DINÂMICA: Forças e Torques

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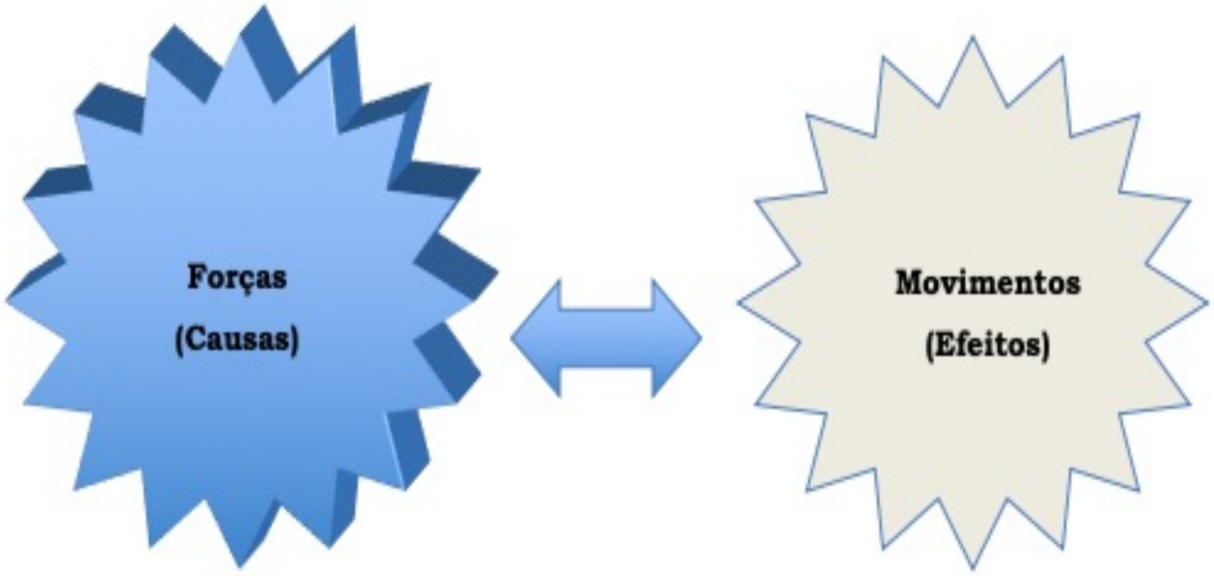
Forças e Torques

- **Força** - tendência de promover o movimento de um corpo: movimento retilíneo e rotação em relação a um ponto qualquer.
- **Momento de uma força**, ou **torque**, está associado à rotação de um corpo.

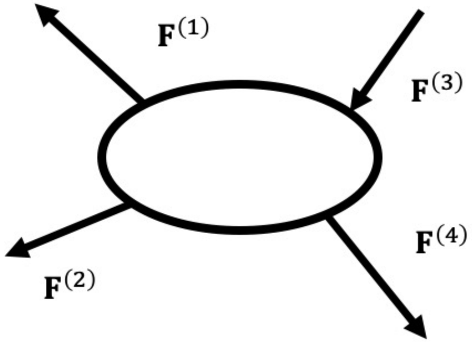


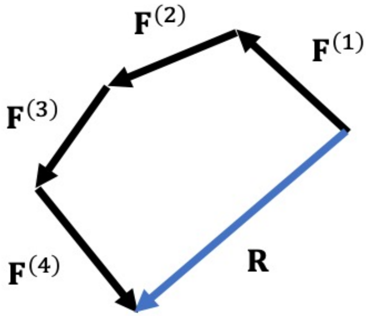
Forças

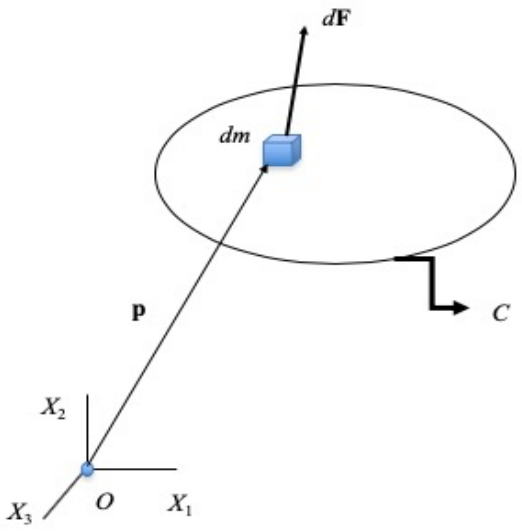
- **Forças:** de contato e de corpo.

