DINÂMICA: Introdução

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Mecânica - Histórico

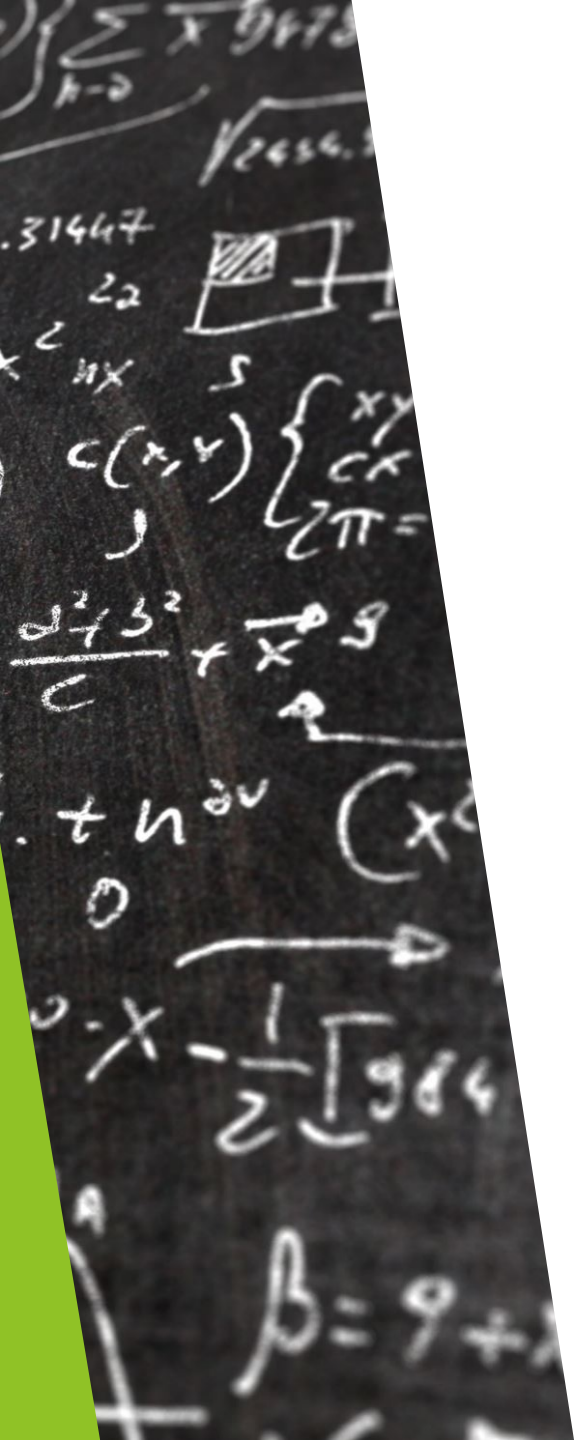
Aristóteles postulou, cerca de 300 AC, que a Terra seria uma esfera e que estaria estática, com o sol, os planetas e as estrelas girando em torno dela com órbitas circulares.

Ptolomeu, no segundo século DC, propôs um modelo baseado na ideia Aristotélica.

A igreja católica adotou essa ideia passando a considerá-la um dogma que se enraizou na cultura humana por muitos séculos.

Nicolau Copérnico (1473-1543), em 1514, propôs um modelo heliocêntrico em que a Terra girava em torno do sol em órbita circular.

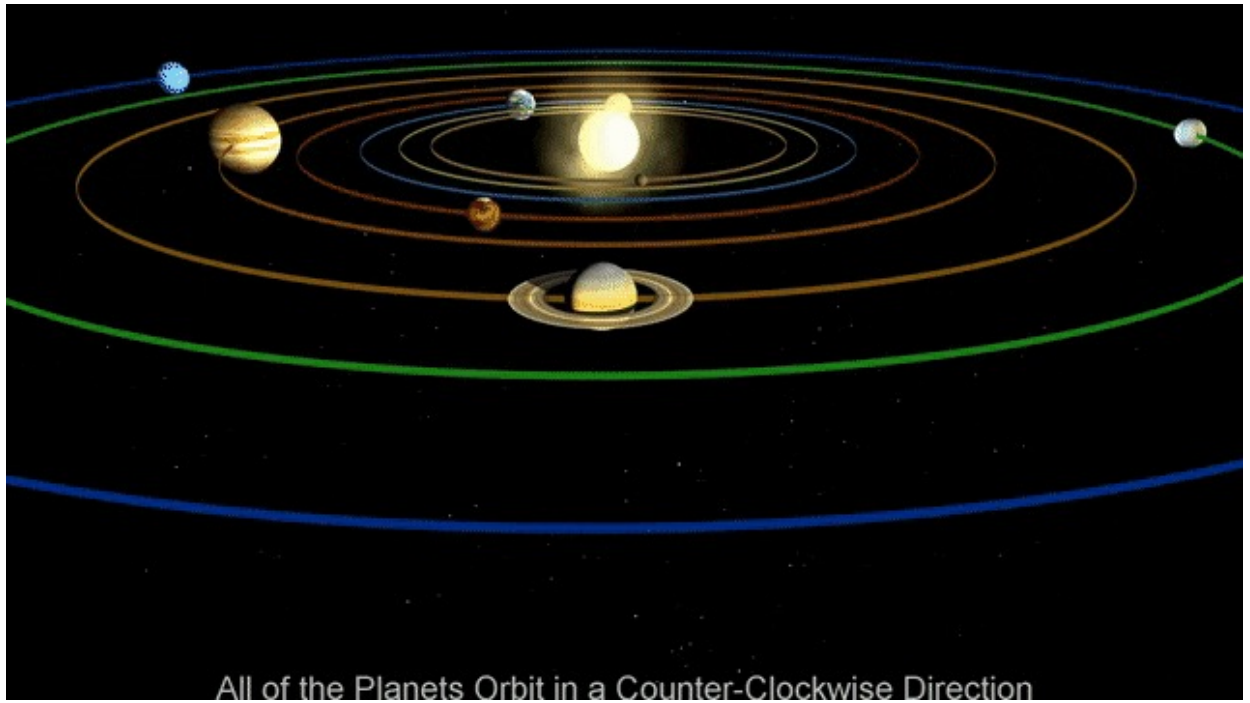
Johann Kepler (1571-1630) e Galileu Galilei (1564-1642) passaram a investigar o tema seguindo a ideia de Copérnico.

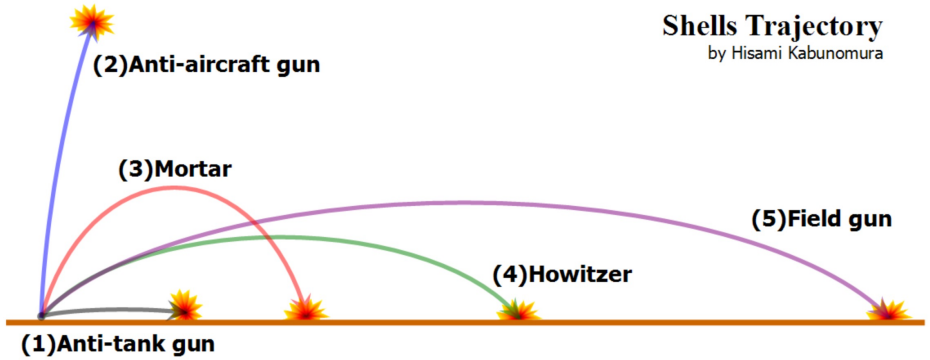


Mecânica - Histórico

- Galileu Galilei (1564–1642) é considerado o pai da ciência moderna: observação experimental como fonte de inspiração para formular teorias.
- Isaac Newton (1642–1727): leis de movimento e *cálculo diferencial*.
- Gottfried Leibniz (1646-1716): cálculo diferencial, em paralelo à Newton.
- Leonhard Euler (1707–1783): mecânica dos fluidos e dinâmica da rotação de corpos rígidos.
- Johann Bernoulli (1667-1748) que também possui contribuições notáveis para a mecânica.
- Joseph Louis Lagrange (1736-1813) consolida a ideia da mecânica analítica, construindo a base matemática para isso: o *cálculo variacional*.
- William Hamilton (1805-1865): mecânica analítica.

