

Adaptive fuzzy sliding mode control of smart structures

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Abstract. Smart structures are usually designed with a stimulus-response mechanism to mimic the autoregulatory process of living systems. In this work, in order to simulate this natural and self-adjustable behavior, an adaptive fuzzy sliding mode controller is applied to a shape memory two-bar truss. This structural system exhibits both constitutive and geometrical nonlinearities presenting the snap-through behavior and chaotic dynamics. On this basis, a variable structure controller is employed for vibration suppression in the chaotic smart truss. The control scheme is primarily based on sliding mode methodology and enhanced by an adaptive fuzzy inference system to cope with modeling inaccuracies and external disturbances. The robustness of this approach against both structured and unstructured uncertainties enables the adoption of simple constitutive models for control purposes. The overall control system performance is evaluated by means of numerical simulations, promoting vibration reduction and avoiding snap-through behavior.

1 Introduction

The term smart structures and systems has been used to identify mechanical systems that are capable of changing their geometry or physical properties with the purpose of performing a specific task. They must be equipped with sensors and actuators that induce such controlled alterations. Several applications in different fields of sciences and engineering have been developed with this innovative idea, employing some of the so-called smart materials. Shape memory alloys (SMAs), piezoelectric materials and magneto-rheological fluids are some of the smart materials largely employed in structural systems.

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Specifically, shape memory alloys are being used in situations where high force, large strain, and low frequency structural control are needed. SMA actuators are easy to manufacture, relatively lightweight, and able of producing high forces or displacements. Self-actuating fasteners, thermally actuator switches and several bioengineering devices are some examples of these SMA applications. Aerospace technology are also using SMAs for distinct purposes as space savings achieved by self-erectable structures, stabilizing mechanisms, non-explosive release devices, among others. Micromanipulators and robotics actuators have been built employing SMAs properties to mimic the smooth motions of human muscles. Moreover, SMAs are being used as actuators for vibration and buckling control of flexible structures.

SMA thermomechanical behavior is related to thermoelastic martensitic transformations. The shape memory effect is a phenomenon where apparent plastically deformed objects may recover their original form after going through a proper heat treatment. The pseudoelastic behavior is characterized by complete strain recovery accompanied by large hysteresis in a loading-unloading cycle [1]. Fibers of shape-memory alloys can be used to fabricate hybrid composites exhibiting these two different but related material behaviors. Detailed description of the shape memory effect and other phenomena associated with martensitic phase transformations, as well as examples of applications in the context of smart structures, may be found in references [2–6].

The investigation of SMA structures has different approaches. The finite element method is an important tool to this aim. Auricchio & Taylor [7] proposed a three-dimensional finite element model. Lagoudas et al. [8] considered the thermomechanical response of a laminate with SMA strips. La Cava et al. [9] considered SMA bars and Bandeira et al. [10] treated truss structures. The response of SMA beams was treated by Collet et al. [11], which analyzes the dynamical response, as well as Auricchio & Sacco [12]. Auricchio & Petrini [13, 14] presented a solid finite element to describe the thermo-electro-mechanical problem that is used to simulate different SMA composite applications. Dual kriging interpolation has been employed with finite element method in order to describe the shape memory behavior in different reports [15]. Masud et al. [16], Bhattacharyya et al. [17], Liu et al. [18] are other contributions in this field.

The two-bar truss, also known as the von Mises truss, is an important archetypal model, largely employed to evaluate stability characteristics of framed structures as well as of flat arches, and of many other physical phenomena associated with bifurcation buckling [19]. The nonlinear dynamics of this system may exhibit a number of interesting, complex behaviors. The snap-through behavior, represented by a displacement jump, is a classical example of the complexity behind this simple structure.

The dynamic behavior of the two-bar truss is even richer when material nonlinearities are considered. In particular, the present contribution deals with two-bar trusses made from shape memory materials. Savi & Nogueira [20] and Savi et al. [21] presented numerical investigations of this kind of structure by considering different constitutive models to describe the thermomechanical behavior of the SMAs.

