Journal of Sound and Vibration 438 (2019) 220–237

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Drill-string vibration analysis considering an axial-torsional-lateral nonsmooth model

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A R T I C L E I N F O

Article history:
Received 23 June 2017
Received in revised form 11 May 2018
Accepted 27 August 2018
Available online 7 September 2018
Handling Editor: L.N. Virgin

Keywords:
Nonlinear dynamics
Drilling
Oil and gas
Nonsmooth models

A B S T R A C T

Oil drilling operations is usually associated with drill-string severe vibration conditions that lead to an onerous and inefficient process. In order to avoid or minimize the impact of vibrations on operation conditions it is essential a deep dynamical investigation that allows a proper understanding of system dynamics. Drill-string dynamics may be analyzed by considering different vibration modes: axial, torsional and lateral. The coupled analysis of these modes gives a proper comprehension of the system dynamics, elucidating several critical vibration responses. In general, lumped models present a proper description of the system dynamics. This paper deals with a coupled drill-string vibration considering a four-degree of freedom nonsmooth model that presents axial-torsional-lateral coupling. Bit-rock and wellbore interactions, eccentricity and hydrodynamic forces due to fluid resistance to lateral bending are the main system coupling aspects. A parametric study is carried out treating bit-bounce, stick-slip, whirl, and the combined effects. Numerical results present qualitative agreement with experimental field observations. Critical operational conditions are discussed especially those related to interaction among bit-bounce, stick-slip and whirl conditions.

1. Introduction

Deep-water oil exploration has several technological challenges and drill-string vibrations appear as a critical situation. In brief, drill-string vibrations can be analyzed from three modes: axial, torsional and lateral. The coupling between them is an essential system nonlinearity being associated with several causes. Axial-torsional coupling can be provided by drill-bit cutting process that establishes a combination of rotation and compression on the rock. Besides, compressive loads represent a source of drill-string flexion, characterizing another coupling mode. Unbalance excitations also contribute to torsional-lateral coupling, producing effects that are similar to rotor dynamic systems. Contact phenomenon is a nonsmooth source of nonlinearity that can occur either due to the bit-rock interaction or between the wellbore and the drill-string. Refs. [1,2] presented a general perspective about the vibration of oil and gas drilling.

Critical vibrations deserve special attention due to its dangerous influence on drill-string dynamics. Among the most common types it should be pointed out: bit-bounce, associated with drill-bit contact lost; stick-slip, related to drill-string

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https://doi.org/10.1016/j.jsv.2018.08.054

rotating stops; whirl, characterized by a rotational response of a deflected drill-string around the well center. Each one of these modes is respectively associated with axial, torsional and lateral vibration modes, and although they may occur isolated, it is common to have them together.

In this regard, drill-string vibration is a complex phenomenon that motivates different approaches for its analysis. Lumped parameter systems have been proving to be effective in order to describe drill-string dynamics. In general, they provide low computational cost results qualitatively coherent with field observations. Besides that, drill-string systems have strong nonlinearities that are associated with complex responses. Nonsmoothness seems to be the essential nonlinearity of drill-string dynamics. Discontinuous contact and friction are typical sources of nonsmooth nonlinearities being usually related to complicated description. Refs. [3–6] presented details about nonsmooth models, their numerical procedures, and their nonlinear dynamics. Divenyi et al. [17,27] and [7] studied nonsmooth dynamics of a single-degree of freedom oscillator with discontinuous support. Grazing bifurcation was identified, which can turn a periodic response into a chaotic one due to tinny system parameter change. Andreaus and Casini [8] investigated contact-impact problems between interacting rigid bodies. Refs. [9,10] presented other investigations related to single-degree of freedom nonsmooth systems.

Christoforou and Yigit [11] proposed a lumped parameter model that became classical for the description of axial-torsional-lateral (3-mode) drill-string vibrations. The investigation also considered an active control system showing that feedback control was effective in suppressing stick-slip vibrations. A comparative analysis between 3-mode (axial-torsion-lateral) and 2-mode coupled models (neglecting the torsional mode) was conducted by Ref. [12]. Conclusions pointed out that the 3-mode coupled model presented less lateral impacts than the 2-mode model. Since there is no torsional mode, there is no dissipation during the contact with borehole wall, and therefore, the drill-string is still in critical rotation providing more impacts. Based on that, the full-coupled model showed to be more interesting for a proper description of drill-string vibrations.

The mode couplings constitute the essential point to obtain a proper description of drill-string vibrations. The analysis of bit-rock interaction and cutting process are some of these points. Usually, bit-rock interaction is represented by an equivalent friction law [37-39,42]. Phenomenological models can also be proposed being usually based on kinematically controlled single cutter tests. In essence, this test is carried out with a constant speed cutting tool advance, maintaining a constant cutting depth. Time-delayed approach is usually employed in order to generalize the depth of cut of a single cutter to a bit with multiple blades. This is based on the fact that the depth of cut is dependent on the previous revolution. Detournay & Defourny 40 and Detournay et al. 41] presented discussions about phenomenological models for bit-rock interaction. A comparison between two different bit-rock interaction theories is carried out on Ref. [13] for axial-torsional vibrations.

Wiercigroch et al. [14] discussed relations among depth of cut, force cut and regenerative effects due to time delay dependence. Experimental and modeling efforts showed that the hypothesis of uniform angular distribution of blades on drag drill-bits leads to regenerative types of instabilities that does not correspond to real drag drill-bit behavior. A non-uniform distribution was considered on the model and the state-dependent nature of the time delay and number of blades was significantly modified. Pyalchenkov et al. [15] presented similar investigation for triconic drill-bits. Experimental study pointed that the axial load has an irregular distribution while the load is expected a regular distribution along the teeth rows.

Kapitaniak et al. [16] treated an experimental rig with real drill-bits and rock samples proposing an equivalent friction model accounting the cutting and friction contact during bit-rock interactions. Torsional vibration of helically buckled drill-strings was of concern. The data were used to calibrate a finite element model employed for the three modes of vibration. Numerical and experimental results presented good agreement allowing important conclusions related to drill-string vibrations.

A deep investigation of drill-string vibrations points to nonlinear dynamics perspective analysis. In this regard, nonlinear tools are employed for the better understanding of system dynamics. Leine [4] employed bifurcation theory to analyze a coupled torsional-lateral model. Comparison with experimental field data showed that a combination of stick-slip and whirl is, if not impossible, at least very difficult to happen. On the other hand, Divenyi et al. [36] presented a nonlinear analysis of the axial-torsional coupling dynamics. A parametric analysis based on bifurcations showed to be effective to analyze a wide range of operational conditions. Liao et al. [31] presented another parametric analysis of drill-string motions..

Gupta and Wahi [18] investigated stability considering a lumped parameter axial-torsional model. Results showed that it is possible to have chaotic solutions co-existing with period-1 and period-2 solutions for certain parameters. Numerical simulations were carried out to show this kind of behavior. Bakhtiari-Nejadz and Hosseinzadeh [19] investigated drill-string stability showing the effect of proportional damping coefficients and the intrinsic energy of the rock on the stable drilling zones. Basically, the rock intrinsic energy may be considered as a key parameter to the bit-rock interaction model proposed by Detournay et al. (2008). Results showed that the minimum value of rotation for a stable drilling is approximately independent of the system damping. The increasing of intrinsic energy moved the boundary stable area downward and maximum allowable WOB were reduced. Finally, another important conclusion is related to the simplification assumption of constant axial velocity for the top of drill-string showing that this kind of hypothesis may affect the stability analysis.

In many drilling operations, the drill-string has to vary its inclination tending sometimes to a horizontal configuration. Therefore, modeling of non-vertical well is an important subject related to drill-string vibration and the use of reduced-order models seems to be also effective. Under the same operational parameters, Liu & Gao [20] investigated the influence of different well inclination. For this, a four degree-of-freedom model was proposed accounting for the lateral and torsional modes. It was observed that horizontal well has a characteristic that the fluid mass contributes for the increase of lateral contact that can make stick-slip a major problem. The stick-slip gradually turns to a pure rotational motion confined by a borehole wall when the drill-string becomes closer to vertical position. The greater the inclination the more susceptible to stick-slip and wear the drill-string becomes.

Due to the problem presented by vertical drilling and susceptibility of horizontal drilling to lateral contacts with borehole wall, Tian et al. [21] proposed an antifriction oscillator. Basically, the device uses high-pressure mud and variation of the flow area to produce a periodic axial force. The antifriction oscillator is able to change the friction status of the drill-string and the borehole wall. A review of the evaluation, control and technologies for drill-string vibrations and shock is found on Ref. [22].

Uncertainty analysis of drill-string vibration is another subject that has been investigated regarding the drilling process. Ritto et al. [23] considered the uncertainty of friction force interaction on horizontal drill-string system. Results show that there is a limit frictional force for which the axial speed of the drill-string increases significantly.

The goal of this work is to present a dynamical investigation of axial-torsional-lateral drill-string system. The idea is to investigate details of the system dynamics giving special attention to the mode interactions, critical vibrations and possible ways to mitigate them. A nonsmooth four-degree of freedom system is employed based on the original model of Ref. [11]. A parametric study identifying bit-bounce, stick-slip and whirl, and the interaction among them is carried out under different operational conditions. Bifurcation analysis is performed identifying the critical vibration responses. Numerical results showed to be qualitatively coherent with experimental field observations confirming the main conclusions. The parametric analysis allows one to imagine mitigation strategies for critical vibrations and some strategies are discussed under the light of these analyses.

2. Mathematical model

Mathematical description of the drill-string vibration is proposed assuming oscillators for each one of the vibration modes (axial, torsional and lateral) that are coupled by different phenomena. The following assumptions are adopted: the lateral motion is restrict to the lower part of the drill-string (drill-collars); drill-collars is the only part under compression being susceptible to deflexion [1]; the upper part of the drill-string (drill-pipes) is subjected to torsional vibrations and drill-collars can be assumed rigid in terms of torsion [11].

Fig. 1 presents a schematic picture of the drill-string oscillators. The origin of the axial displacement (z) is at the seabed being positive to higher depths. Concerning the lateral mode, the well center is the origin of the coordinate system and the lateral displacement (r) is positive towards the borehole wall. The driving forces for cutting process are the table rotation ($\dot{\varnothing}_{rt}$) and the compressive load, represented by weight on bit (W_{ob}). Drill-string rotation ($\dot{\varnothing}$) may differ significantly to the rotation source ($\dot{\varnothing}_{rt}$) due to resistive torques associated with friction and contact forces during drilling.