Partícula e Corpo

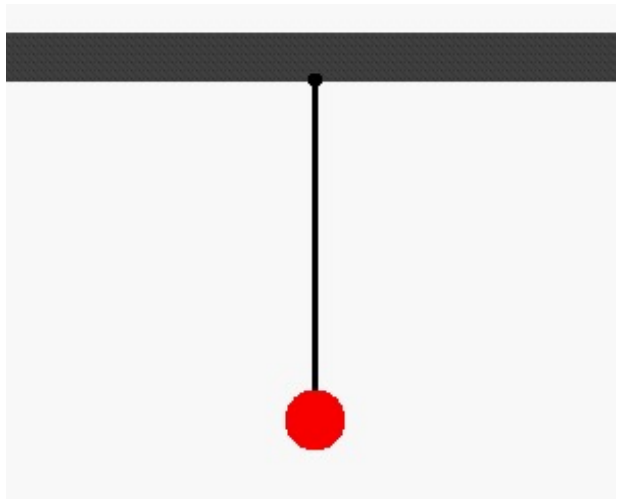


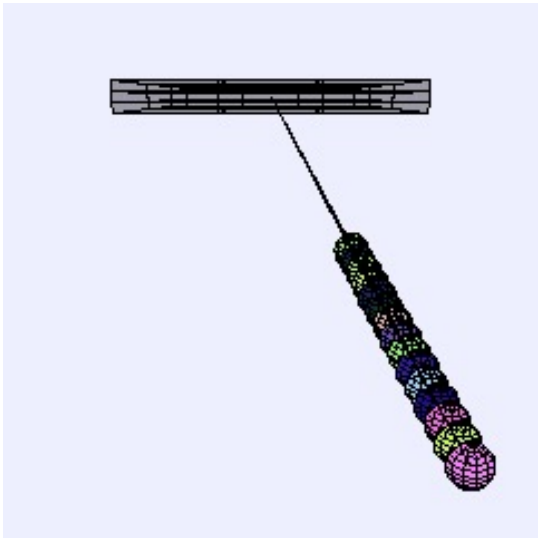


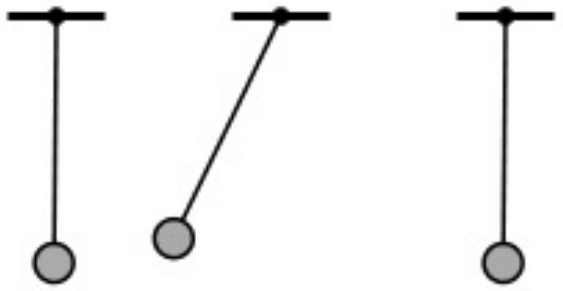
Partícula e Corpo: Football

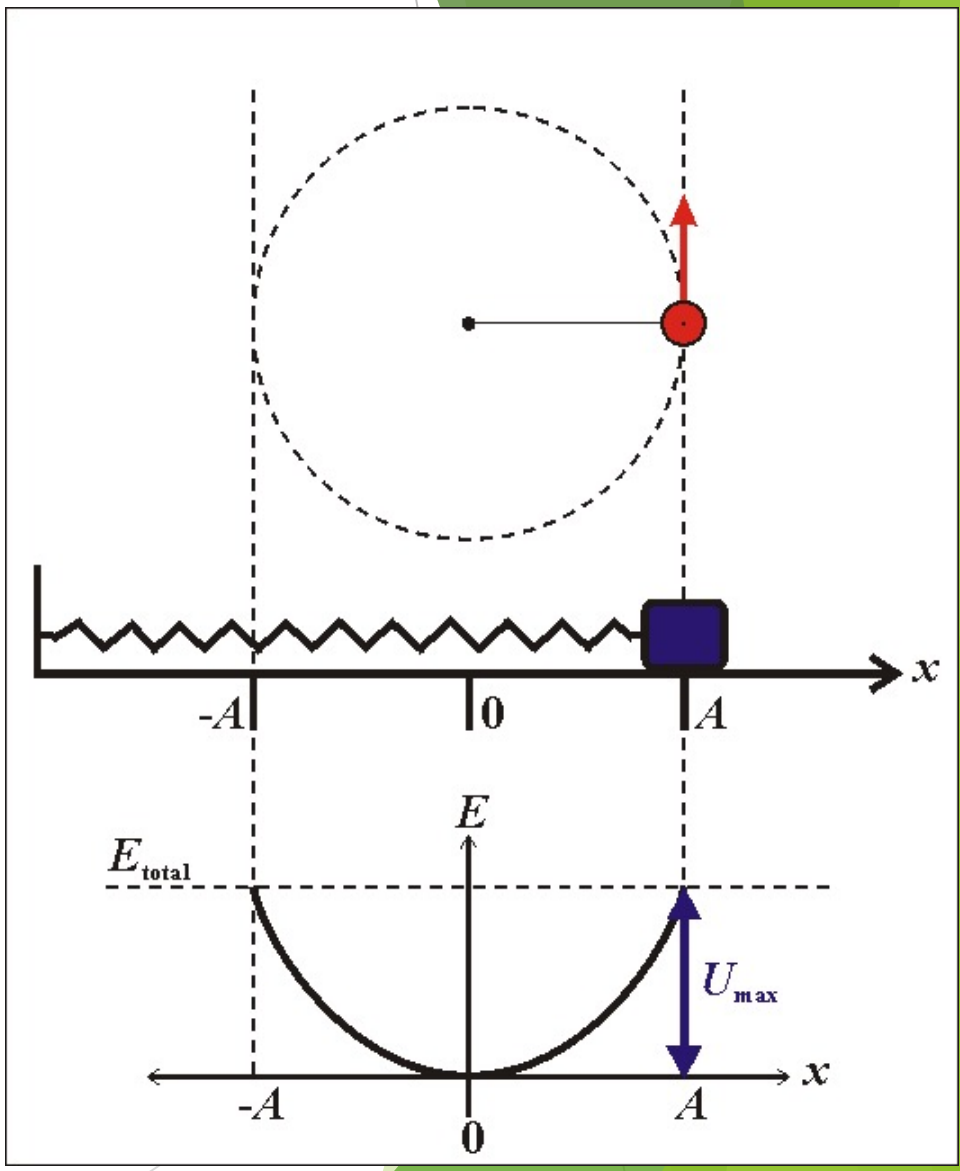
Partícula e Corpo: Soccer

Vibrações

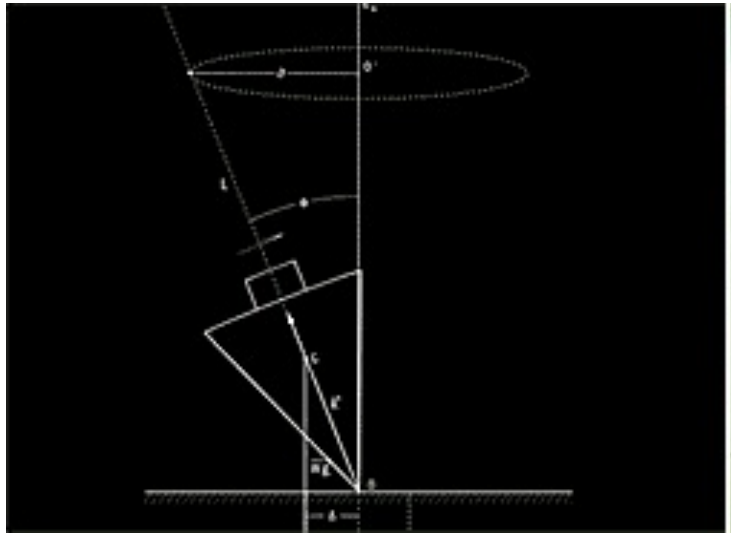