In this work, an adaptive fuzzy sliding mode controller is proposed for vibration suppression in an SMA two-bar truss. A polynomial constitutive model is assumed to describe the behavior of the shape memory bars. Although this model is simple and does not present a proper description of the hysteretic behavior, it can qualitatively represent the general SMA behavior. This system has a rich dynamic response and can easily reach a chaotic behavior even at moderate loads and frequencies [21]. Regarding the adopted control scheme, due to its robustness to modeling imprecisions and adaptive feature [22], a smooth sliding mode controller with an embedded fuzzy inference system is considered. The adoption of a robust and adaptive control law allows simple constitutive models, such as the polynomial equation considered in

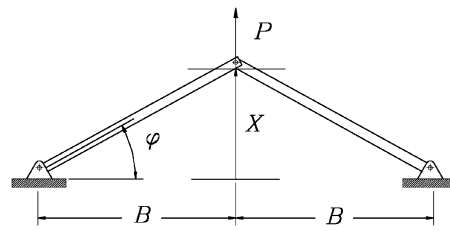


Fig. 1. Two-bar truss (von Mises truss).

this work, to be used for control purposes. Numerical simulations are carried out in order to demonstrate the control system performance.

The main goal is the vibration reduction, avoiding some critical responses as snap-through behavior. A linear actuator is employed to help this control procedure and therefore, the SMA actuation is not employed for the control purposes. In this regard, we are investigating an SMA structure that needs an appropriate control using external actuators. It is important to highlight that SMA properties are being used to achieve other goals than control. This situation is common in distinct applications that include aerospace systems as self-erectable structures.

2 Dynamic model

The two-bar truss is depicted in Fig. 1. This plane, framed structure, is formed by two identical bars, free to rotate around their supports and at the joint.

In the present investigation, we consider a shape memory two-bar truss where each bar presents the shape memory and pseudoelastic effects. The two identical bars have length L and cross-sectional area A . They form an angle φ with a horizontal line and are free to rotate around their supports and at the joint, but only on the plane formed by the two bars (Fig. 1). The critical Euler load of both bars is assumed to be sufficiently large so that buckling will not occur in the simulations reported here.

We further assume that the structure's mass is entirely concentrated at the junction between the two bars. Hence, the structure is divided into segments without mass, connected by nodes with lumped mass that is determined by static considerations. We consider only symmetric motions of the system, which implies that the concentrated mass, m , can only move vertically. The symmetric, vertical displacement is denoted by X . Under these assumptions, the dynamic behavior is expressed through the following equation of motion

$$-2F \sin \varphi - c\dot{X} + P = m\ddot{X} \quad (1)$$

where F is the force on each bar, P is an external force and $c\dot{X}$ is a linear viscous damping term used to represent all dissipation mechanisms.

There are several works dedicated to the constitutive description of the thermo-mechanical behavior of shape memory alloys [2,3]. In this article, we employ polynomial constitutive model to describe the thermomechanical behavior of the SMA bars [23,24]. Despite the simplicity of this model, it allows an appropriate qualitative description of the dynamical response of the system. Its major drawback is the hysteresis description. In this regard, for control purposes, it should be appropriate with robust controllers that could deal with unmodelled dynamics. Here, dissipation process is represented by an equivalent viscous damping term.

Polynomial model is concerned with one-dimensional media employing a sixth degree polynomial free energy function in terms of the uniaxial strain, ε . The form of

the free energy is chosen in such a way that its minima and maxima are respectively associated with the stability and instability of each phase of the SMA. As it is usual in one-dimensional models proposed for SMAs [25], three phases are considered: austenite (A) and two variants of martensite (M+, M-). Hence, the free energy is chosen such that for high temperatures it has only one minimum at vanishing strain, representing the equilibrium of the austenitic phase. At low temperatures, martensite is stable, and the free energy must have two minima at non-vanishing strains. At intermediate temperatures, the free energy must have equilibrium points corresponding to both phases. Under these restrictions, the uniaxial stress, σ , is a fifth-degree polynomial of the strain [25], i.e.