$$\mathbf{F} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = F_1\mathbf{e}_1 + F_2\mathbf{e}_2 + F_3\mathbf{e}_3$$

- **Resultante de forças**







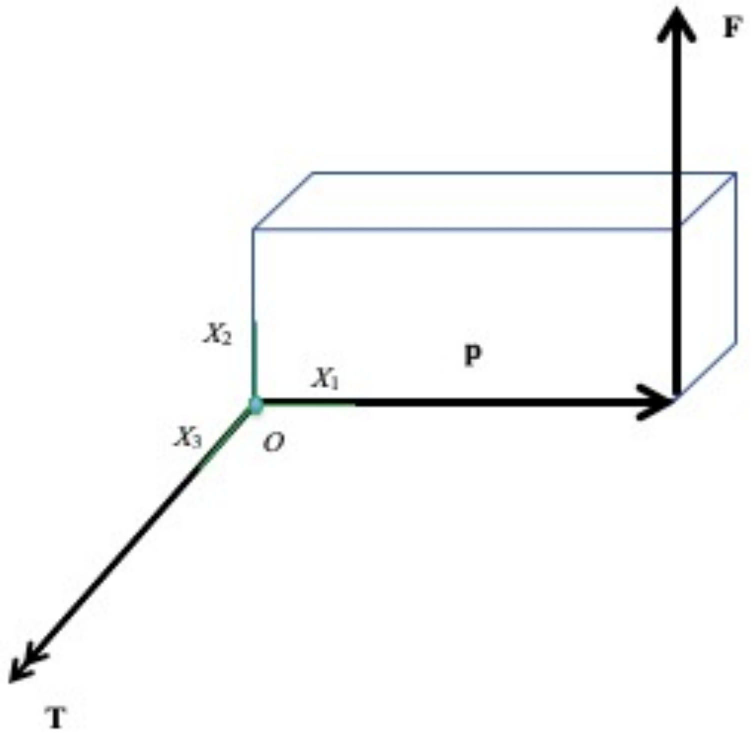
$$\mathbf{R} = \sum_{k=1}^{N} \mathbf{F}^{(k)}$$

$$\mathbf{R} = \int_C d\mathbf{F}$$

Torques

- **Torque:** momento de uma força.

$$\mathbf{M}^{\mathbf{F}/O} = \mathbf{p}\times\mathbf{F} = \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ p_1 & p_2 & p_3 \\ F_1 & F_2 & F_3 \end{vmatrix} = \begin{Bmatrix} p_2F_3 - p_3F_2 \\ p_3F_1 - p_1F_3 \\ p_1F_2 - p_2F_1 \end{Bmatrix}$$