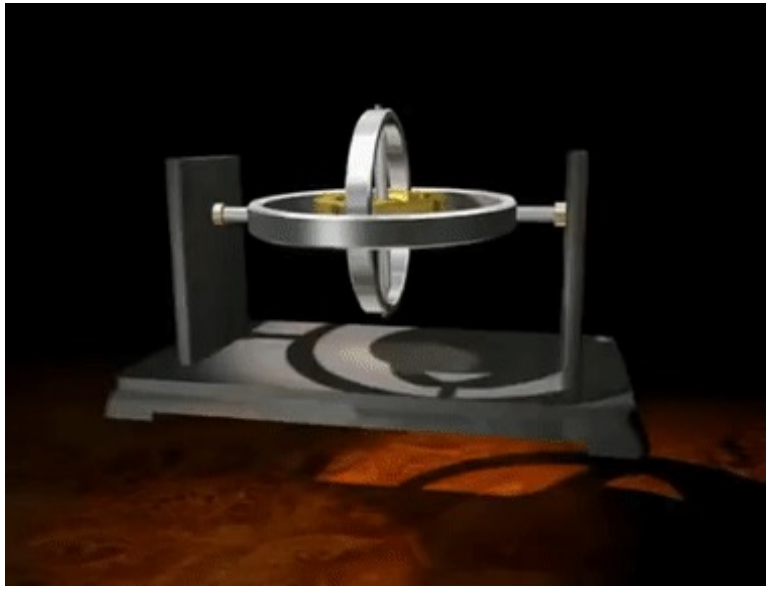




Pião

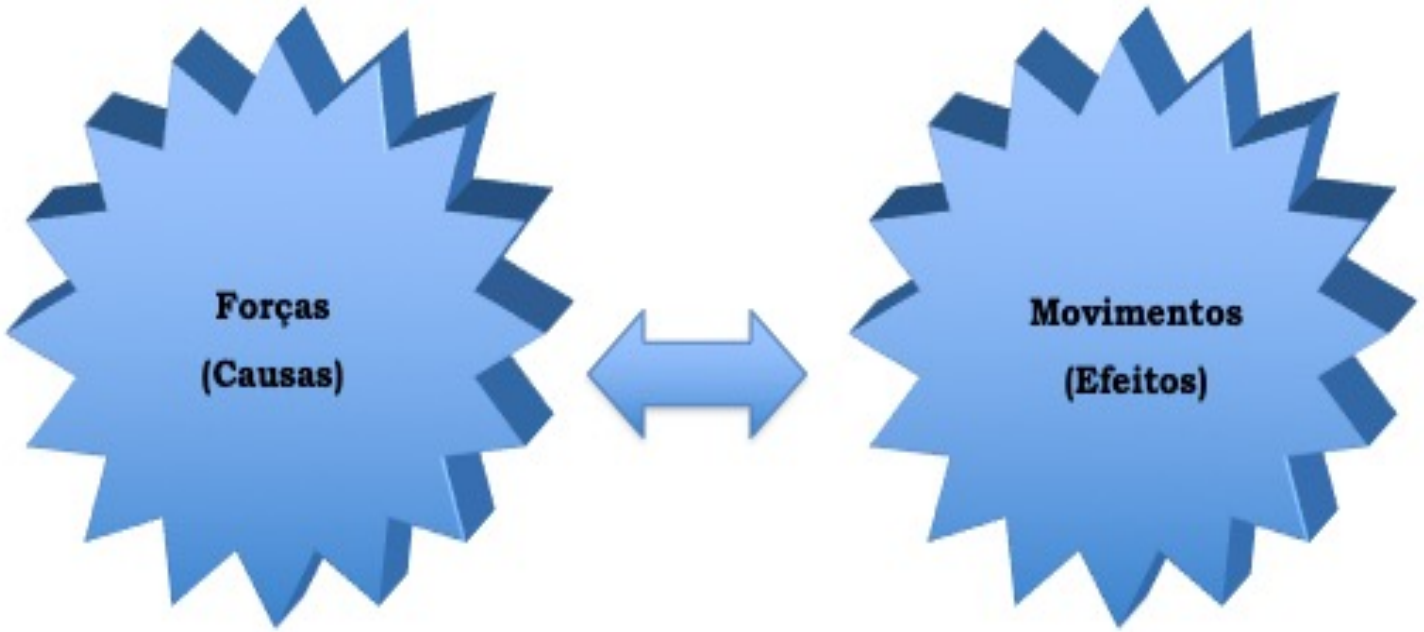


Giroscópio

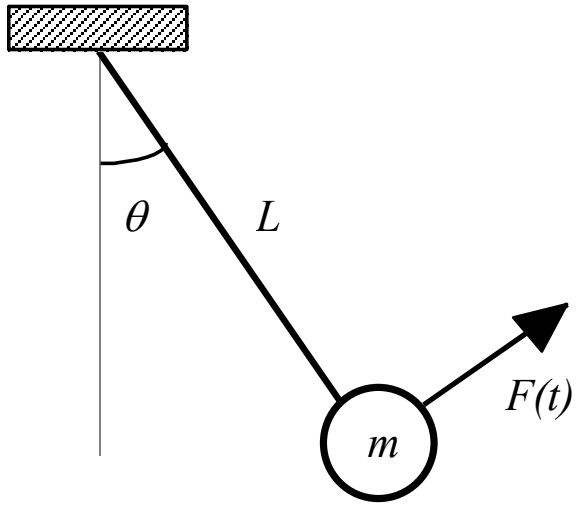


Uma breve fundamentação teórica

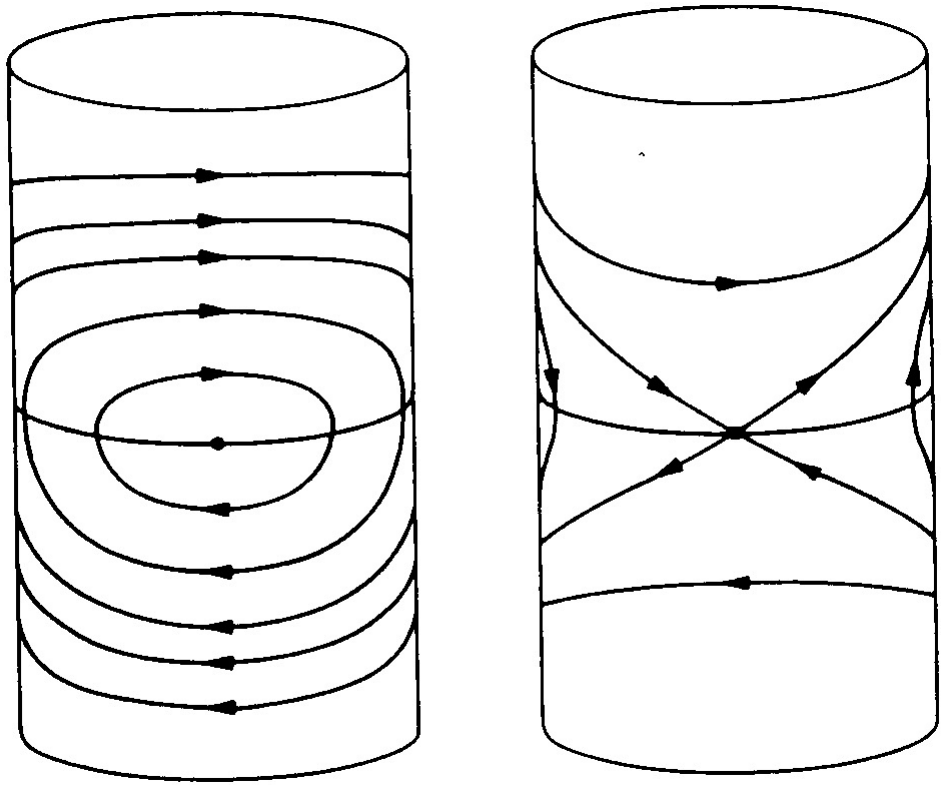
- Mecânica é a ciência que estuda forças e movimentos e as sua interações