$$\sigma = a_1(T - T_M)\varepsilon - a_2\varepsilon^3 + a_3\varepsilon^5 \quad (2)$$

where a_1 , a_2 and a_3 are material constants, and T the temperature, while T_M is the temperature below which the martensitic phase is stable. If T_A is defined as the temperature above which austenite is stable, and the free energy has only one minimum at zero strain, it is possible to write the following condition,

$$T_A = T_M + \frac{1}{4} \frac{a_2^2}{a_1 a_3}. \quad (3)$$

Therefore, the constant a_3 may be expressed in terms of other constants of the material. Now, the following strain definition is considered,

$$\varepsilon = \frac{L}{L_0} - 1 = \frac{\cos \varphi_0}{\cos \varphi} - 1 \quad (4)$$

with L_0 and φ_0 representing the nominal values of L and φ , respectively.

At this point, we can use the constitutive Eq. (2) together with kinematic Eq. (4) into the equation of motion (1), obtaining the governing equation of the SMA two-bar truss:

$$\begin{aligned} m\ddot{X} + c\dot{X} + \frac{2A}{L_0}X \left\{ [a_1(T - T_M) - 3a_2 + 5a_3] + \right. \\ + [-a_1(T - T_M) + a_2 - a_3]L_0(X^2 + B^2)^{-1/2} + \\ + [3a_2 - 10a_3]\frac{1}{L_0}(X^2 + B^2)^{1/2} + \\ + [-a_2 + 10a_3]\frac{1}{L_0^2}(X^2 + B^2) + \\ \left. - \frac{5a_3}{L_0^3}(X^2 + B^2)^{3/2} + \frac{a_3}{L_0^4}(X^2 + B^2)^2 \right\} = P(t) \end{aligned} \quad (5)$$

where B is the horizontal projection of each truss bar (Fig. 1).

Considering a periodic excitation $P = P_0 \sin(\omega t)$, Eq. (5) may be written in non-dimensional form as

$$\begin{aligned} x' &= y \\ y' &= \gamma \sin(\Omega\tau) - \xi y + x \left\{ -[(\theta - 1) - 3\alpha_2 + 5\alpha_3] + \right. \\ &+ [(\theta - 1) - \alpha_2 + \alpha_3](x^2 + b^2)^{-1/2} - [3\alpha_2 - 10\alpha_3](x^2 + b^2)^{1/2} + \\ &\left. + [-\alpha_2 + 10\alpha_3](x^2 + b^2) + 5\alpha_3(x^2 + b^2)^{3/2} - \alpha_3(x^2 + b^2)^2 \right\} \end{aligned} \quad (6)$$

where ξ is a non-dimensional viscous damping coefficient. The dissipation due to hysteretic effect may be considered by assuming an equivalent viscous damping related to

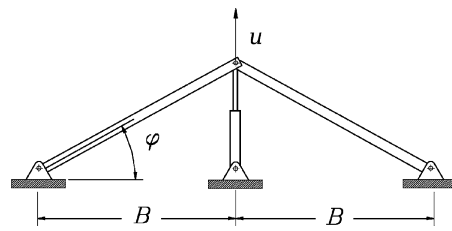


Fig. 2. Actuated truss structure.

this parameter. Moreover, the following non-dimensional parameters are considered:

$$x = \frac{X}{L}, \quad \gamma = \frac{P_0}{mL_0\omega_0^2}, \quad \omega_0^2 = \frac{2Aa_1T_M}{mL_0}, \quad \Omega = \frac{\omega}{\omega_0}, \quad \tau = \omega_0 t,$$

$$\theta = \frac{T}{T_M}, \quad \alpha_2 = \frac{a_2}{a_1T_M}, \quad \alpha_3 = \frac{a_3}{a_1T_M}, \quad b = \frac{B}{L_0} \quad \text{and} \quad (\cdot)' = \frac{d(\cdot)}{d\tau}.$$

3 Controller design

According to Bessa and Barrêto [22], adaptive fuzzy inference systems could be combined with smooth sliding mode controllers to improve the overall performance of the control system. This approach has been successfully employed in several applications, ranging from underwater robotic vehicles [26,27] and electro-hydraulic actuated systems [28] to chaos control in a nonlinear pendulum [29,30].