Equations of motion of the drill-string dynamics are presented in the sequence [11]. It is important to highlight that lateral motion needs two degrees of freedom to be characterized: radial (r) and angular (θ) displacements. The radial motion is related to flexure itself and the angular displacement is associated with the rotation around the well center.

$$m_l\left(\ddot{r} - r\dot{\theta}^2\right) + c_h|V|\dot{r} + k_r r = m_l e_o\left[\dot{\varnothing}^2\cos(\varnothing - \theta) + \ddot{\varnothing}\sin(\varnothing - \theta)\right] - F_{cr} \tag{1}$$

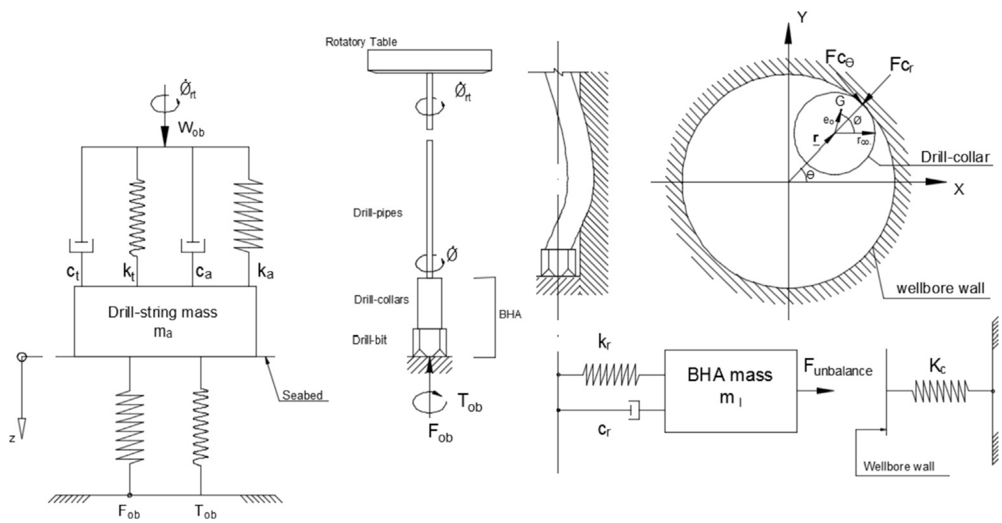

Fig. 1. Drill-string model.

$$m_l\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)+c_h|V|r\dot{\theta}=m_l e_o\left[\dot{\varnothing}^2\sin(\varnothing-\theta)+\ddot{\varnothing}\cos(\varnothing-\theta)\right]-F_{c\theta} \tag{2}$$

$$m_a\ddot{z}+c_a\dot{z}+k_a z=W_{ob}-F_{ob} \tag{3}$$

$$J\ddot{\varnothing}+c_t\dot{\varnothing}+k_t(\varnothing-\varnothing_{rt})+c_h|V|\dot{r}e_o\sin(\varnothing-\theta)-c_h|V|r\dot{\theta}e_o\cos(\varnothing-\theta)=-T_{ob}-F_{cr}e_o\sin(\varnothing-\theta)+F_{c\theta}\left(\frac{De_{co}}{2}-e_o\cos(\varnothing-\theta)\right) \tag{4}$$

The stiffness of axial, torsional and lateral modes are respectively represented by k_a, k_t and k_r. The dissipation associated with lateral mode is represented by a hydrodynamic coefficient c_h and V is the velocity of the drill-string geometric center; c_a is the axial dissipation while c_t is the rotational dissipation. The system has an eccentricity, e_o.

Bit-rock interaction provides the axial-torsional coupling. Refs. [4,11,17] discussed details about forcing terms, F_{ob} and T_{ob}, and the following nonsmooth equations can be adopted to describe nonsmooth system behavior:

$$F_{ob}=\begin{cases} k_c[z-s_o\sin(n_b\varnothing)], & \text{if} \quad z<s_o\sin(n_b\varnothing) \\ 0, & \text{if} \quad z>s_o\sin(n_b\varnothing)\end{cases} \tag{5}$$

$$T_{ob}=\begin{cases} F_{ob}\left[\frac{2}{3}r_w f(\dot{\varnothing})+\zeta\sqrt{\frac{D_w}{2}\delta_c}\right], \quad \text{if} \quad z<s_o\sin(n_b\varnothing)\ 0, \quad \text{if} \quad z>s_o\sin(n_b\varnothing)\end{cases} \tag{6}$$

where s_o is a forcing parameter related to drilling irregularity and n_b is a drill-bit factor, which varies with the type used. Usually, $n_b=3$ is used to represent tri-cone drill-bits and $n_b=1$ is used for PDC bits. Note that the torque on bit is a sum of the friction loss and cutting components, being represented by a friction smoothened function [17],

$$f(\dot{\varphi})=\frac{2}{\pi}\text{atan}(\varepsilon\dot{\varphi})\left(\frac{\mu_e-\mu_d}{1+\tau|\dot{\varphi}|}+\mu_d\right) \tag{7}$$

where ε and τ are coefficients; μ_e and μ_d are the static and dynamic friction coefficients. The cutting contribution is based on an empirical relationship where ζ is a dimensionless factor related to the force necessary to cut the rock and δ_c is the average cutting depth given by

$$\delta_c=\frac{2\pi ROP}{\dot{\varphi}_{rt}} \tag{8}$$

here, the average rate of penetration (ROP) is defined by an empirical function dependent on the weight on bit (W_{ob}) and rotatory table rotation ($\dot{\varphi}_{rt}$) as follows.

$$ROP=c_1 W_{ob}\sqrt{\dot{\varphi}_{rt}}+c_2 \tag{9}$$

where c_1 and c_2 are dimensionless empirical factors.

Wellbore contact is represented by two nonsmooth forces, radial (F_{cr}) and tangential ($F_{c\theta}$), being defined as follows:

$$F_{cr}=\begin{cases} k_c\left(r+\frac{De_{co}-D_w}{2}\right), & \text{if} \quad r+\frac{De_{co}}{2}>\frac{D_w}{2} \\ 0, & \text{if} \quad r+\frac{De_{co}}{2}<\frac{D_w}{2}\end{cases} \tag{10}$$

$$F_{c\theta}=\begin{cases} \mu F_{cr}=f(V_{rel})F_{cr}, & \text{if} \quad r+\frac{De_{co}}{2}>\frac{D_w}{2} \\ 0, & \text{if} \quad r+\frac{De_{co}}{2}<\frac{D_w}{2}\end{cases} \tag{11}$$

where k_c is the formation stiffness, De_{co} and D_w are respectively the outer drill collar diameter and wellbore diameters. Lateral contact is described by an adaptation of the axial-torsional contact proposed by Ref. [17]; Eq. (7), being expressed as follows

$$f(V_{rel})=\frac{2}{\pi}\text{atan}(\varepsilon V_{rel})\left(\frac{\mu_e-\mu_d}{1+\tau|V_{rel}|}+\mu_d\right) \tag{12}$$

Here, V_{rel} is the relative velocity during the wellbore contact, being a sum of two tangential velocities components: one related to the whirling motion ($\dot{\theta}r$) and the other one associated with the rotation of the drill-string ($\dot{\varnothing}r_{co}$),

$$V_{rel} = \dot{\theta}r + \dot{\varnothing}r_{co} \tag{13}$$

2.1. Parameter definitions

Governing equations present parameters that need to be calculated from well-bore characteristics. In this regard, it is important to define the main definitions [11]. The axial mass, m_a, is given by a summation of the BHA mass, m_{BHA}, the fluid added mass, m_{ad}, and the equivalent drill-pipes mass, m_{dp}.

$$m_a = \frac{\rho\pi\left(De_{co}^2 - Di_{co}^2\right)l_{BHA}}{4} + \frac{\rho_{fl}\pi\left(Di_{co}^2 + C_{am}De_{co}^2\right)l_{BHA}}{4} + \frac{\rho\pi\left(de_{dp}^2 - di_{dp}^2\right)l_{dp}}{12} \tag{14}$$

where ρ is the steel density, De_{co} and Di_{co} are the drill collars external and internal diameters and l_{BHA} is the BHA length. C_{am} is the added mass coefficient and ρ_{fl} is the drilling fluid density. Finally, for the drill-pipe mass parameter, de_{dp} and di_{dp} are the drill pipes internal and external diameters and l_{dp} is the drill-pipe length. For the sake of simplicity, the BHA is composed only by drill-collars.

Lateral vibration is restricted to the lowest part of the drill-string, the BHA [1]. Therefore, the lateral mass, m_l, is defined by a summation of the BHA mass, m_{BHA}, and the added fluid mass, m_{ad}.

$$m_l = \frac{\rho\pi\left(De_{co}^2 - Di_{co}^2\right)l_{BHA}}{4} + \frac{\rho_{fl}\pi\left(Di_{co}^2 + C_{am}De_{co}^2\right)l_{BHA}}{4} \tag{15}$$

The torsional inertia, J, is given by the sum of the BHA inertia and the equivalent drill-pipe inertia.

$$J = \frac{\rho\pi\left(De_{co}^4 - Di_{co}^4\right)l_{BHA}}{32} + \frac{1}{3}\frac{\rho\pi\left(de_{dp}^4 - di_{dp}^4\right)l_{dp}}{32} \tag{16}$$

Stiffness of the axial, torsional and lateral modes are respectively estimated by the following equations,

$$k_a = \frac{E\pi\left(de_{dp}^2 - di_{dp}^2\right)}{4l_{dp}} \tag{17}$$

$$k_t = \frac{G\pi\left(de_{dp}^4 - di_{dp}^4\right)}{32l_{dp}} \tag{18}$$

$$k_r = \frac{EI\pi^4}{2l_{BHA}^2} - \frac{T_{ob}\pi^3}{2l_{BHA}^3} - \frac{F_{ob}\pi^2}{2l_{BHA}} \tag{19}$$

where E and G are the Young's and shear moduli; I is the moment of inertia, given by

$$I = \frac{\pi\left(De_{co}^4 - Di_{co}^4\right)}{64} \tag{20}$$

Note that stiffness k_r depends on the torque on bit and force on bit, T_{ob} and F_{ob}, representing stiffness changes due to torsion and compression.

The dissipation due to the fluid interaction is given by c_h and c_ν.

$$c_h = \frac{2}{3\pi}\left(\rho_{fl}C_d De_{co} l_{BHA}\right) \tag{21}$$

$$c_t = \frac{\pi\mu_{fl} l_{BHA} De_{co}^3}{2(D_w - De_{co})} \tag{22}$$

where C_d is the drag coefficient and μ_{fl} is the fluid friction coefficient.

Table 1
Model parameters.