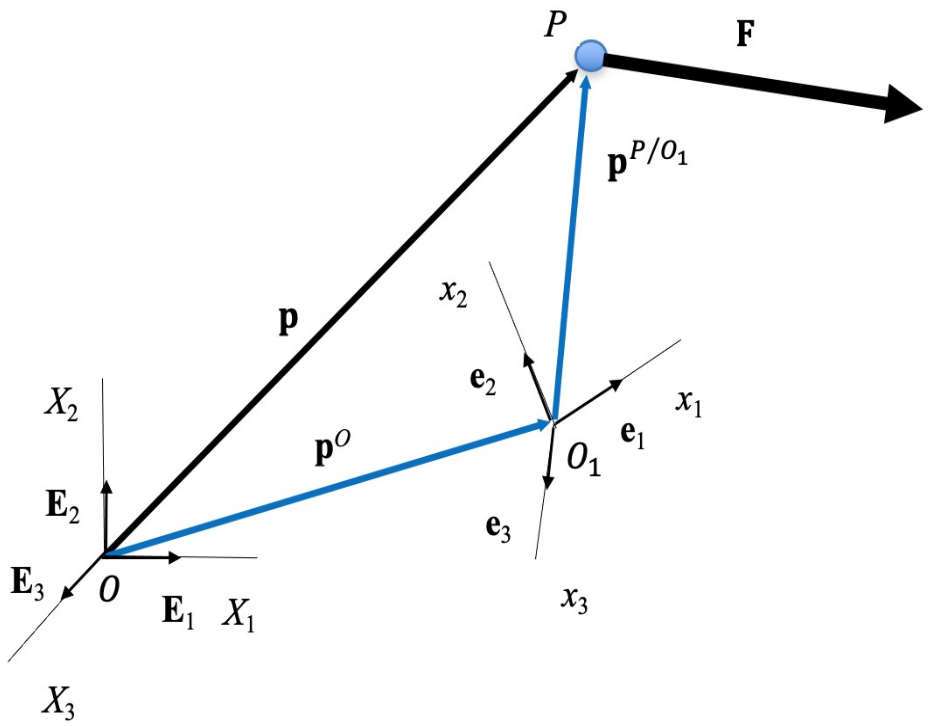


- **Resultante de momentos**

$$\mathbf{M}^{\mathbf{F}^{(k)}} = \sum_{k=1}^{N} \mathbf{M}^{(k)} = \sum_{k=1}^{N} \mathbf{p}^{(k)}\times\mathbf{F}^{(k)} = \mathbf{p}\times\sum_{k=1}^{N} \mathbf{F}^{(k)} = \mathbf{p}\times\mathbf{R}$$

$$\mathbf{M}^{\mathbf{R}/0} = \int_C \mathbf{p}\times d\mathbf{F}$$

Transporte de Momentos



$$\mathbf{M}^{\mathbf{F}/O} = \mathbf{p}\times\mathbf{F} = \left(\mathbf{p}^O + \mathbf{p}^{P/O_1}\right)\times\mathbf{F} = \mathbf{p}^O\times\mathbf{F} + \mathbf{p}^{P/O_1}\times\mathbf{F} = \mathbf{p}^O\times\mathbf{F} + \mathbf{M}^{\mathbf{F}/O_1}$$

$$\mathbf{M}^{\mathbf{F}/O} = \mathbf{M}^{\mathbf{F}/O_1} + \mathbf{p}^O\times\mathbf{F}$$

Sistemas de Forças e Torques

Dois sistemas de forças $\mathfrak{I}_1$ e $\mathfrak{I}_2$ são equivalentes se suas resultantes forem iguais e se seus momentos resultantes em relação a um ponto forem iguais