In this work, the proposed control problem is to ensure that, even in the presence of modeling inaccuracies and external disturbances, the state vector $\mathbf{x} = [x, y]$ will be stabilized in a desired state $\mathbf{x}_d = [x_d, y_d]$, i.e. the error vector $\tilde{\mathbf{x}} = [\tilde{x}, \tilde{y}] = [x - x_d, y - y_d] \rightarrow \mathbf{0}$ as $\tau \rightarrow \infty$.

In order to ensure the stabilization, a linear actuator is supposed to be installed vertically at the junction between the two bars, as illustrated in Fig. 2. The combination of linear actuators with shape memory elements enables the development of variable geometry trusses that also have the ability of self-attenuate their vibration levels. This kind of adaptive structure could be very useful, for example, in aerospace applications.

On this basis, the related control variable u must be added to the equation of motion (6), which for control purposes could be simply rewritten as

$$\begin{aligned} x' &= y \\ y' &= f + d + u \end{aligned} \tag{7}$$

where u is the control action, $d = \gamma \sin(\Omega\tau)$ is an external disturbance assumed to be unknown, and $f = -\xi y + x\{ -[(\theta - 1) - 3\alpha_2 + 5\alpha_3] + [(\theta - 1) - \alpha_2 + \alpha_3](x^2 + b^2)^{-1/2} - [3\alpha_2 - 10\alpha_3](x^2 + b^2)^{1/2} + [-\alpha_2 + 10\alpha_3](x^2 + b^2) + 5\alpha_3(x^2 + b^2)^{3/2} - \alpha_3(x^2 + b^2)^2 \}$.

Regarding the development of the control law, the following assumptions must be made:

Assumption 1. *The state vector \mathbf{x} is available.*

Assumption 2. *The function f is unknown but bounded by a known function of \mathbf{x} , i.e. $|\hat{f}(\mathbf{x}) - f(\mathbf{x})| \leq \mathcal{F}(\mathbf{x})$ where \hat{f} is an estimate of f .*

Assumption 3. *The disturbance d is unknown but bounded, i.e. $|d| \leq \mathcal{D}$.*

Now, according to the control strategy described in [22] and considering the switching variable as $s = \tilde{y} + \lambda\tilde{x}$, the adaptive fuzzy sliding mode controller for the shape memory two-bar truss can be defined with the combination of an equivalent control $\hat{u} = -\hat{f} - \hat{d}(s) - \lambda\tilde{y}$ and another term, $-K\text{sat}(s/\phi)$, to confer robustness to the system:

$$u = -\hat{f} - \hat{d}(s) - \lambda\tilde{y} - K\text{sat}(s/\phi) \quad (8)$$

where \hat{d} is an estimate of d , K is a positive gain, ϕ is a strictly positive constant that represents the boundary layer thickness and $\text{sat}(s/\phi)$ is defined as

$$\text{sat}(s/\phi) = \begin{cases} \text{sgn}(s) & \text{if } |s/\phi| \geq 1 \\ s/\phi & \text{if } |s/\phi| < 1 \end{cases}$$

Considering Assumptions 2 and 3, the robustness of the adopted adaptive fuzzy sliding mode controller against parametric uncertainties, modeling inaccuracies and external disturbances is assured by defining the gain K according to [22]:

$$K \geq \eta + \mathcal{F} + \mathcal{D} + |\hat{d}| \quad (9)$$

where η is a strictly positive constant related to the time required to reach the sliding surface.

In order to obtain a good approximation to the disturbance d , the estimate \hat{d} is computed directly by an adaptive fuzzy algorithm. According to Kosko [31], fuzzy systems can be considered as universal approximator, and hence, they can approximate any function on a compact set to an arbitrary accuracy. The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang) [32], whose rules can be stated in a linguistic manner as follows:

$$\text{If } s \text{ is } S_r \text{ then } \hat{d}_r = \hat{D}_r; \quad r = 1, 2, \dots, N$$

where S_r are fuzzy sets, whose membership functions could be properly chosen, and \hat{D}_r is the output value of each one of the N fuzzy rules.