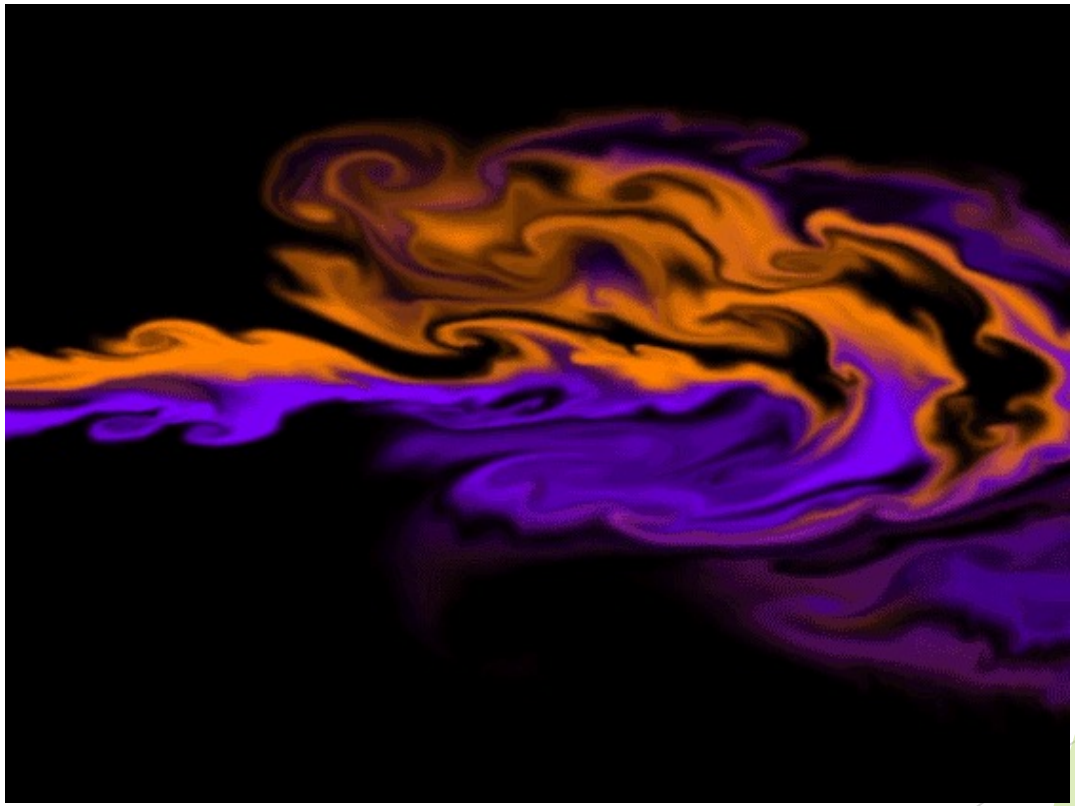
PÊNDULO



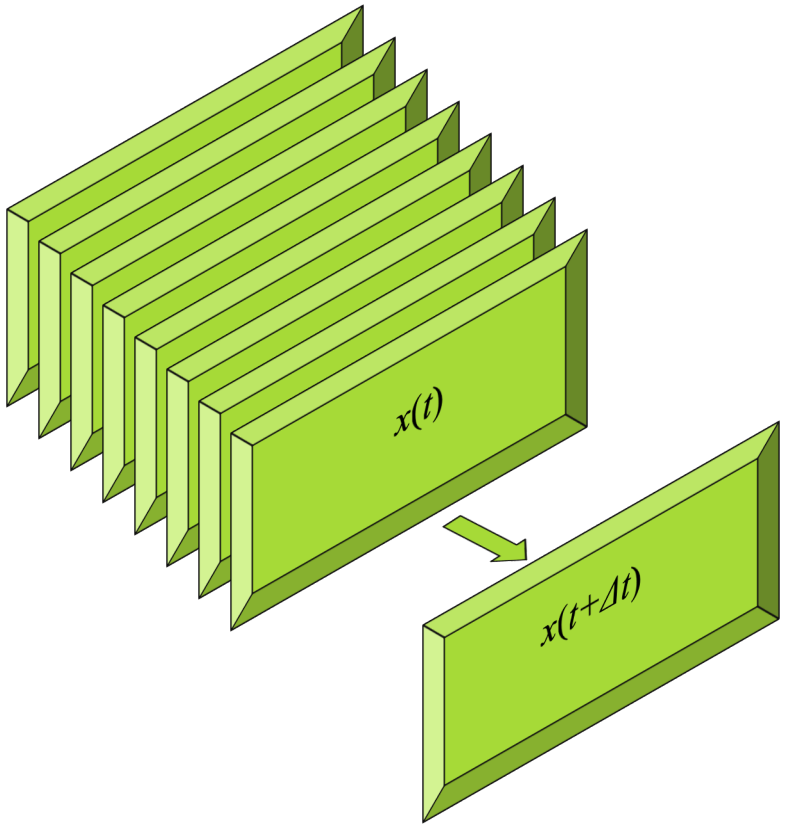
$$\ddot{\theta} + \alpha\dot{\theta} + \omega_0^2 \sin(\theta) = \mu \sin(\Omega t)$$