$$\mathbf{R}(\mathfrak{I}_1) = \mathbf{R}(\mathfrak{I}_2)$$

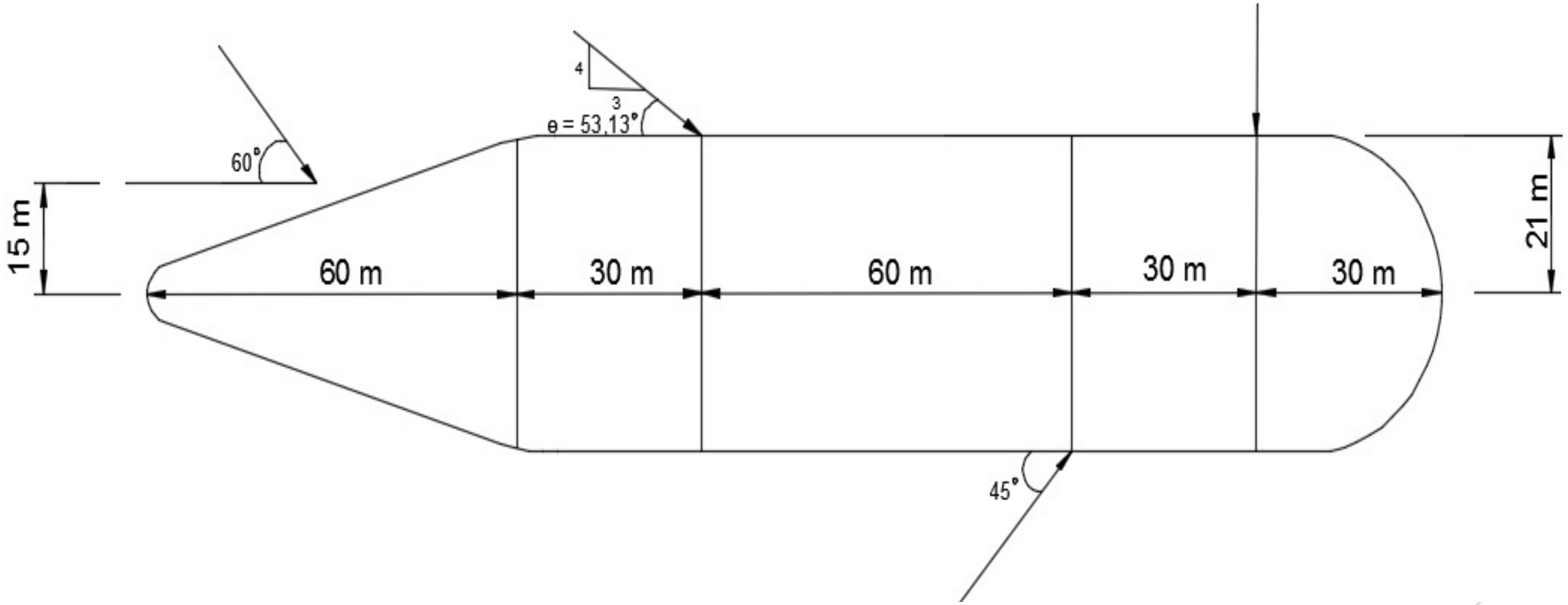
$$\mathbf{M}^{\mathfrak{I}_1/O} = \mathbf{M}^{\mathfrak{I}_2/O}$$

Rebocadores

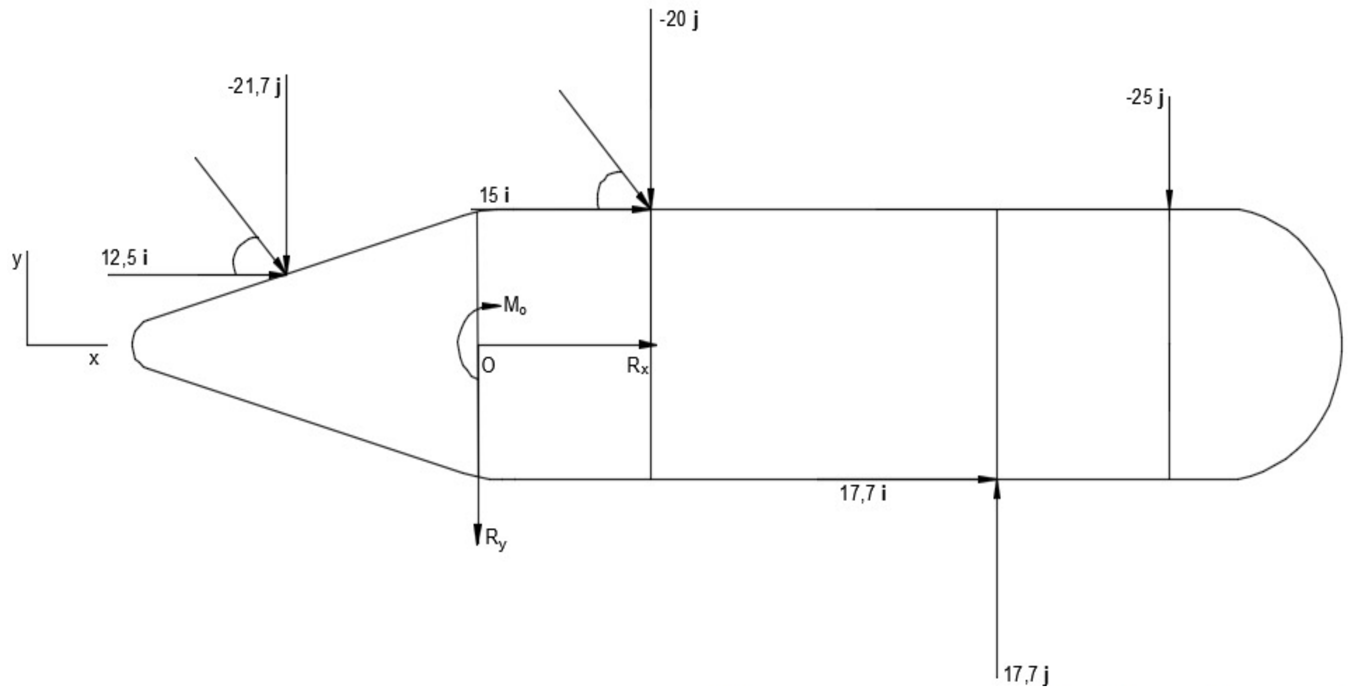
Quatro rebocadores são usados para trazer um navio ao cais. Considerando que cada rebocador exerce uma força de $F = 25N$ avalie:

a) O sistema força-binário equivalente no mastro O.

b) O ponto no casco onde apenas um rebocador maior poderia produzir o mesmo efeito dos quatro originais.



Rebocadores: sistema equivalente



$$\mathbf{F}^{(1)} = 12{,}5\ \mathbf{e}_1 - 21{,}7\mathbf{e}_2$$

$$\mathbf{F}^{(2)} = 15\ \mathbf{e}_1 - 20\mathbf{e}_2$$

$$\mathbf{F}^{(3)} = -25\mathbf{e}_2$$

$$\mathbf{F}^{(4)} = 17{,}7\ \mathbf{e}_1 + 17{,}7\mathbf{e}_2$$

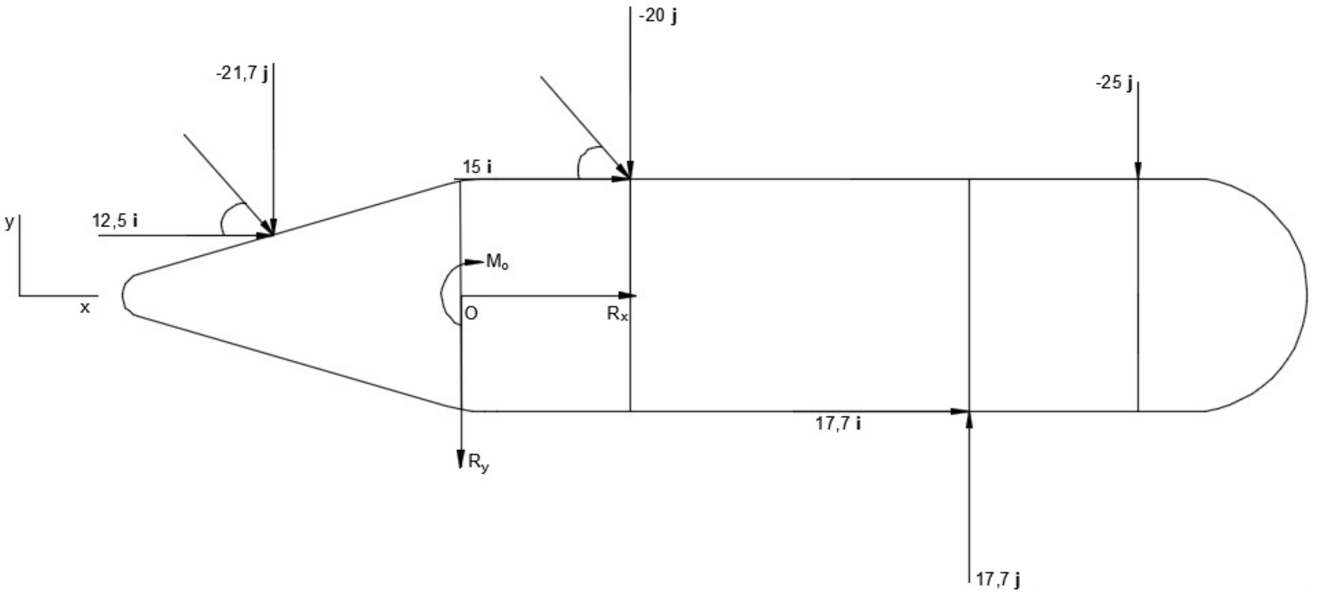
$$\mathbf{R} = (12{,}5\ \mathbf{e}_1 - 21{,}7\ \mathbf{e}_2) + (15\mathbf{e}_1 - 20\ \mathbf{e}_2) - (25\mathbf{e}_2) + (17{,}7\ \mathbf{e}_1 + 17{,}7\mathbf{e}_2)$$