E	210	GPa	c_2	-1.9×10^{-4}	
G	78	GPa	μ_{fl}	0.2	N s/m^2
ρ	7850	Kg/m^3	μ_e	0.35	
ρ_{fl}	1500	Kg/m^3	μ_d	0.3	
e_o	0.5	pol	c_a	4000	N s/m
c_1	1.35×10^{-8}		C_{am}	1.7	
ζ	0.1		C_d	1.0	
k_c	25,000	kN/m	s_o	0.001	m

3. Typical responses

This section presents numerical simulations performed by employing the fourth order Runge-Kutta method with adaptive time steps in order to deal with nonsmooth nonlinearities. Results are presented in the form of phase spaces. A normal operating situation is initially presented being followed by cases in which critical vibrations are developed. Especial attention is dedicated to the combination of critical vibrations. In this regard, three scenarios are discussed: normal operation conditions; bit-bounce and stick-slip combination; bit-bounce and whirl combination. The stick-slip and whirl combination does not occur due to the highly rotation changes, which do not sustain unbalance forces for the necessary time to develop whirl [4].

Table 1 shows parameters employed in all simulations. These parameters can be altered in order to present other kinds of response, being indicated through the text.

3.1. Normal operation condition

The regular and desirable response of a drill-string is represented by a periodic behavior with low, acceptable amplitudes. Such dynamics is achieved by considering parameters presented in Table 2.

Fig. 2 presents normal condition response showing axial, torsion, radial and angular phase spaces. All responses are periodic presenting closed curve phase spaces. Fig. 2a shows axial phase space with vibration amplitude associated with values smaller than the admitted irregularity (s_o), representing a motion without loss of contact. Fig. 2b shows the torsion phase space that also presents periodic behavior. It should be pointed out that the torsional phase space is built using ($\dot{\varphi}_{rt}$ - $\dot{\varphi}$) as the abscissa axis instead of $\dot{\varphi}$ that presents a better motion characterization [17]. The radial phase space is shown in Fig. 2c indicating a small displacement drill-string rotation around the well center. The periodicity of other modes within suitable values and the small radial displacement amplitude suggest that this rotation is a direct consequence of unbalance. This kind of lateral unbalance, with small amplitudes, does not cause major disturbances on system dynamics. The evaluation of the phase space regarding the rotation around the well center (whirling velocity), shown in Fig. 2d, confirms this argue since it is very close to the rotatory table velocity.

3.2. Bit-bounce and stick-slip combination

A combination of bit-bounce and stick-slip phenomena is now discussed. This is achieved considering the drill-string length of 5000 m and the rotation $\dot{\varnothing}_{rt} = 20$ RPM. These changes make the drill-string less stiff and more susceptible to stick-slip. It is also expected that the high rotation value during slip phase induces the bit-bounce. Fig. 3 presents results related to this set of parameters. The axial phase space is presented in Fig. 3a showing a typical curve where contact occurs. Note a two-region phase space due to stiffness changes. The right side of the curve has a flattened configuration due to the high stiffness of the formation. The left part is related to the stiffness of the drill-string, which is significantly smaller. The negative values of the axial displacement are also indicating contact loss since it exceeds the irregularity value. This is a characteristic behavior of the bit-bounce phenomenon. Fig. 3b shows the torsional phase space. It should be pointed out that this response differs from those related to the isolated stick-slip, without bit-bounce. The typical isolated stick-slip curve is characterized by a regular curve varying from zero to the double speed of rotatory table [1]. However, the combination of stick-slip with bit-bounce also makes the drill-string rotation to tend to the rotatory table speed. Therefore, three distinct speeds characterize the torsional phase space: the minimum speed (zero), the intermediate (rotatory table speed) and the maximum (double speed). The radial phase space is shown in Fig. 3c. The drill-string is subjected to large rotational

Table 2
Model parameters for normal operation condition.

W_{ob}	10,000 lbf (44.763 KN)	de_{dp}	6.625 in (0.1683 m)
$\dot{\varnothing}_{rt}$	30 RPM (3.14 rad/s)	Di_{co}	3 in (0.0762 m)
l_{BHA}	914.4 ft (300 m)	De_{co}	9 in (0.2286 m)
l_{dp}	658.2 m (2000 m)	D_w	15 in (0.3810 m)
di_{dp}	5.3545 in (0.1358 m)	n_b	3

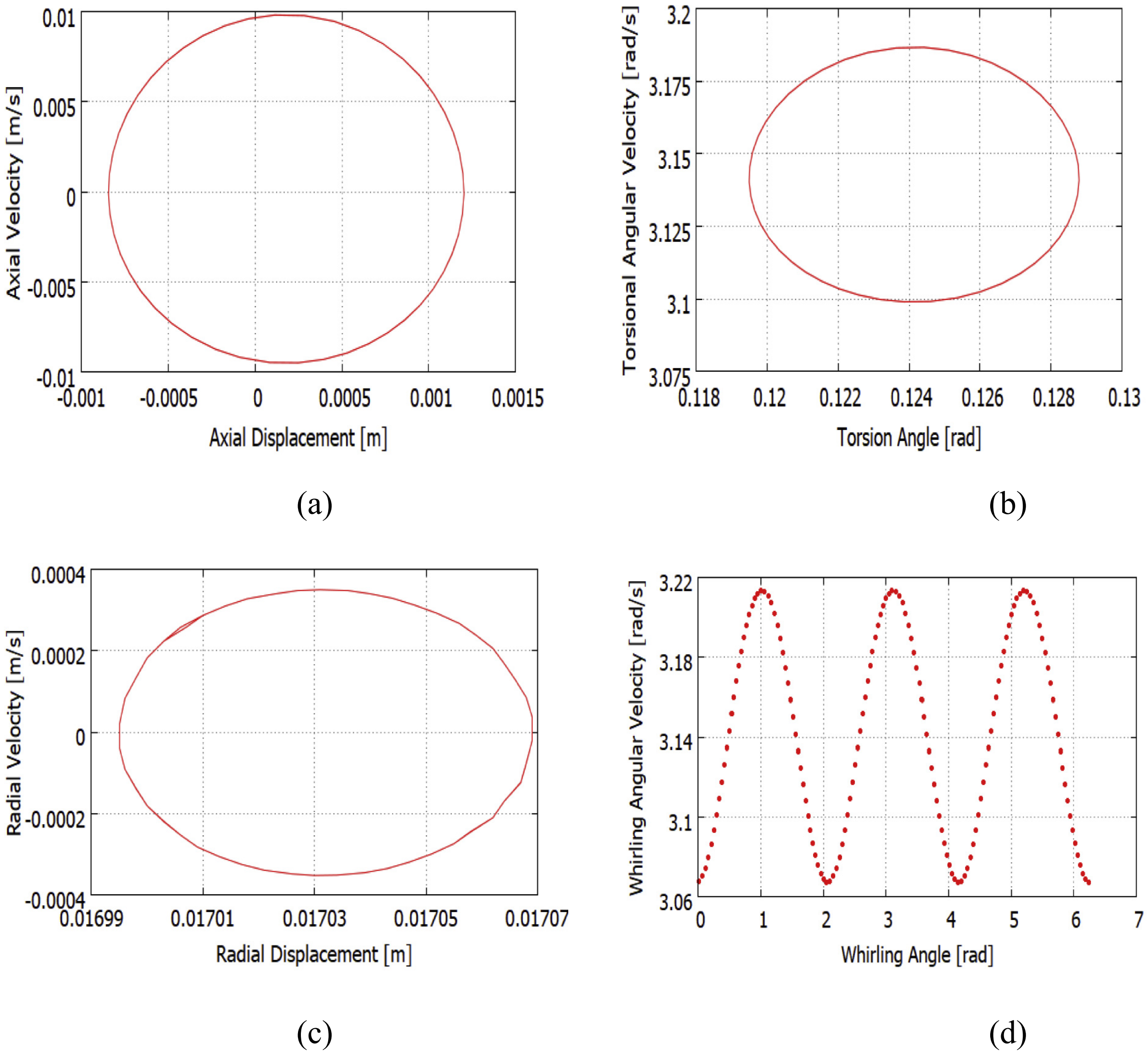

Fig. 2. Normal condition operation: (a) axial; (b) torsion; (c) radial; (d) whirl angle.

fluctuations and axial impacts. These conditions hinder the presence of whirl since the deflection is constantly modified by these fluctuations. This behavior is characterized by large radial velocity variations at small deflection intervals identified on its phase space. Fig. 3d presents the phase space regarding the whirling motion. Note that the whirling angular velocity is strongly influenced by rotation and its values are restricted to rotation close to the slip phase values. The whirling velocity is not influenced by low values of rotation due to this dissipative mechanism and by the high frequency rotation changes.

3.3. Bit-bounce and whirl combination

The bit-bounce and whirl combination is now of concern considering the same set of parameters of the previous case (bit-bounce and stick-slip) but the table rotation is increased to 40 RPM. This increase eliminates stick-slip and intensifies unbalance effects. Under these conditions, it is expected that the drill-string be subjected to bit-bounce but inducing high whirl amplitude. Fig. 4 presents the system response under these conditions.

The axial phase space is shown in Fig. 4a and confirms the bit-bounce. Fig. 4c presents the radial phase space that is similar to the axial one. This reveals that drill-string axial impacts are able to induce borehole wall lateral impacts. Fig. 4d shows the whirl phase space where the negative angular velocities indicate that backward whirl is occurring. Note that there are large angular velocity fluctuations, which are associated with variations in deflection and rotation during axial contact loss. The torsional phase space is shown in Fig. 4b where it is perceptible the effects of axial and lateral modes. It is observed great