MECÂNICA

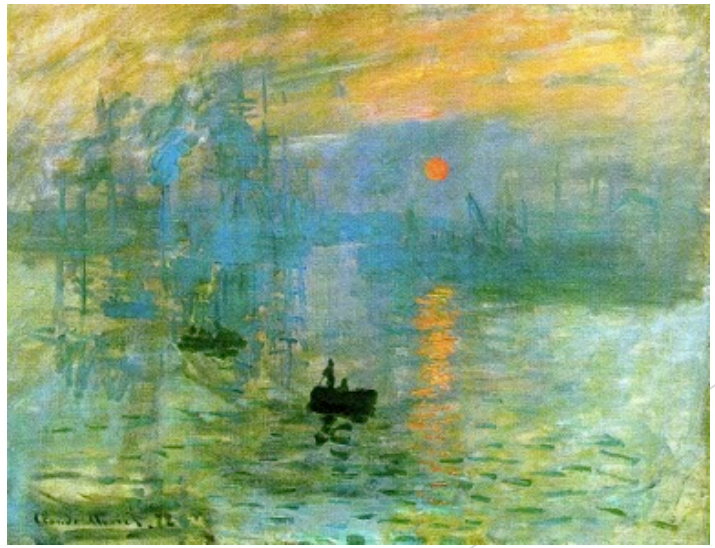


Equações Diferenciais:
Descrição quadro-a-quadro da realidade



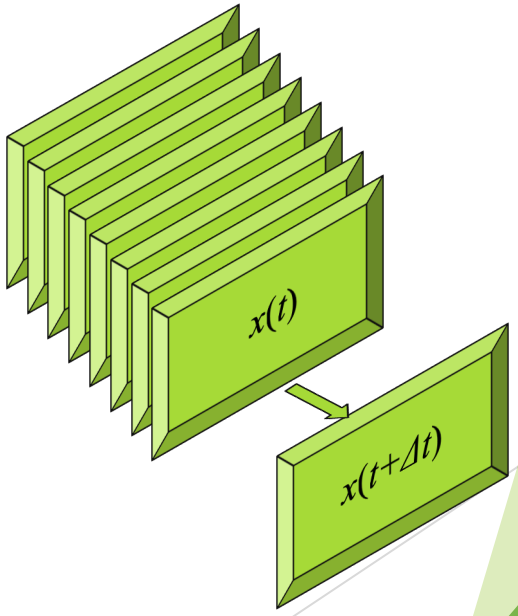
$$\dot{x} = f(x)\,,\ x \in R^n$$

Modelando a Realidade



Modelando a Realidade:
Descrição quadro-a-quadro da realidade

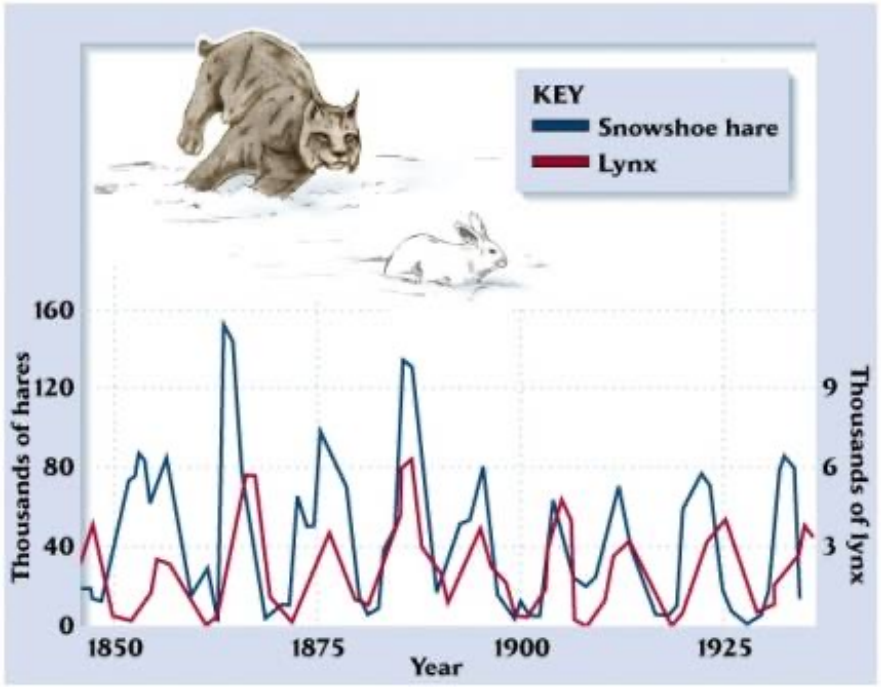
$$\dot{x} = f(x),\ x \in R^n$$



LOTKA-VOLTERRA: Predator-Presa

$$\dot{u} = \gamma u - \alpha uv$$

$$\dot{v} = \beta uv - \mu v$$



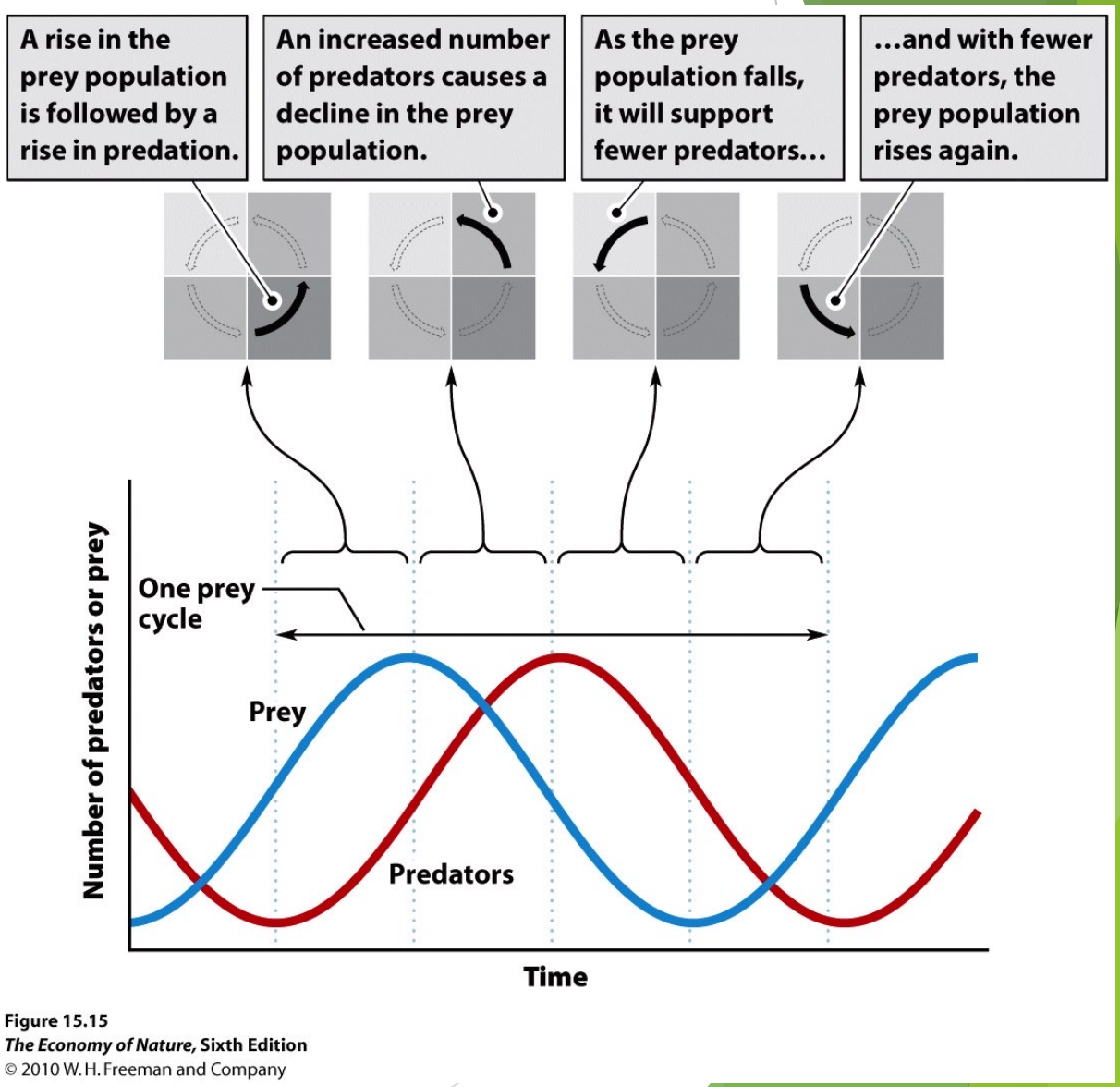
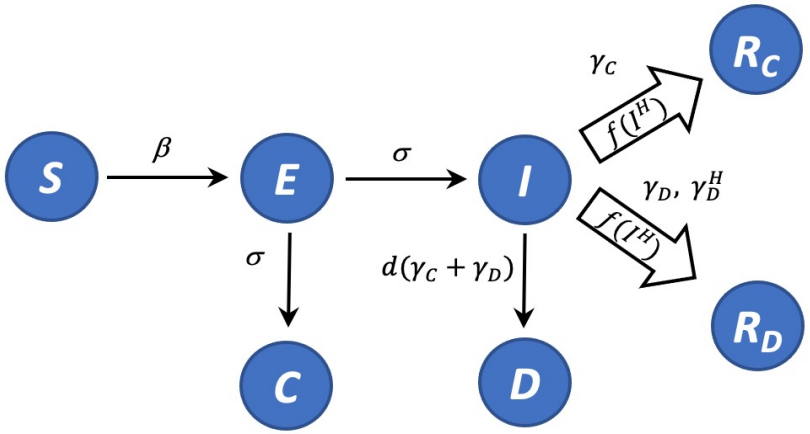
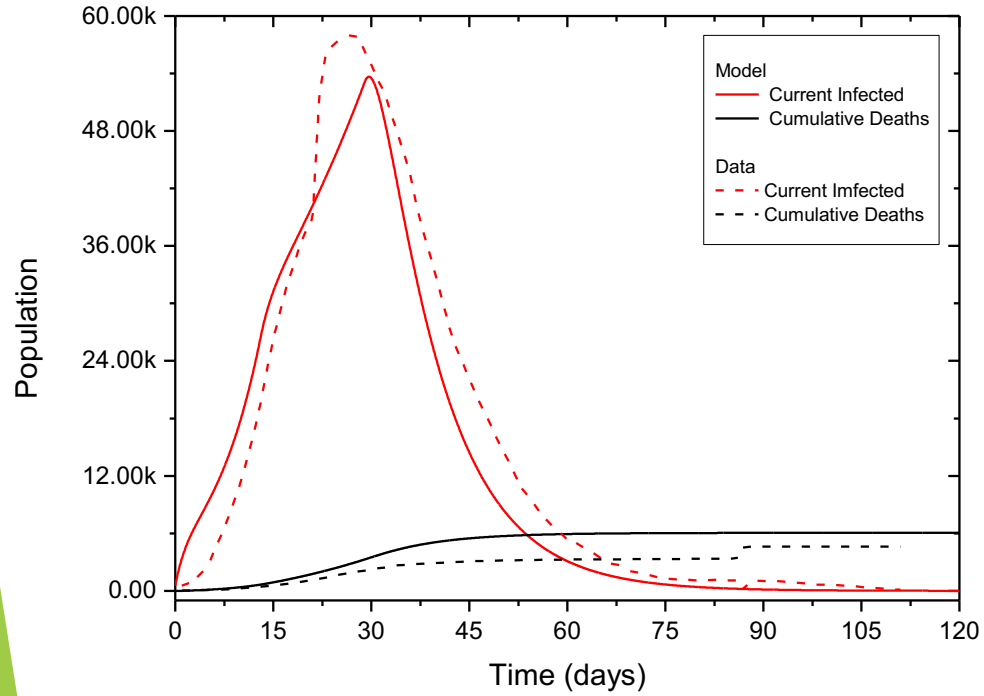


Figure 15.15
The Economy of Nature, Sixth Edition

DINÂMICA DO COVID-19: Modelo SEIR





$$\dot{S} = -\beta \frac{SI}{N}$$

$$\dot{E} = \beta \frac{SI}{N} - \sigma E$$

$$\dot{I} = \sigma E - (\gamma_C + \gamma_D)(I - I^H) - \gamma_D^H I^H$$

$$\dot{R}_C = \gamma_C (I - I^H)$$

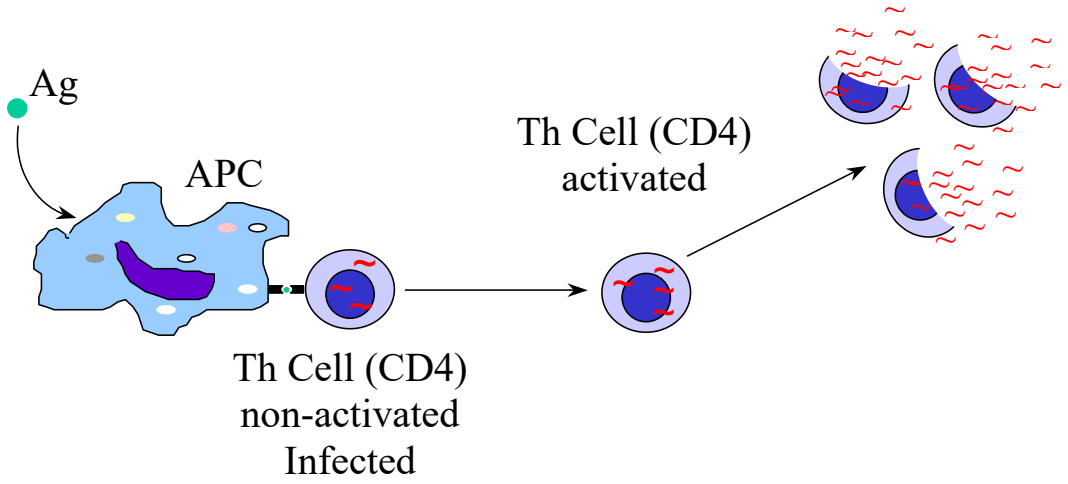
$$\dot{R}_D = \gamma_D (I - I^H) + \gamma_D^H I^H$$