$$\mathbf{R} = \begin{Bmatrix} 45{,}2 \\ -49 \\ 0 \end{Bmatrix}$$

$$\mathbf{M}^{\mathbf{F}/O} = \sum \mathbf{p}^{(k)} \times \mathbf{F}^{(k)}$$
$$= (-27\mathbf{e}_1 + 15\mathbf{e}_2)\times(12{,}5\mathbf{e}_1 - 21{,}7\mathbf{e}_2) + (30\mathbf{e}_1 + 21\mathbf{e}_2)\times(15\mathbf{e}_1 - 20\mathbf{e}_2)$$
$$+ (120\mathbf{e}_1 + 21\mathbf{e}_2)\times(-25\mathbf{e}_2) + (90\mathbf{e}_1 - 21\mathbf{e}_2)\times(17{,}7\mathbf{e}_1 + 17{,}7\mathbf{e}_2)$$

$$\mathbf{M}^{\mathbf{F}/O} = \begin{Bmatrix} 0 \\ 0 \\ -1552 \end{Bmatrix}$$

Rebocadores: rebocador único



$$\mathbf{R} = \begin{Bmatrix} 45{,}2 \\ -49 \\ 0 \end{Bmatrix}$$

$$\mathbf{M}^{\mathbf{F}/O} = \begin{Bmatrix} 0 \\ 0 \\ -1552 \end{Bmatrix}$$

$$\mathbf{p} = x\mathbf{e}_1 + 21\mathbf{e}_2$$

$$\mathbf{M}^{\mathbf{F}/O} = \mathbf{p}\times\mathbf{R} = (x\,\mathbf{e}_1 + 21\mathbf{e}_2)\times(45{,}2\mathbf{e}_1 - 49\mathbf{e}_2) = -1552\,\mathbf{e}_3$$

$$x = 12{,}3\text{m}$$

Equilíbrio

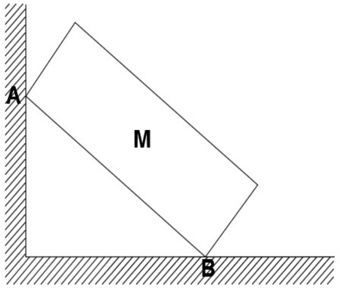
$$\mathbf{R} = \sum \mathbf{F}^{(k)} = 0$$