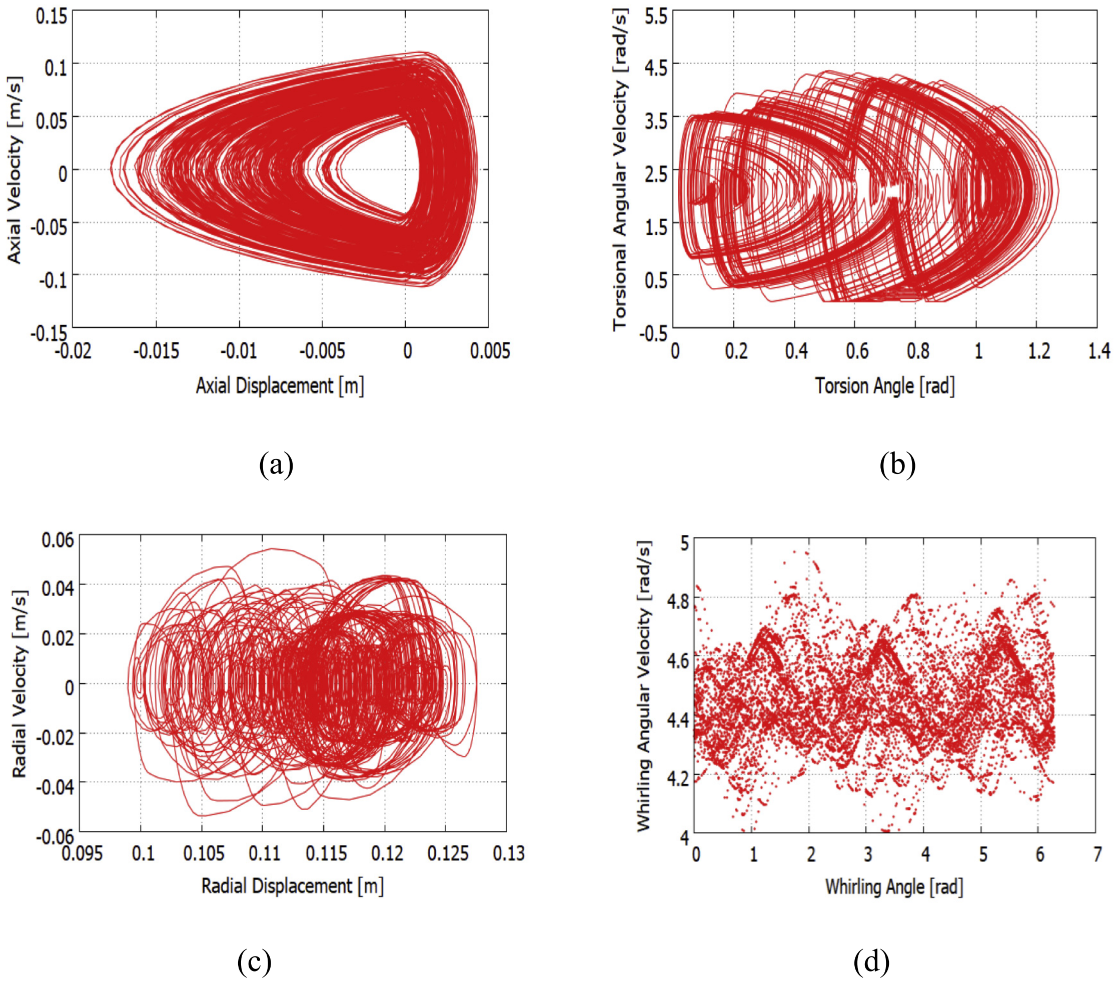

Fig. 3. Bit-bounce and stick-slip combination: (a) axial; (b) torsion; (c) radial; (d) whirl angle.

variations with respect to table rotation. Smaller values are consequence of the high resistive friction torque from the axial load of impact during the bit-bounce. On the other hand, greater values are essentially provided by the stored torsional energy that is suddenly released during axial contact loss [17].

4. Parametric analysis

After the characterization of critical behaviors, a parametric analysis is carried out. Bifurcation diagrams are constructed evaluating the effect of parameter variation on some system variable that represents system response. Basically, the maximum displacement values are analyzed allowing one to evaluate the influence of operational conditions and the identification of critical vibrations. Hence, axial, torsional and radial drill-string motions are monitored by their displacement and velocity peaks.

Two procedures are adopted to build the diagrams concerning initial conditions: reset initial conditions for each parameter change; and using the last result as initial condition for the next parameter value. Besides, this diagram indicates when critical behaviors, namely, bit-bounce, stick-slip and whirl occur. The criterion for each one of them is based on displacement analysis, considering a tolerance to define contact condition. Hence, stick-slip occurs with velocity small values associated with this tolerance. The same is done to characterize bit-bounce, where tolerance is associated with contact loss.

Parameters used in all simulations are in agreement with those practiced in Ref. [24] and in other works as [12]. In the sequence, the influence of some operational conditions is analyzed changing the following parameters: drill-string length and table rotation. Concerning drill-string length, different conditions are treated related to distinct values of weight on bit and well diameter.

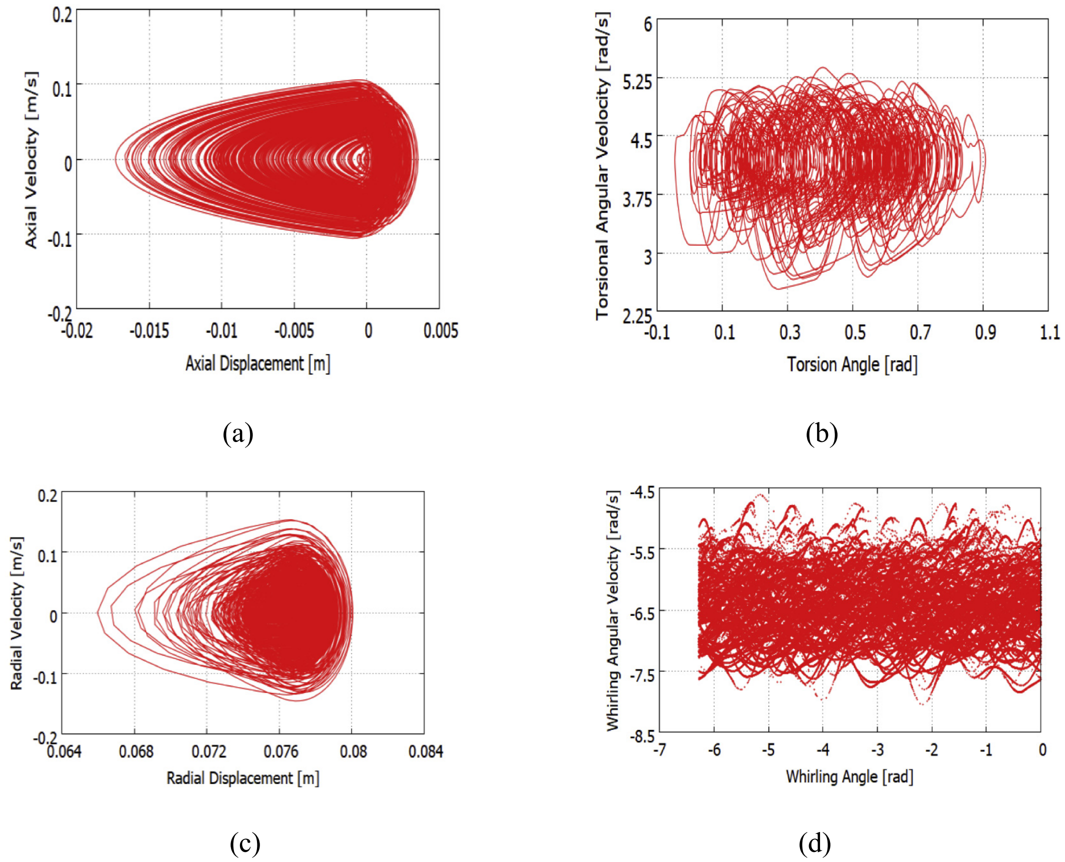

Fig. 4. Bit-bounce and whirl combination: (a) axial; (b) torsional; (c) radial; (d) whirl angle.

4.1. Influence of drill-string length

Consider a drill-string with 0.0127 m (0.5″) of eccentricity and 0.381 m (15″) of wellbore diameter operating under a weight on bit of 22.24 kN. All other parameters are presented in Table 3.

Fig. 5 presents results of the displacement maximum values under the variation of drill-string length. They are built with the same initial conditions for each parameter and also indicate critical behaviors. Fig. 5a presents results related to axial displacements showing an interval with efficient responses around 1500 m. After that value, bit-bounce dominates. Moreover, at higher depths, whirl becomes relevant and shows a great influence on axial response, which can be observed around 7000 m and over 8000 m. Fig. 5b presents the torsional diagrams and also points to an abrupt decrease at 7000 m. [17] showed similar results except for this behavior due to the absence of lateral coupling. This highlights the importance of a 3-mode coupled model to describe drill-string vibrations. The decrease observed on the angular velocity is a consequence of the coupling between torsional and lateral modes during contact. Consequently, this affects the axial mode through bit-rock interaction, as it couples the torsional and axial modes. The lateral diagrams are presented in Fig. 5c, showing a crescent aspect that is associated with the decreasing drill-string stiffness when its length is increased. This explains the reason for the critical phenomena relevance at higher depths.

Fig. 6 presents the same diagrams but considering different initial conditions, defined from the last value of the previous parameter, for each parameter value. It is evident that the initial conditions have significant influence on system dynamics. In

Table 3
Model parameters for parametric analysis.

$\dot{\O}_{rt}$	30 RPM | 4.188 rad/s
n_b	3
l_{BHA}	914.4 ft | 300 m
di_{dp}	5.3545 in | 0.1358 m
de_{dp}	6.625 in | 0.1683 m
D_{ico}	3 in | 0.0762 m
De_{co}	9 in | 0.2286 m
D_w	15 in | 0.3810 m

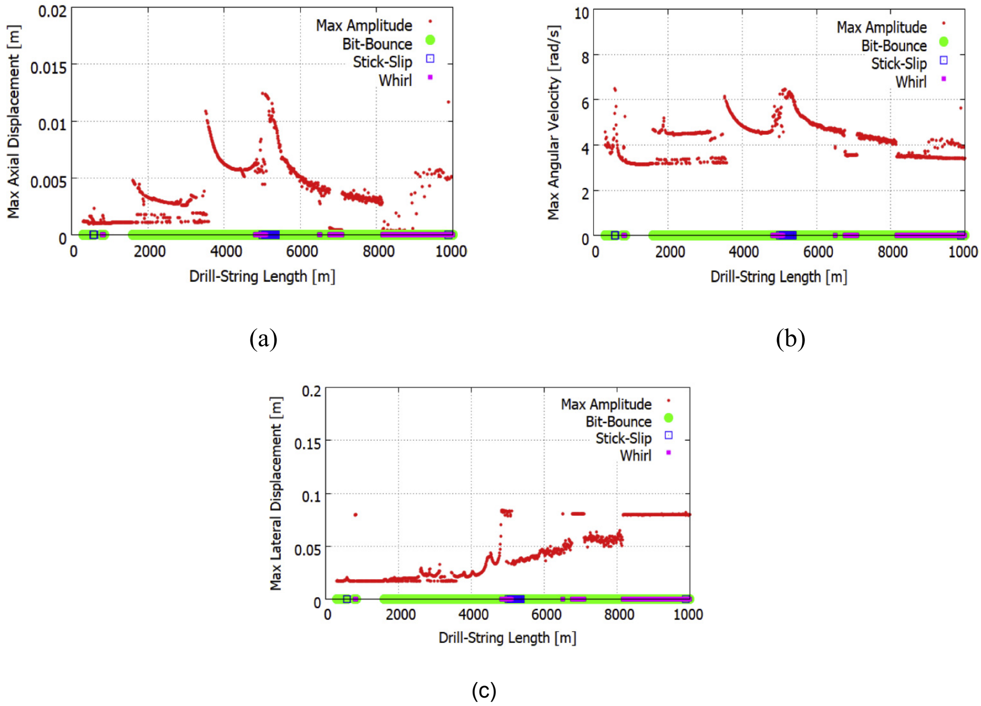

Fig. 5. Diagrams varying drill-string length with the same initial conditions (resetting): (a) axial; (b) torsional; (c) lateral.