$$\dot{D} = d(\gamma_C + \gamma_D) I - \lambda D$$

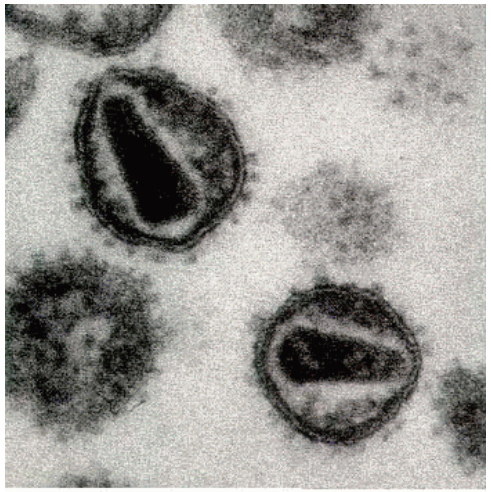
$$\dot{C} = \sigma E$$

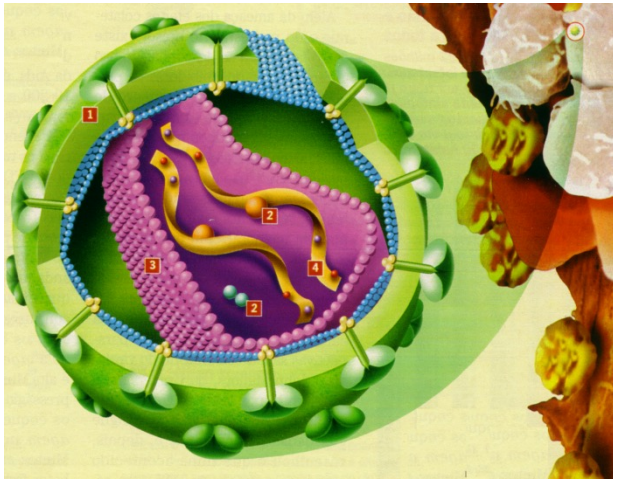
$$\dot{N} = -\dot{R}_D$$

DINÂMICA HIV-SISTEMA IMUNE



$$\dot{P} = \Lambda - \mu P - \gamma PV$$
$$\dot{X} = \gamma PV + rX + kIX - \beta XV - dX^2$$
$$\dot{Y} = \beta XV - \alpha Y$$
$$\dot{V} = \lambda\alpha Y - bV - \delta(X+Y)V - \alpha XV$$
$$\dot{I} = cI - hIX$$





P- Nonactivated cells

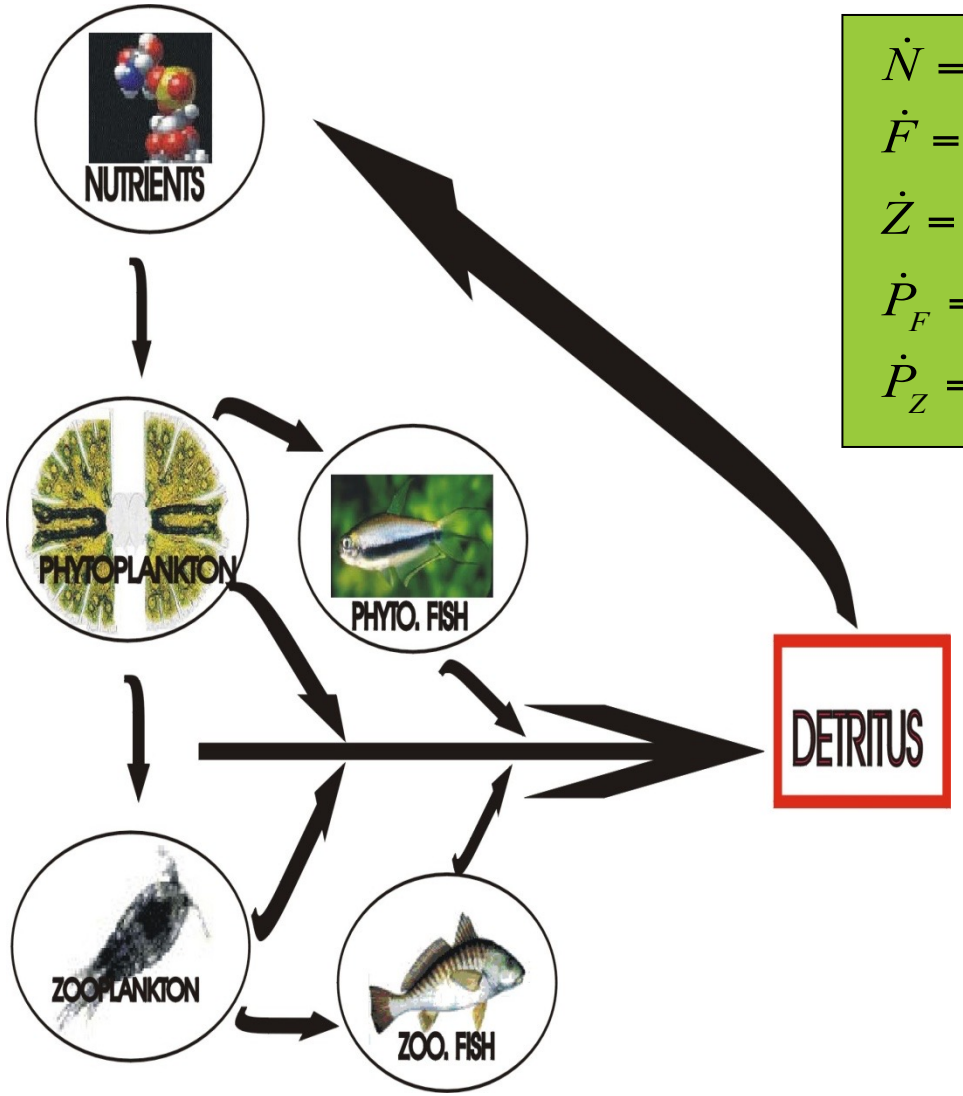
X- Activated but uninfected cells

Y- Infected cells

V- HIV virus

I- Infectious agent

EUTROFIZAÇÃO DOS CORPOS D'ÁGUA



$$\dot{N} = \alpha(t) - \beta PF(1+kF) - \delta N$$
$$\dot{F} = \varepsilon + \gamma\beta NF(1+kF) - \eta FZ(1+cZ) - \varphi P_F F - \lambda F$$
$$\dot{Z} = \mu + \nu\eta_1 F(1+cZ) - \zeta P_Z Z - \eta_2 P_Z F^2 - \psi Z$$
$$\dot{P}_F = \rho + \tau_1\varphi P_F F - \tau_2 P_F F^2 - \tau_3 P_F^2 - \theta P_F$$
$$\dot{P}_Z = \varsigma + \upsilon_1\zeta P_Z Z - \upsilon_2 P_Z F^2 - \upsilon_3 P_Z^2 - \sigma P_Z$$

N – Nutrients

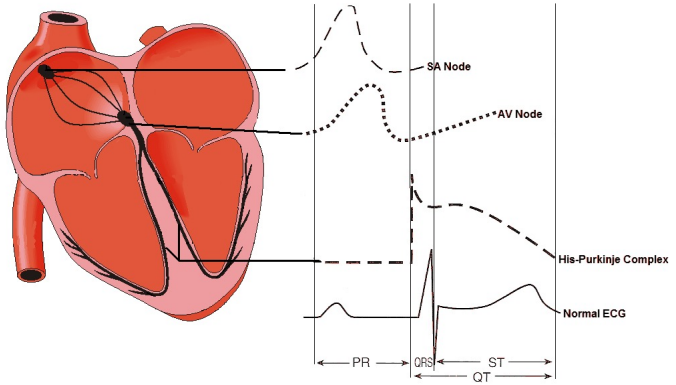
F – Phytoplankton

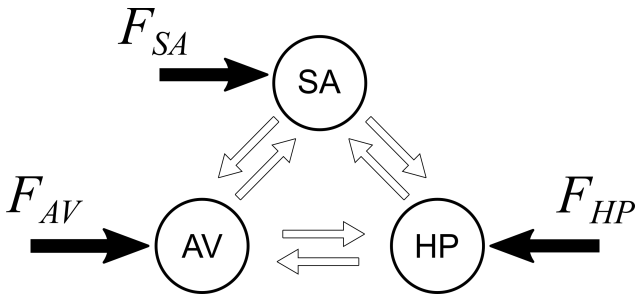
Z – Zooplankton

P_F - Phytoplanktivorous fishes

P_Z - Zooplanktivorous fishes

DINÂMICA DO CORAÇÃO





$$\dot{x}_1 = x_2$$

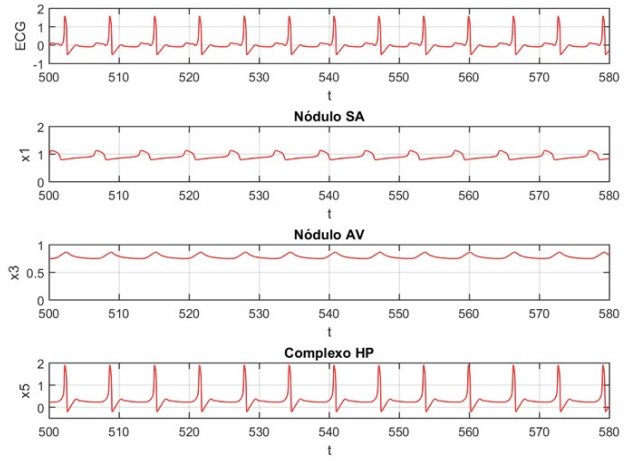
$$\dot{x}_2 = F_{SA}(t) - \alpha_{SA} x_2 (x_1 - v_{SA_1})(x_1 - v_{SA_2}) - \frac{x_1(x_1 + d_{SA})(x_1 + e_{SA})}{d_{SA} e_{SA}} - k_{AV-SA} x_1 + k^{\tau}_{AV-SA} x_3^{\tau_{AV-SA}} - k_{HP-SA} x_1 + k^{\tau}_{HP-SA} x_5^{\tau_{HP-SA}}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = F_{AV}(t) - \alpha_{AV} x_4 (x_3 - v_{AV_1})(x_3 - v_{AV_2}) - \frac{x_3(x_3 + d_{AV})(x_3 + e_{AV})}{d_{AV} e_{AV}} - k_{SA-AV} x_3 + k^{\tau}_{SA-AV} x_1^{\tau_{SA-AV}} - k_{HP-AV} x_3 + k^{\tau}_{HP-AV} x_5^{\tau_{HP-AV}}$$