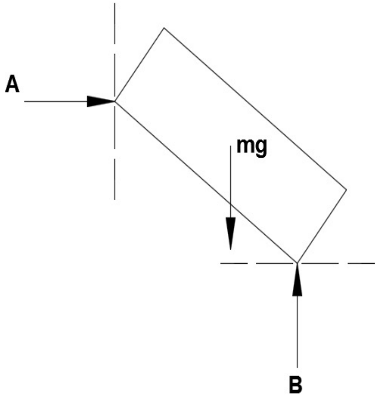
$$\mathbf{T} = \sum \mathbf{M}^{(k)}$$

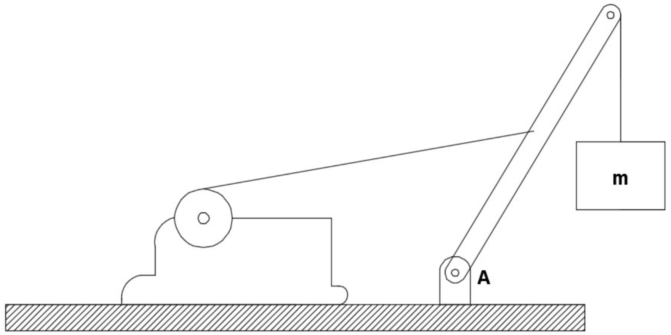
$$R_i = 0$$

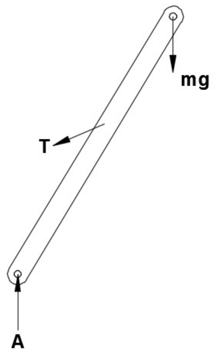
$$T_i = 0$$

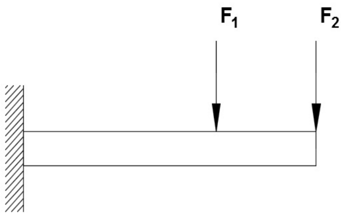
- **Diagrama de Corpo Livre (DCL)**

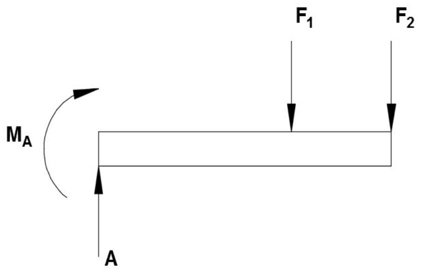


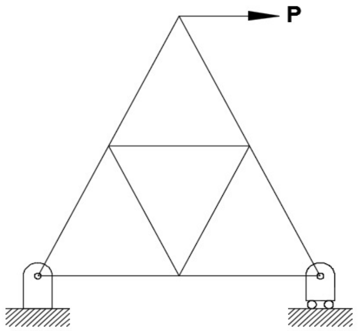


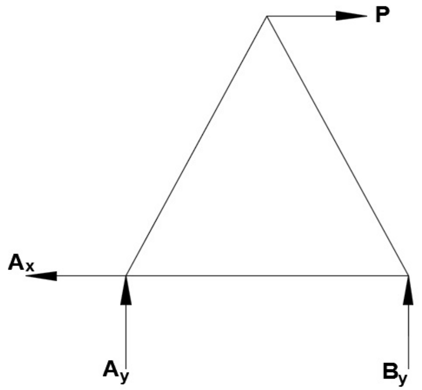




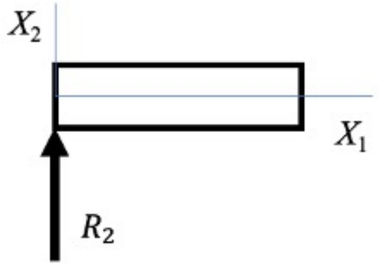
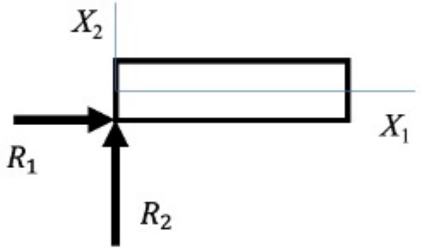
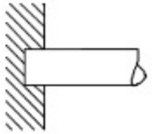
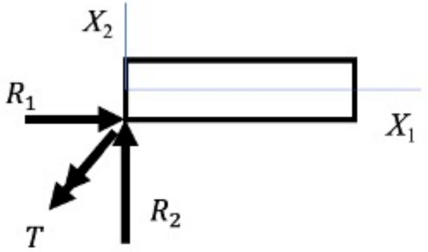