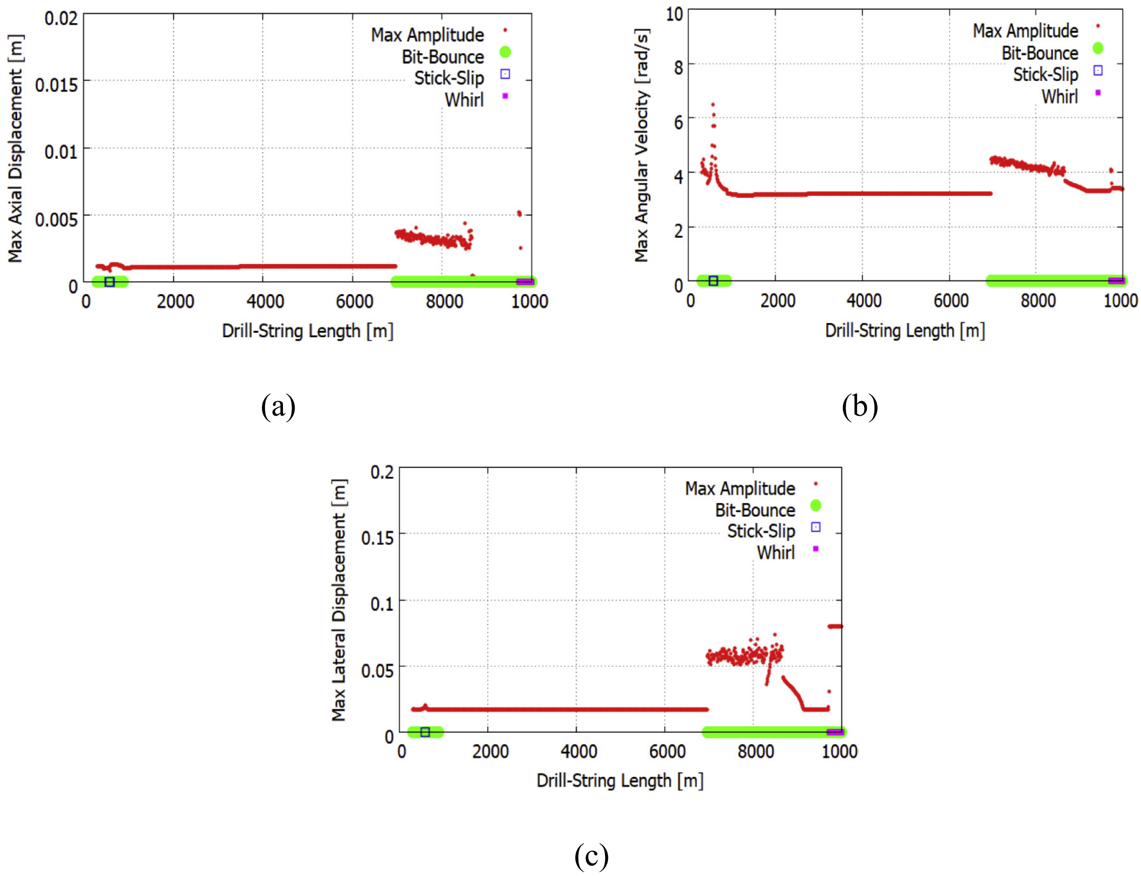

Fig. 6. Diagrams varying drill-string length with different initial conditions (previous): (a) axial; (b) torsional; (c) lateral.

general, there is an evident reduction of critical behaviors, showing the importance to properly choose the drilling history before reach the operational conditions and can be properly exploited for mitigation purposes. Conclusions related to initial conditions constitute an essential aspect in order to obtain a wider range of desirable operation conditions, which agrees with [17]. Note that results extend appropriate conditions from 1500 to 7000 m, approximately.

The influence of weight on bit is now of concern. In brief, it is possible to say that the weight on bit increase makes the drill-string more vulnerable to stick-slip and whirl for high depths but, on the other hand, can sustain a desirable behavior over a wider range of low depths. Hence, in order to analyze the weight on bit influence, consider $W_{ob} = 67.72$ kN.

Fig. 7 shows bifurcation diagrams considering the same initial conditions. Fig. 7a presents the axial diagram showing that bit-bounce starts at 3000 m. Based on that, it can be concluded that weight on bit increase is effective to avoid severe vibrations for low and medium depths. However, it is not effective for greater values. The increase of weight on bit at high depths is responsible for severe vibrations in other modes, especially the lateral one. The peaks of the torsional and lateral mode diagrams shown in Fig. 7b and c are significantly increased as well as their incidence range.

Fig. 8 shows the same situations of Fig. 7 using different initial conditions for each parameter. The effect of weight on bit increase is even more pronounced in these diagrams. Note that, except for the important peak close to 500 m due to the presence of stick-slip and bit-bounce, system response does not present critical vibrations. Other critical behaviors are only occurring close to 10,000 m, first due to bit-bounce and later to whirl.

The analysis of the well diameter influence on drill-string vibration is now in focus. A higher value of well diameter is considered, corresponding to situations related to the well opening where 26″ is a typical value. The weight on bit is 22.24 kN and all other parameters remain the same.

Results are presented in Fig. 9 showing axial, torsional and lateral diagrams. It is noticeable that stick-slip is preponderant for all values of well diameter. The other responses are dominated by bit-bounce. This means that, under these conditions, there are not desirable operational dynamics. It should be highlighted that the existence of stick-slip induces bit-bounce and, the vulnerability for stick-slip is a consequence of the drill-bit with larger contact surface with the formation. With a higher

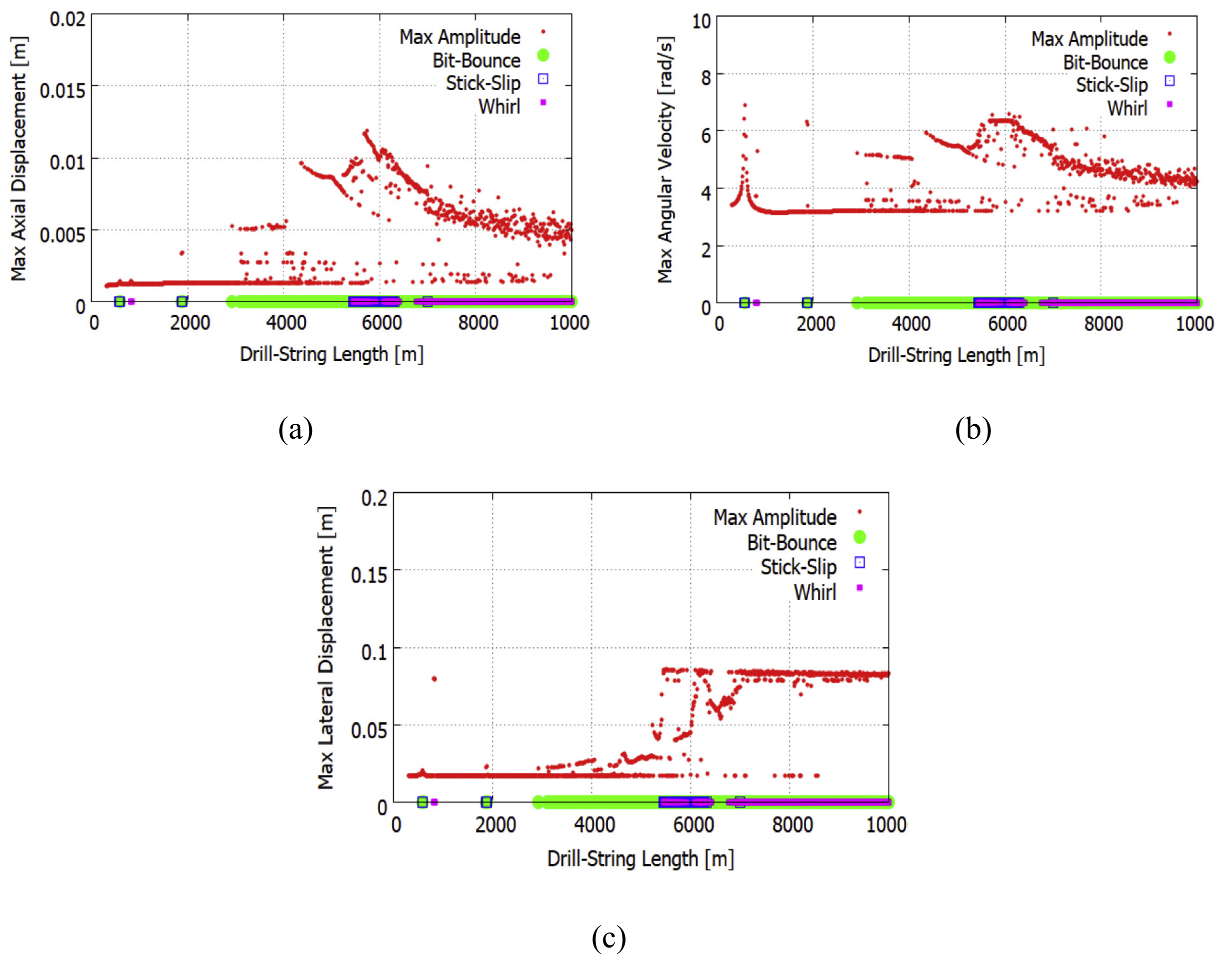

Fig. 7. Diagrams varying drill-string length with the same initial conditions (resetting): (a) axial; (b) torsional; (c) lateral.