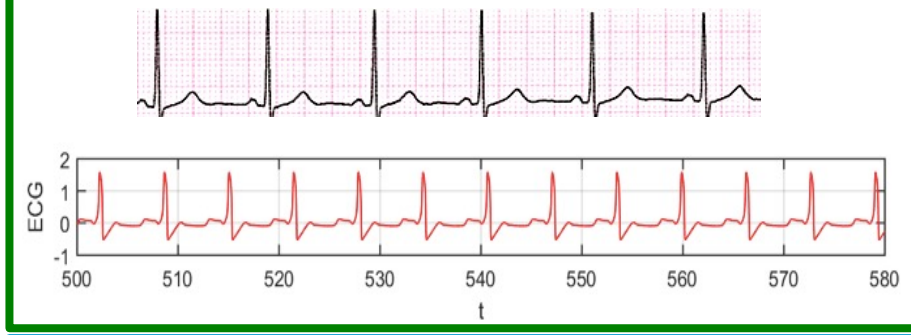
$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = F_{HP}(t) - \alpha_{HP} x_6 (x_5 - v_{HP_1})(x_5 - v_{HP_2}) - \frac{x_5(x_5 + d_{HP})(x_5 + e_{HP})}{d_{HP} e_{HP}} - k_{SA-HP} x_5 + k^{\tau}_{SA-HP} x_1^{\tau_{SA-HP}} - k_{AV-HP} x_5 + k^{\tau}_{AV-HP} x_3^{\tau_{AV-HP}}$$

