusions

This paper deals with a numerical investigation of the influence of nonlinear effects on piezoelectric vibration-based energy harvesting. The main goal is to use nonlinear effects as alternative to enhance energy harvesting capacity. Mechanical nonlinearity is treated by considering a Duffing-type oscillator analyzing either monostable (single-well potential) or bistable (double-well potential) systems. Piezoelectric electro-mechanical nonlinearity is treated by considering a quadratic constitutive equation. Numerical simulations are carried out exploring different kinds of dynamical behaviors and results are analyzed monitoring input power, generated power, and the efficiency defined from the relation between them. Nonlinearities are responsible for the rich response of energy harvesting systems, including chaotic and multistable solutions. In general, monostable systems present better performance under resonant conditions. On the other hand, bistable systems present more possibilities due to their richer behavior. The main point is the possibility to os-

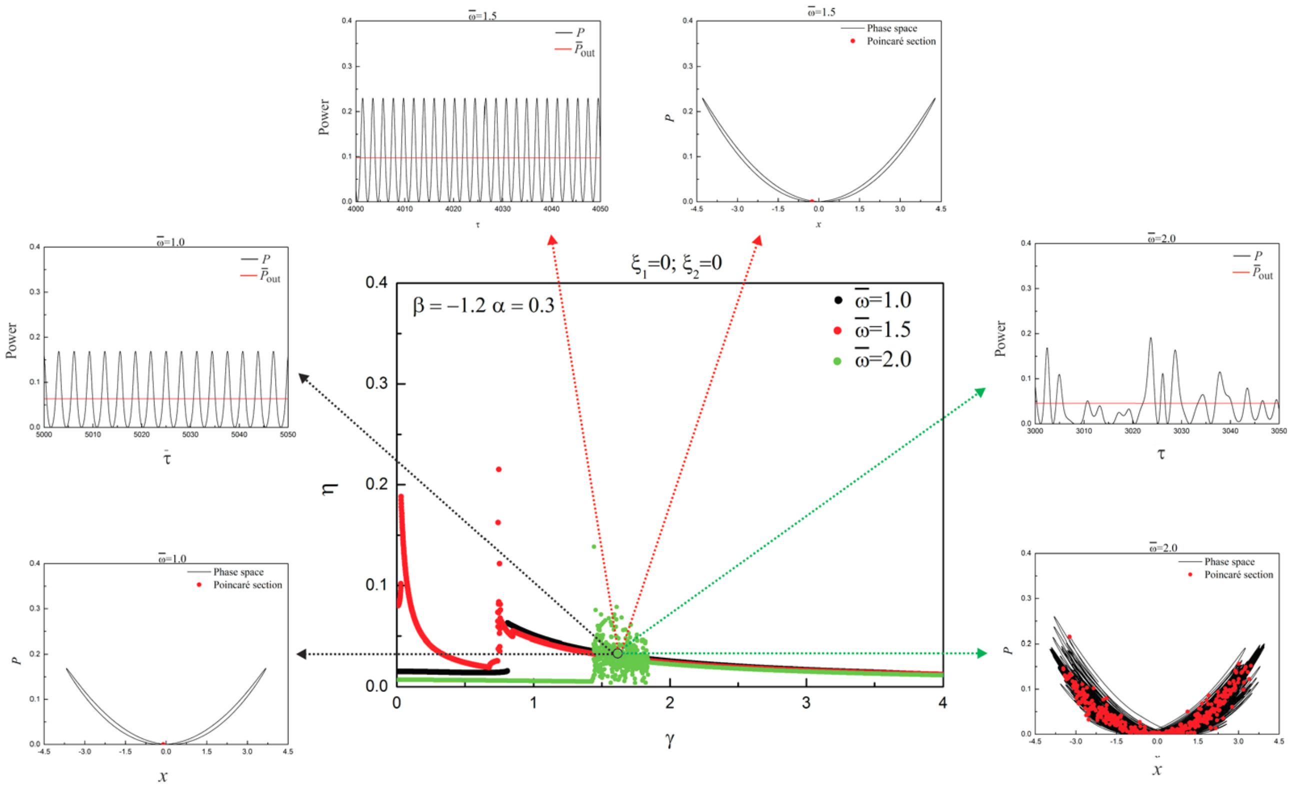

Figure 21: Details of the dynamics of a bistable system ($\beta = -1.2$; $\alpha = 0.3$; $\xi_1 = \xi_2 = 0$) for different excitation frequencies, $\overline{\omega} = 1.0$, 1.5 and 2.0.