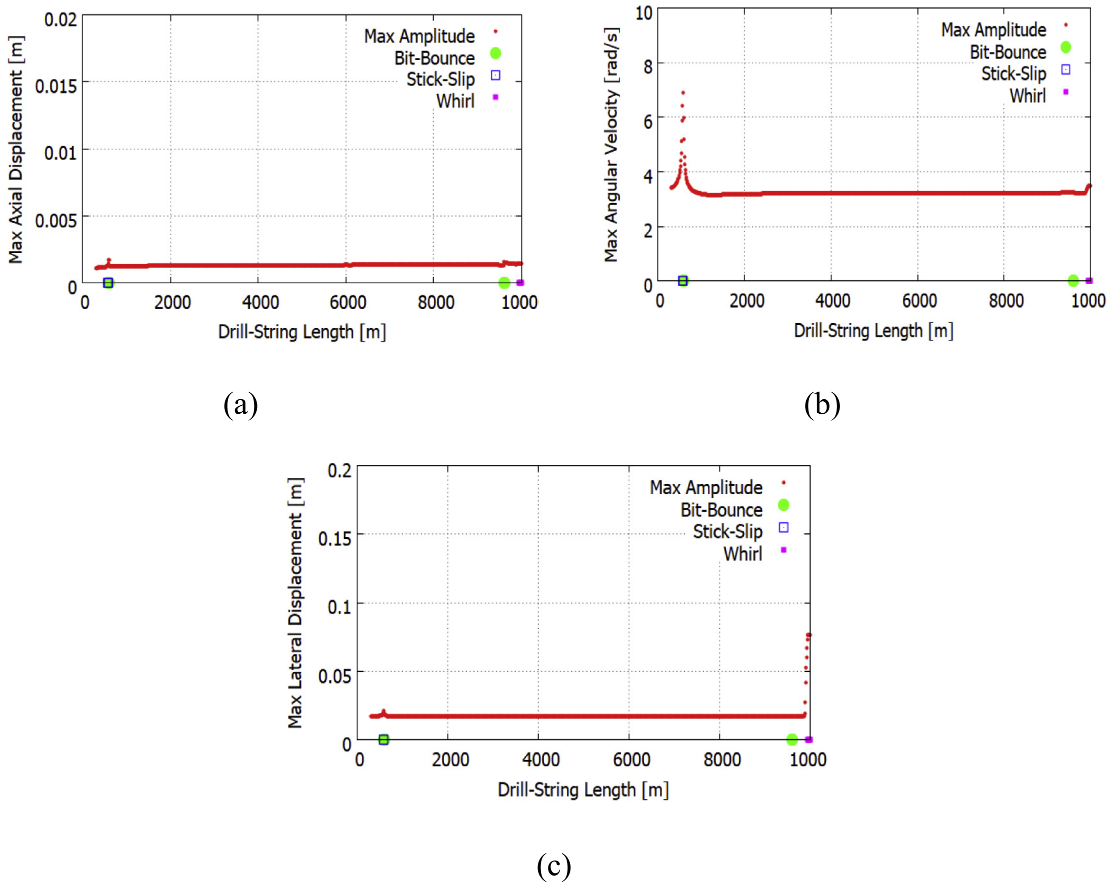

Fig. 8. Diagrams varying drill-string length with different initial conditions (previous): (a) axial; (b) torsional; (c) lateral.

moment arm, any change on the axial drill-string displacement produces a more significant change on T_{ob} and on its rotation, which may induce bit-bounce. When this phenomenon happens, any torsional energy stored during drilling is released with the axial contact loss and generates an increase on drill-string rotation. The torsional diagram, Fig. 9b, shows that even when stick-slip is not occurring, the rotation value is significantly higher than the table rotation.

Another important characteristic of these responses is the lack of whirl. Fig. 9c shows that there are only three peaks where the deflection is significant. However, such values are not enough to induce lateral contact, which means that whirl is not occurring for this range of parameters. The weight on the bit increase is not a correct strategy to reduce bit-bounce since this phenomenon is basically induced by stick-slip. In this kind of situation, a more interesting strategy would be the change of table rotation values or restart the operation [17].

4.2. Influence of rotation

The influence of rotation on drilling dynamics is now evaluated. For this purpose, a specific length is analyzed under different weight on bit while the rotatory table speed is changing and critical behaviors are identified. Two different situations are investigated to represent the rotation variation: up-sweep and down-sweep. The objective of this analysis is to evaluate conditions where smooth variations of rotation parameter are enough to mitigate critical vibrations.

Initially, consider a configuration with length 5000 m and weight on bit 22.24 kN. Fig. 10 presents axial and lateral diagrams considering up-sweep and down-sweep. Critical vibrations are marked at the inferior horizontal axis for the up-sweep simulations and at the superior horizontal axis for down-sweep. An interval without critical vibrations is observed until rotation is approximately for 30 RPM for up-sweep test. The same situation occurs for values smaller than 20 RPM during down-sweep test. For greater values, bit-bounce and whirl dominates drilling dynamics. Sensitivity of initial conditions is clearly indicated from the discrepancies between up-sweep and down-sweep.

Fig. 11 presents similar simulations increasing the weigh on bit to 66.72 kN. This increase is responsible for the increase of the desirable operation conditions range, related to non-critical responses. Moreover, a weight on bit increase induces stick-slip and some situations of whirl for low table rotation, especially in down-sweep test.

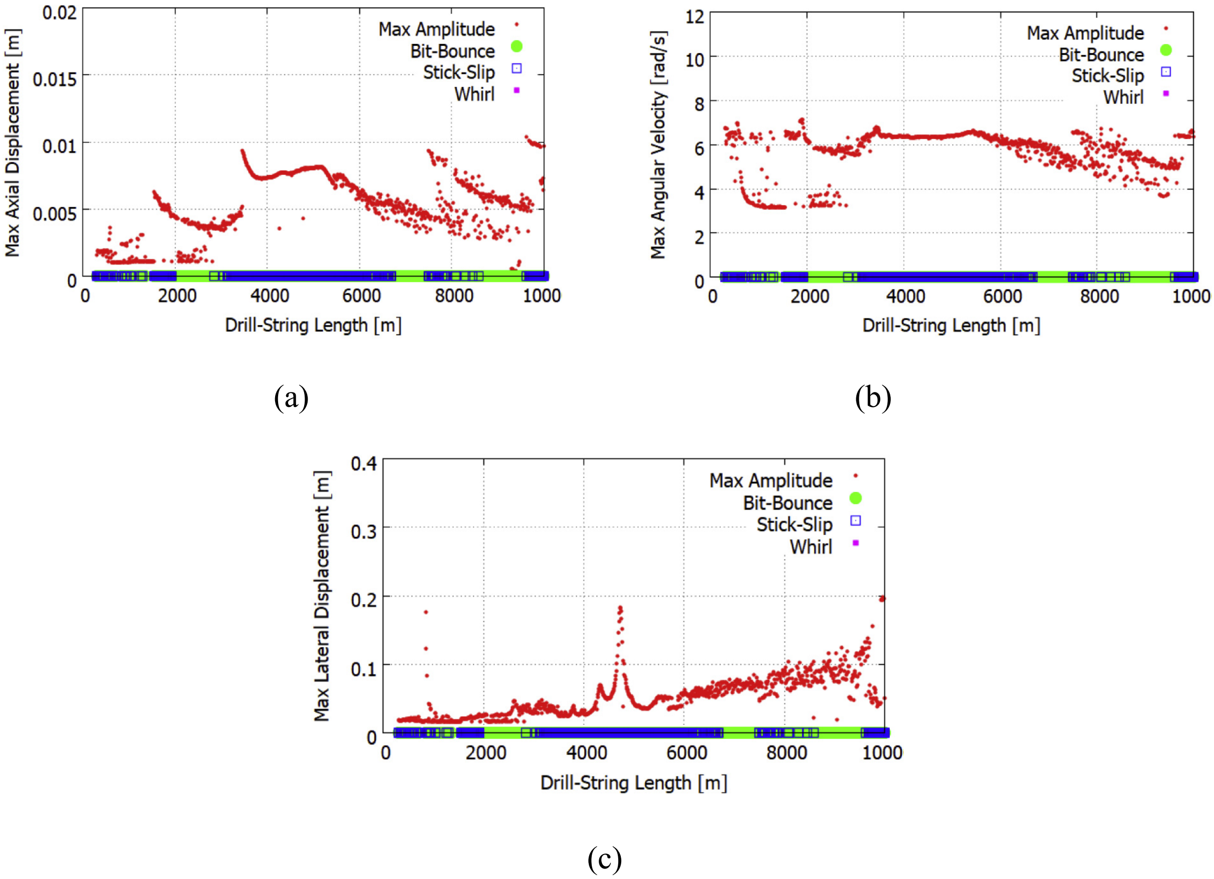

Fig. 9. Diagrams varying well diameter with the same initial conditions (resetting): (a) axial; (b) torsional; (c) lateral.

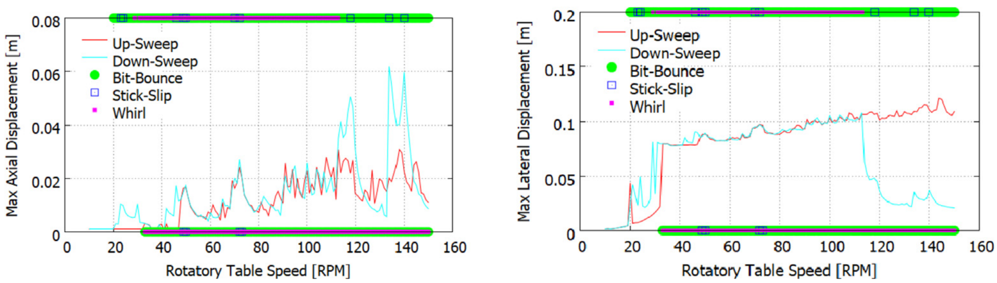

Fig. 10. Diagram for different rotation, $W_{ob} = 22.24$ kN: (a) axial; (b) lateral.

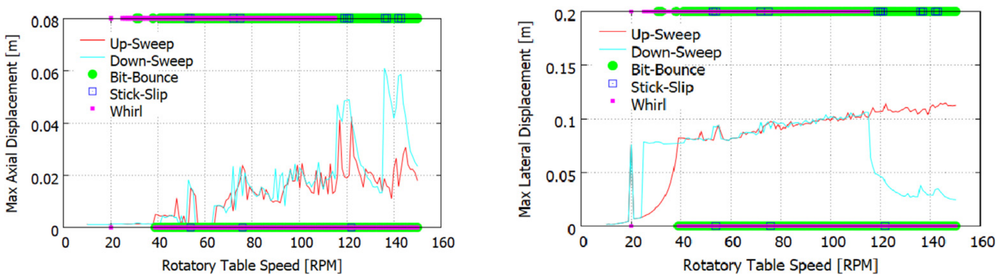

Fig. 11. Diagram for different rotation, $W_{ob} = 66.72$ kN: (a) axial; (b) lateral.