Considering that each rule defines a numerical value as output \hat{D}_r , the final output \hat{d} can be computed by a weighted average:

$$\hat{d}(s) = \hat{\mathbf{D}}^T \Psi(s) \quad (10)$$

where $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, \dots, \hat{D}_N]$ is the vector containing the attributed values \hat{D}_r to each rule r , $\Psi(s) = [\psi_1(s), \psi_2(s), \dots, \psi_N(s)]$ is a vector with components $\psi_r(s) = w_r / \sum_{r=1}^N w_r$ and w_r is the firing strength of each rule.

To ensure the best possible estimate $\hat{d}(s)$ to the disturbance d , the vector of adjustable parameters can be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{D}}} = \vartheta s \Psi(s) \quad (11)$$

where ϑ is a strictly positive constant related to the adaptation rate.

A detailed discussion on the boundedness of all closed-loop signals and the convergence properties of the adaptive fuzzy sliding mode control of n^{th} -order uncertain nonlinear systems is presented in [22].

4 Numerical simulations

Numerical simulations are now in focus exploring the controller capability to perform vibration reduction in smart structures. A fourth-order Runge-Kutta scheme is

Table 1. Material properties.

a_1 (MPa/K)	a_2 (MPa)	a_3 (MPa)	T_M (K)	T_A (K)
523.29	1.868×10^7	2.186×10^9	288	364.3

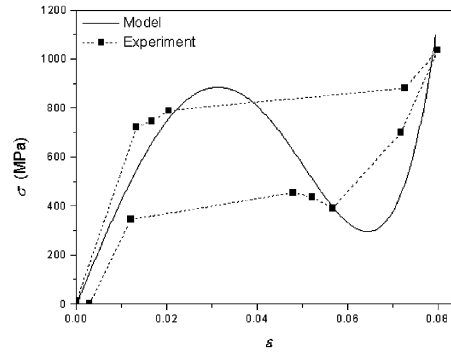


Fig. 3. Stress-strain curve: experimental and predicted by polynomial model.

adopted. Initially, we illustrate the rich dynamic behavior of the shape memory two-bar truss. Time steps are chosen according to $\Delta\tau = \pi/1000\Omega$. Uncontrolled dynamics of the two-bar truss was previous analyzed by Savi et al. [21], and some of these results are used here as references. In all simulations, the material properties presented in Table 1 are used. These values are chosen in order to match experimental data obtained by Sittner et al. [33] for a Cu-Zn-Al-Ni alloy at 373 K (see Fig. 3). For the data in Table 1, the parameters defined in Eq. (6) assume the values: $\alpha_2 = 1.240 \times 10^2$ and $\alpha_3 = 1.450 \times 10^4$. We further let $b = 0.866$, corresponding to a two-bar truss with an initial position $\varphi_0 = 30^\circ$.

Free vibration analysis of the uncontrolled SMA two-bar truss shows that it has several equilibrium points that are temperature dependent. It is well known that the elastic von Mises truss presents three equilibrium points due to kinematics nonlinearity. Of those, two are stable while the other one is unstable. In the case of a shape memory two-bar truss, constitutive nonlinearity introduces a different behavior. At low temperatures, where the martensitic phase is stable, there are seven equilibrium points, four of them stable while the others are unstable. By considering a higher temperature, where both martensitic and austenitic phases may coexist, the system exhibits five unstable and six stable equilibrium points. At an even higher temperature, where only the austenitic phase is present, the system has one unstable and two stable equilibrium points. See Savi et al. [21] for more details about free vibration analysis. These scenarios give an idea about the complex behavior of the SMA two-bar truss, especially when forced vibration is of concern.

A high temperature forced vibration behavior ($\theta = 1.30$), where the austenitic phase is stable, is now in focus. Figure 4(a) shows the bifurcation diagram for γ , with $\Omega = 0.1$ and $\xi = 0.01$. It could be easily observed that the shape memory two-bar truss presents both periodic and chaotic behavior for different values of γ . Considering, as for instance $\gamma = 0.015$, Fig. 4(b) presents the Poincaré section with a strange attractor, which illustrates the chaotic motion of the system.