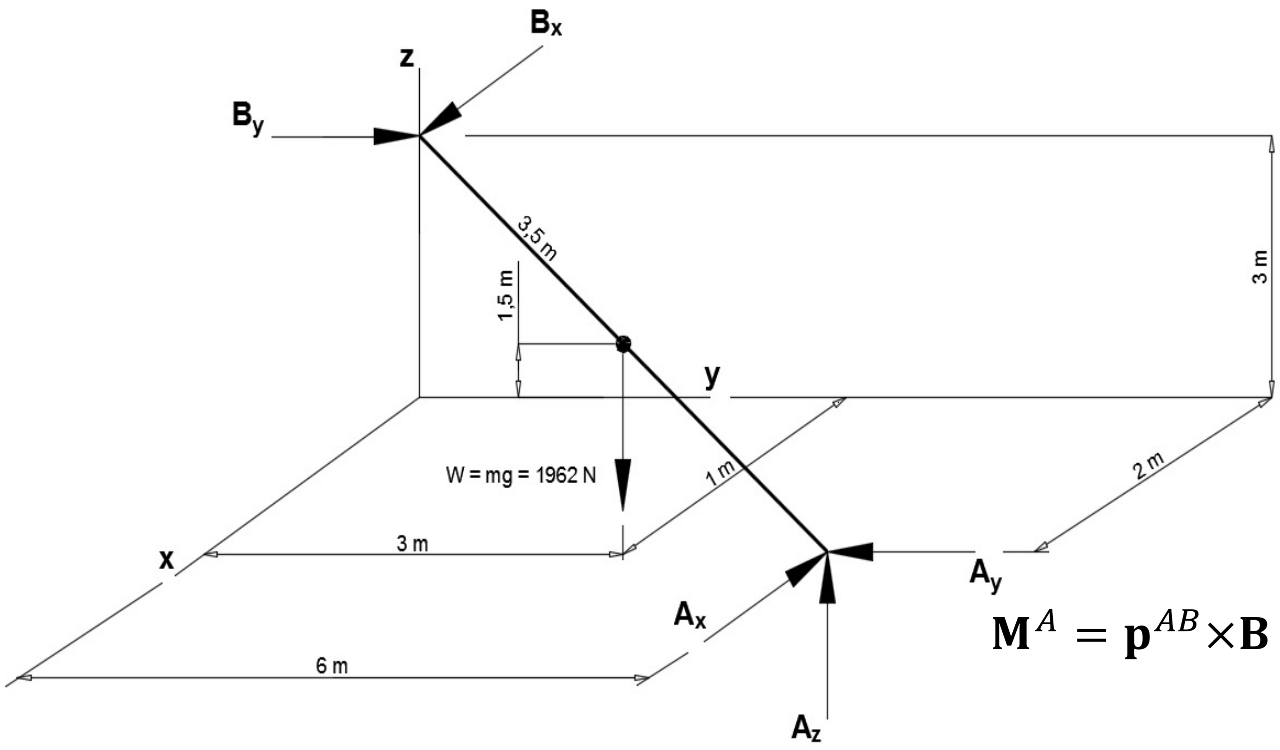




Vínculos

Vínculo	Representação	DCL
Apoio deslizante		
Apoio		
Engaste		

Equilíbrio



$$\mathbf{A} + \mathbf{B} - \mathbf{P} = 0$$

$$\mathbf{M}^A = \mathbf{p}^{AB} \times \mathbf{B} + \mathbf{p}^{AG} \times \mathbf{P} = 0$$

$$\mathbf{p}^{AB} = -2\mathbf{e}_1 - 6\mathbf{e}_2 + 3\mathbf{e}_3$$

$$\mathbf{p}^{AG} = -\mathbf{e}_1 - 3\mathbf{e}_2 + 1{,}5\mathbf{e}_3$$

$$\mathbf{M}^A = \mathbf{p}^{AB} \times \mathbf{B} + \mathbf{p}^{AG} \times \mathbf{P} = \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ -2 & -6 & 3 \\ B_1 & B_2 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ -1 & -3 & 1{,}5 \\ 0 & 0 & -1962 \end{vmatrix}$$

$$B_1 = 654$$

$$B_2 = 1962$$

$$\mathbf{M}^A = (-3B_2 + 5886)\mathbf{e}_1 + (-3B_1 + 1962)\mathbf{e}_2 + (-2B_2 + 6B_1)\mathbf{e}_3$$

$$\mathbf{A} + \mathbf{B} - \mathbf{P} = (A_1\mathbf{e}_1 + A_2\mathbf{e}_2 + A_3\mathbf{e}_3) + (B_1\mathbf{e}_1 + B_2\mathbf{e}_2) - (P\mathbf{e}_3) = 0$$

$$A_1 = -B_1 = 654$$

$$A_2 = -B_2 = 1962$$

$$A_3 = 1962$$