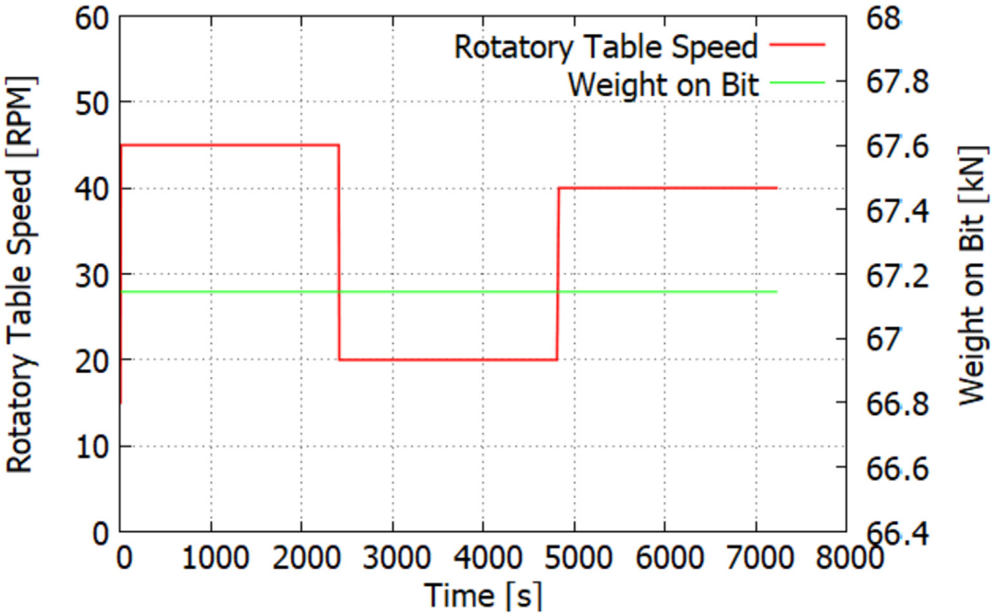

Fig. 12. Mitigation strategy varying rotary table speed.

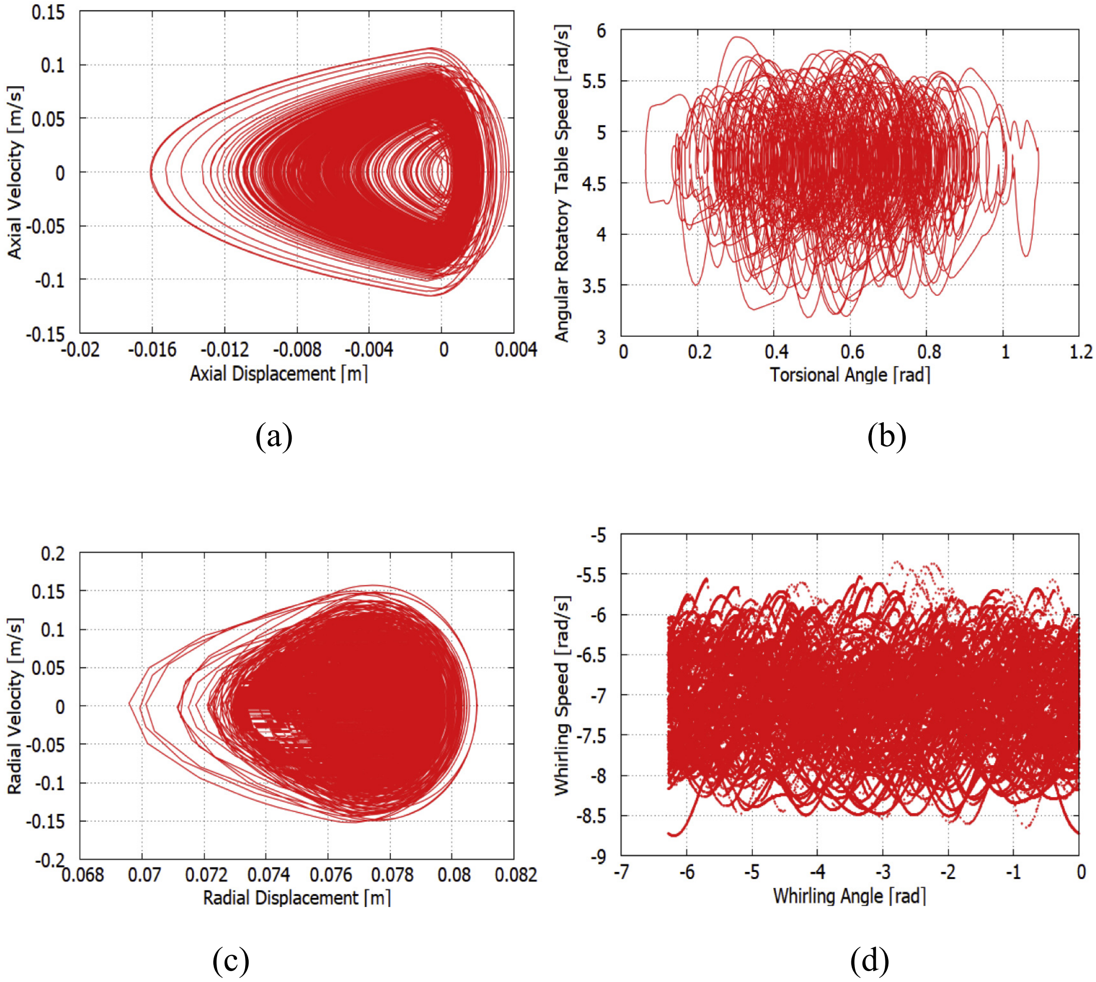

Fig. 13. Bit-bounce and whirl combination for a rotation of 45RPM: (a) axial; (b) torsion; (c) radial; (d) whirl angle.

5. Mitigation strategies

The parametric analysis leads to a set of possible vibration mitigation strategies during drilling operations. Some mitigation procedures are available on drilling field manuals, without a proper explanation. Dynamical perspective can help to explain these approaches and, in addition, help one to imagine new strategies. Rotation increase following some stability paths is a possible strategy to avoid critical vibrations. Sometimes, it is interesting to restart the operation in order to eliminate undesirable responses. Weigh on bit change is another important parameter to be considered for mitigation purposes. Mechanical resistance usually limits the increase of weight on bit. In addition, a combined changed on rotation and weight on bit does not assure an improvement on the drilling response. Field manuals usually limits these changes is three times, indicating the operation restart otherwise. Results discussed in the preceding sections showed some of these situations. Rotation change is useful to mitigate whirl and stick-slip. Weight on bit is interesting to mitigate bit-bounce and stick-slip. Therefore, it is possible to combine some of these variations in order to obtain interesting drilling conditions.

An example of possible mitigation strategies is now evaluated using numerical simulations. Consider a drill-string of 5000 m long using the same parameters associated with Fig. 5, Tables 1 and 3, but varying some parameters. A mitigation strategy is presented in Fig. 12 showing a rotary table speed variation considering a constant value of weight on bit. Fig. 13 presents system response with a rotation $\dot{\varnothing}_{rt} = 45$ RPM and a weight on bit of $W_{ob} = 66.72$ kN, showing a combination of bit-bounce and whirl. Bit-bounce is identified by axial displacements less than $s_o = -0.001$. By decreasing rotation to 20 RPM the drill-string dynamics changed to a more desirable response, as presented in Fig. 14. Note that both bit-bounce and whirl disappeared. In the sequence, rotation is increased again for 40 RPM, as presented in Fig. 15. Results are less critical than the one obtained with $\dot{\varnothing}_{rt} = 45$ RPM, Fig. 13, which is noticeable observing the values of axial displacements. Although bit-bounce is not eliminated, its is clear a less critical behavior than the previous one.

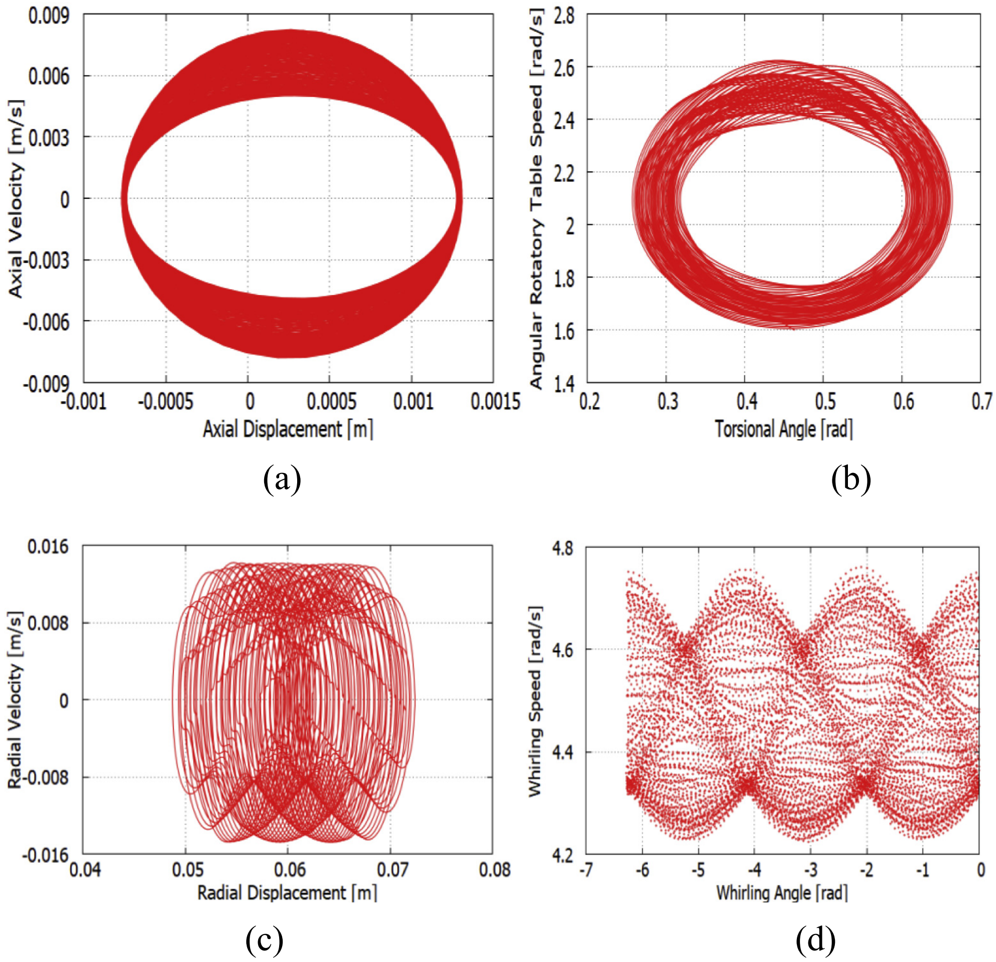

Fig. 14. Normal response for a rotation of 20RPM: (a) axial; (b) torsional; (c) radial; (d) whirl angle.