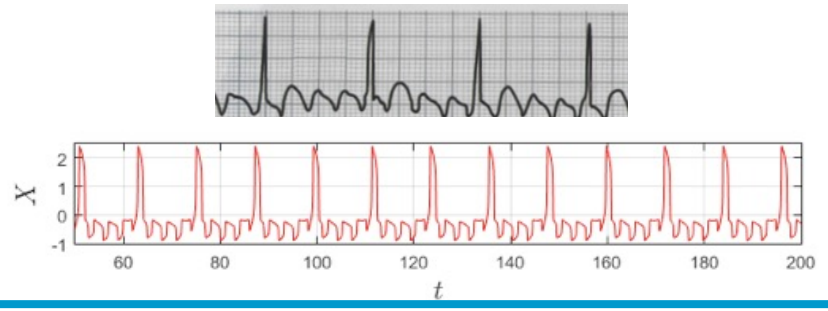


DINÂMICA DO CORAÇÃO

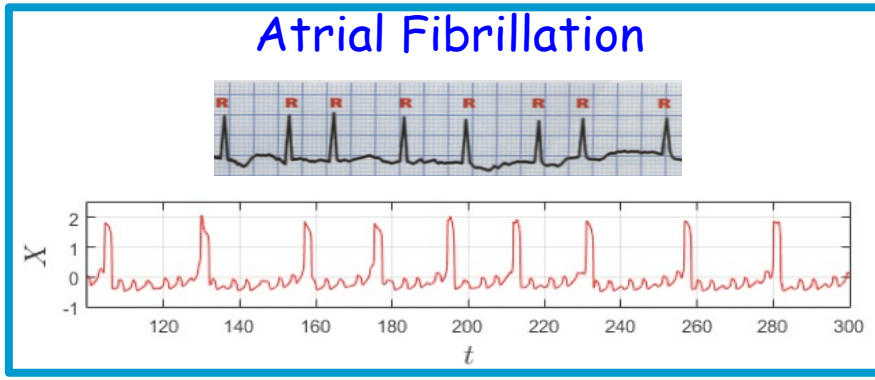
Normal



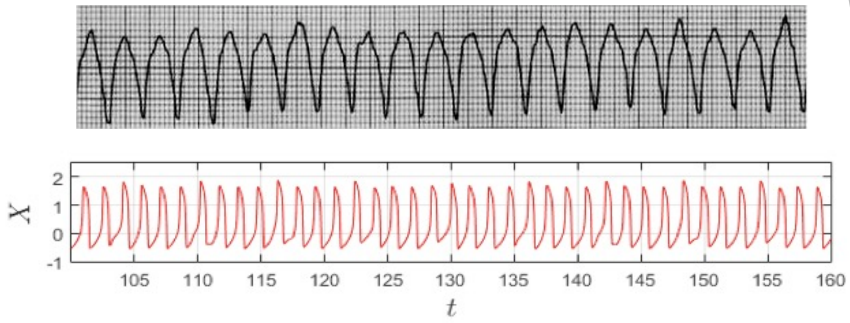
Atrial Flutter



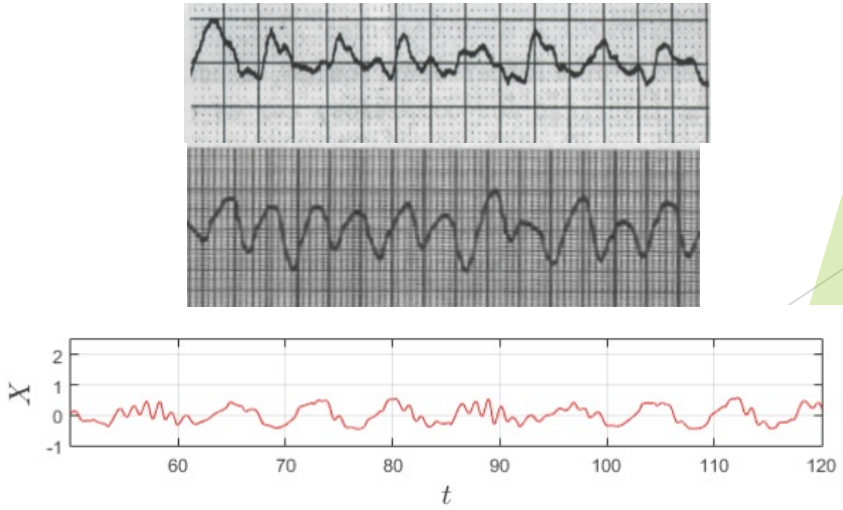
Atrial Fibrillation



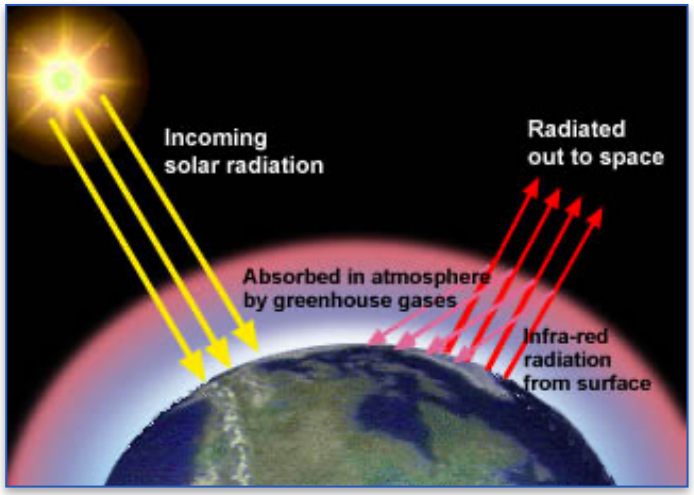
Ventricular Flutter

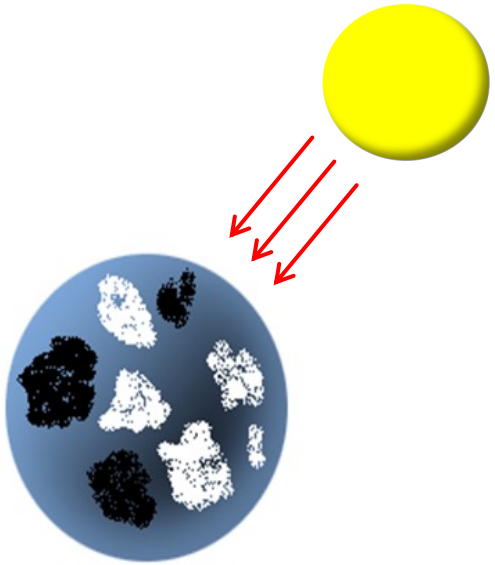


Ventricular Fibrillation



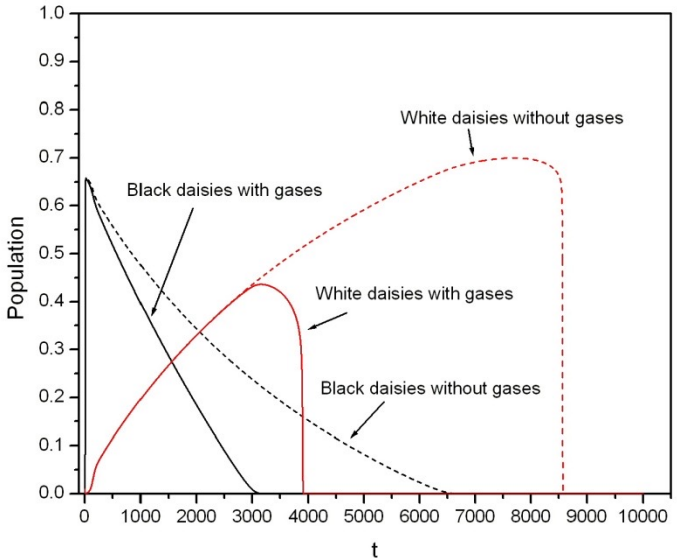
DINÂMICA DO AQUECIMENTO GLOBAL: Planeta das Margaridas

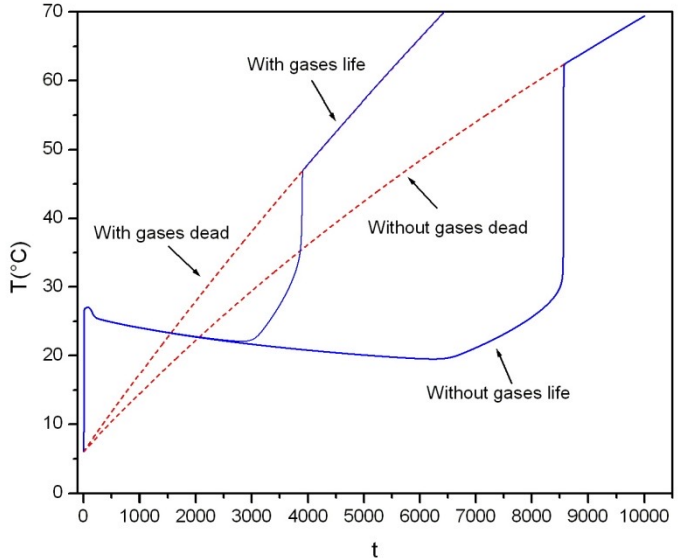




$$\dot{W} = W\left[\alpha_g\,\beta(T_W) - \gamma\right]$$
$$\dot{B} = B\left[\alpha_g\,\beta(T_B) - \gamma\right]$$
$$\dot{T} = \left(\frac{1}{c}\right)\left[SL(1-A) - \sigma T^4\right]$$

$$\alpha_g = p - W - B - G$$
$$A = W a_w + B a_b + \alpha_g a_g + G a_G$$
$$\beta(T_i) = Be^{-\frac{1}{s}(T_i - T_{\text{opt}})^2}$$
$$T_i^4 = q(A - a_i) + T^4$$



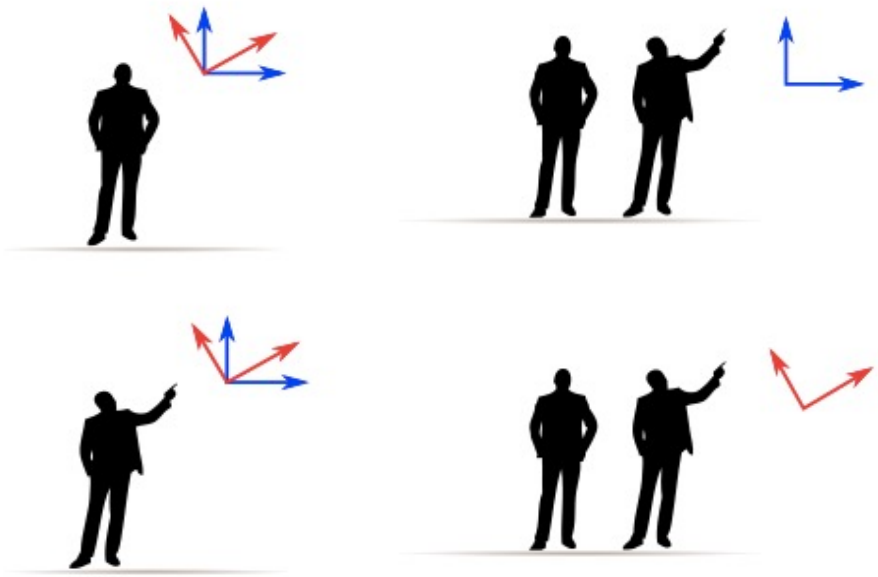


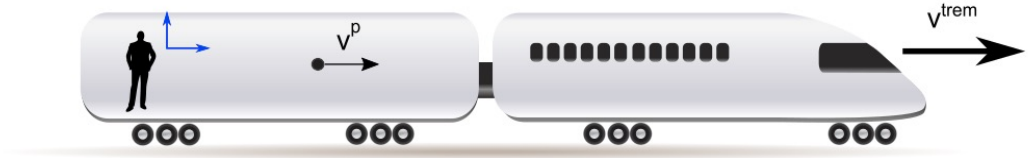
Cinemática

- A cinemática descreve a geometria do movimento não se importando com as suas causas.
- O estado de uma partícula é definido a partir de sua posição e velocidade: grandezas vetoriais.

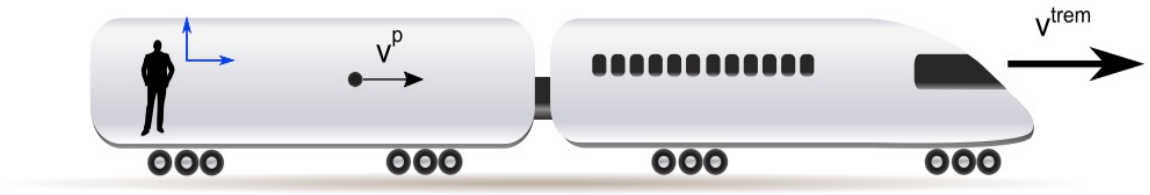
Referencial e Obervador

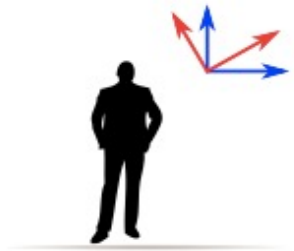
- Referencial: físico
- Observador (base): geométrico

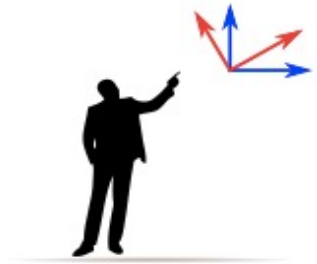


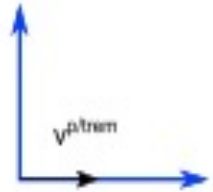


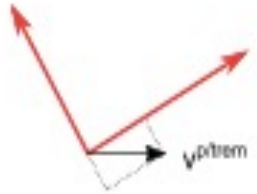
Cinemática

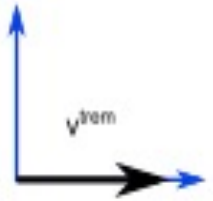


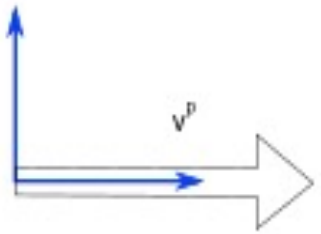


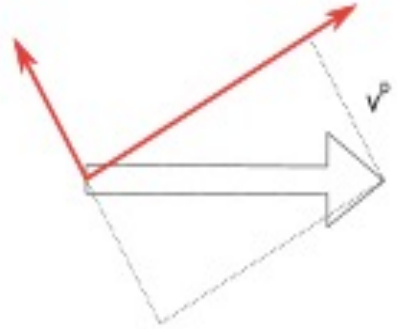












Dinâmica