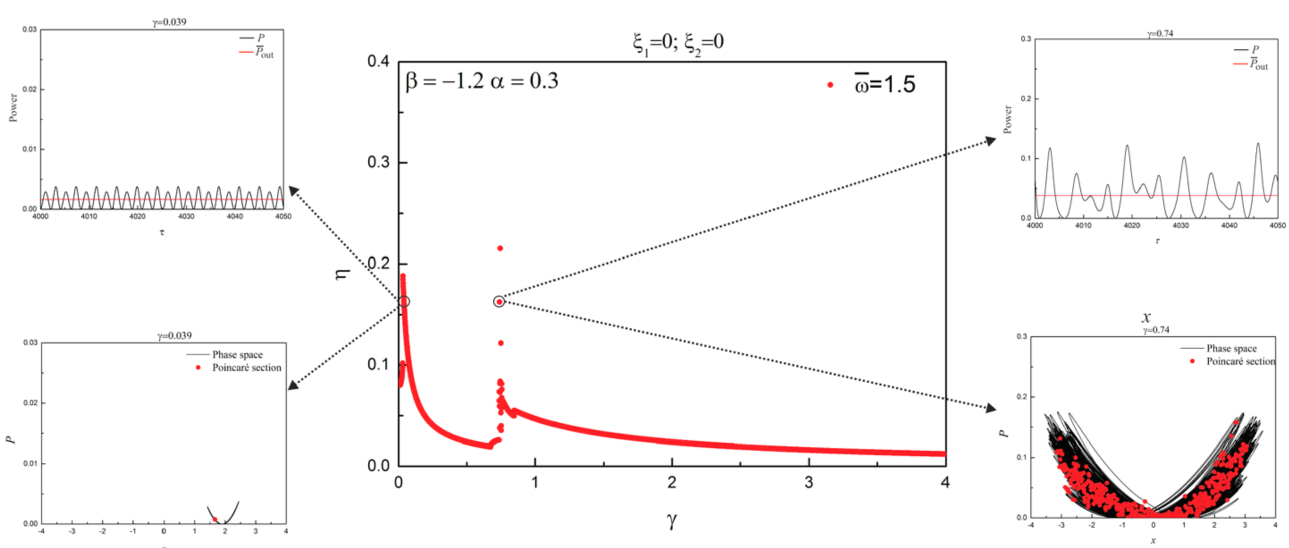

Figure 22: Comparison between different responses with similar efficiency for a bistable system with linear piezoelectric coupling ($\beta = -1.2$; $\alpha = 0.3$; $\xi_1 = \xi_2 = 0$; $\overline{\omega} = 1.5$).

cillate visiting the two potentials that increase oscillation amplitude and, as a consequence, power harvested. Piezoelectric nonlinearity has a significant influence on the system performance especially under resonant conditions and usually, enhances the power harvesting efficiency. Another important aspect about nonlinearity influence is related to the kind of response that can generate more energy. Results show that the comparison between periodic and chaotic responses needs to be evaluated together with response amplitude. In general, it is possible to conclude that nonlinear aspects introduce interesting alternatives for energy harvesting purposes either by enhancing energy harvested or increasing the general system capacity due to richer behaviors. Although this paper considers numerical investigations of the influence of nonlinear effects on energy harvesting system, the main conclusions are in qualitative agreement with experimental observations available in literature.

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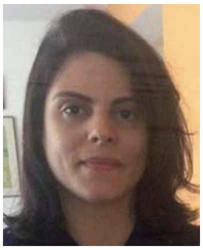

Ana Carolina Cellular
Universidade Federal do Rio de Janeiro, COPPE – Department of Mechanical Engineering, Center for Nonlinear Mechanics, 21.941.972, Rio de Janeiro, Brazil; P.O. Box 68.503
anacellular1@yahoo.com.br

Ana Carolina S. Cellular is Ph.D. in Mechanical Engineering and Mathematician. Nowadays she is teaching at Universidade Candido Mendes and Centro Universitário La Salle. Research interests are related to nonlinear dynamics and energy harvesting.

Luciana L. da Silva Monteiro
CEFET/RJ, Department of Mechanical Engineering, 20.271.110, Rio de Janeiro, Brazil
lucianals.monteiro@gmail.com

Luciana L. Silva Monteiro is Ph.D. in Metallurgical and Materials Science Engineering and Physicist. Professor at CEFET/RJ, she develops research and teaching activities. Research interests are related to nonlinear dynamics, energy harvesting, and smart materials and structures.

Marcelo A. Savi
Universidade Federal do Rio de Janeiro, COPPE – Department of Mechanical Engineering, Center for Nonlinear Mechanics, 21.941.972, Rio de Janeiro, Brazil; P.O. Box 68.503
savi@mecanica.ufrj.br

Marcelo A. Savi is Ph.D. in Mechanical Engineering and Professor at Federal University of Rio de Janeiro (COPPE – Department of Mechanical Engineering) where develops research and teaching activities, being the Head of the Center for Nonlinear Mechanics. Research interests are related to nonlinear mechanics where it should be highlighted smart material and structures; nonlinear dynamics, chaos and control; biomechanics and ecology.