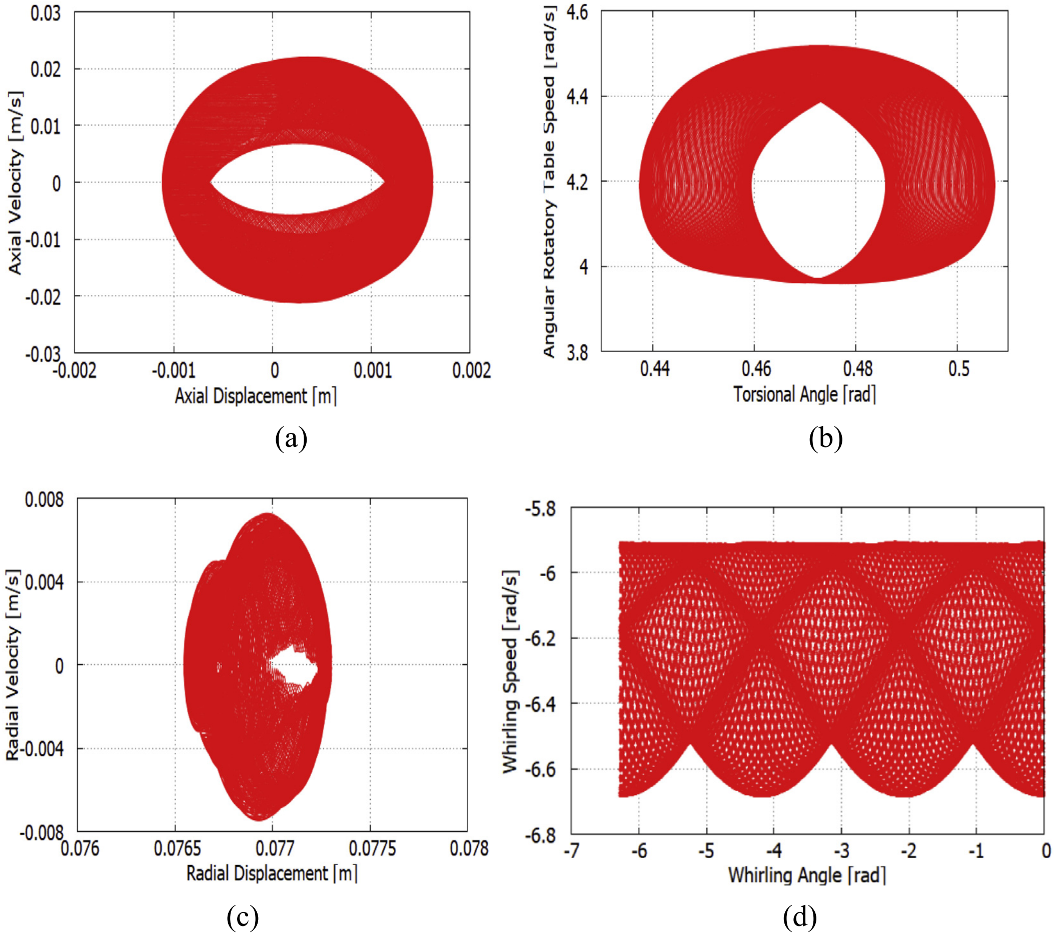

Fig. 15. Response for a rotation of 40RPM: (a) axial; (b) torsional; (c) radial; (d) whirl angle.

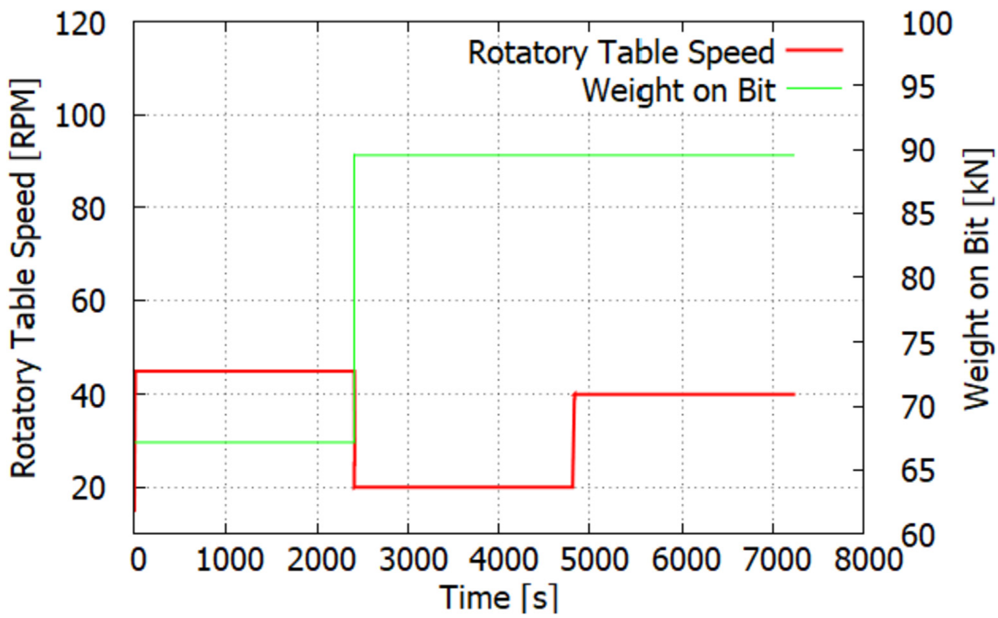

Fig. 16. Mitigation strategy varying rotary table speed and weight on bit.

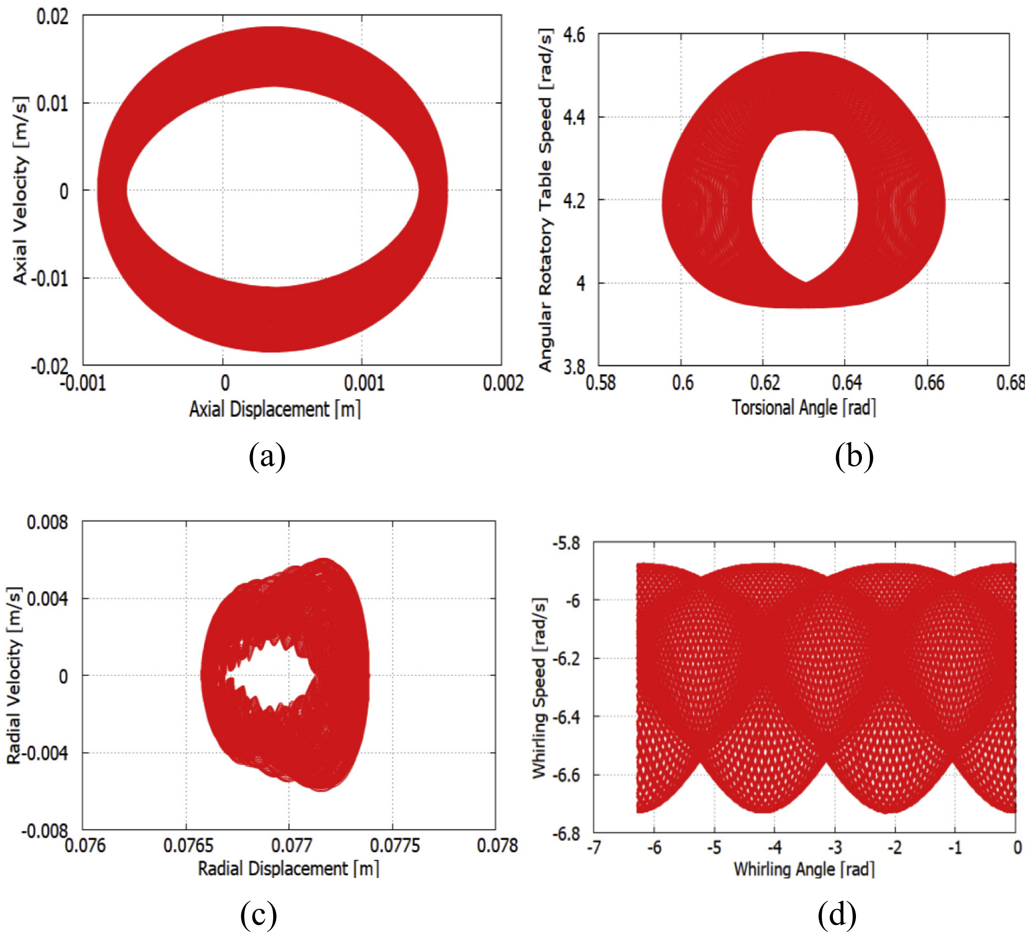

Fig. 17. Response for a rotation of 40RPM and weight on bit 88.96 kN: (a) axial; (b) torsional; (c) radial; (d) whirl angle.

A similar strategy is now considered, but changing the weight on bit, Fig. 16. The starting point is the same of the one presented in Fig. 13, with 45 RPM, showing bit-bounce and whirl. The next step decreases the speed to 20RPM, together with an increase of the weight on bit to 88.96 kN. Under this condition, the system presents similar results for the one obtained in Fig. 14. But, by increasing the rotation again to 40RPM, a different response is observed, Fig. 17. Note that bit-bounce is now eliminated, preserving the same level of whirl.

6. Conclusions

Drill-string vibration is investigated considering a four-degree of freedom nonsmooth model. Operational conditions are investigated from a parametric analysis, proposing alternatives to mitigate critical behaviors. The proposed model is interesting to offer a comprehension of system dynamics, evaluating critical responses, especially the couplings among them. The weight on bit increase may induce a combination of bit-bounce and stick-slip. The same can occur with rotation decrease. The increase of rotation tends to induce whirl and bit-bounce phenomena. The parametric analysis allows one to imagine mitigation strategies for critical vibrations. Some of these conclusions include the following observations. An adequate increase of weight on bit is an efficient strategy to mitigate bit-bounce. The change of initial conditions, restarting the operation with a higher rotation, showed to be a solution to mitigate whirl. Due to strong nonlinearities involved, huge rotation increase can induce other phenomena as stick-slip and bit-bounce. Nevertheless, a desirable dynamics may be obtained by a better combination of rotation and weight on the bit. The continuous increase of rotation also showed to be effective against critical vibration until a certain level. Nonlinear characteristics make the system response sensitive to initial conditions. Based on that, the path to reach some set of parameters is critical and needs to be properly evaluated, pointing to new strategies to mitigate undesirable responses. Results show that dynamical perspective is essential to define mitigation strategies.

Acknowledgments

The authors would like to acknowledge the support of the Brazilian Research Agencies CNPq, CAPES and FAPERJ. The Air Force Office of Scientific Research (AFOSR) is also acknowledged.

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