By assuming low temperature forced vibration behavior ($\theta = 0.69$), where the martensitic phase is stable, Fig. 5(a) shows the related bifurcation diagram for γ , with $\Omega = 0.5$ and $\xi = 0.05$. Periodic and chaotic behavior are again observed for

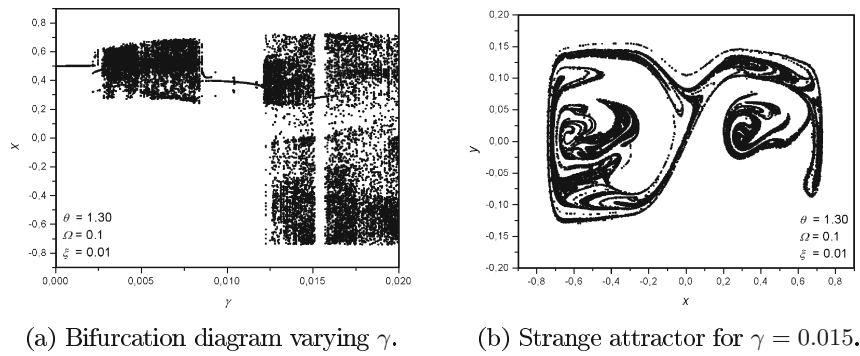


Fig. 4. Chaotic response for $\theta = 1.30$, $\Omega = 0.1$ and $\xi = 0.01$.

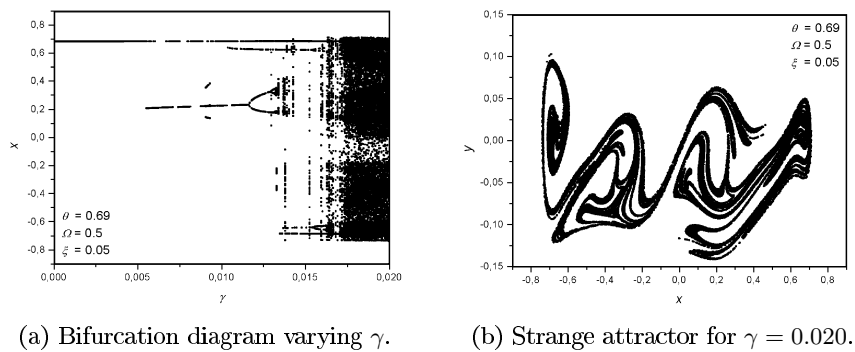


Fig. 5. Chaotic response for $\theta = 0.69$, $\Omega = 0.5$ and $\xi = 0.05$.

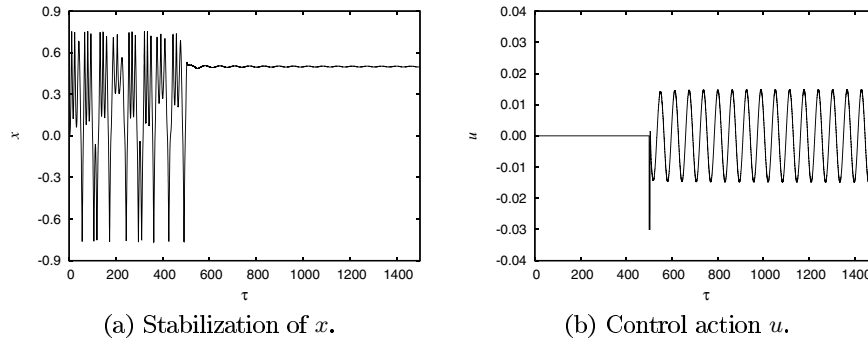


Fig. 6. Controller performance for $\theta = 1.30$, $\Omega = 0.1$, $\xi = 0.01$ and $\gamma = 0.015$.

different values of γ . Figure 5(b), for example, presents the Poincaré section for $\gamma = 0.020$ with the strange attractor corresponding to chaotic dynamics.

The controller capabilities are now investigated. Sampling rates of $200\Omega/\pi$ for control system and $1000\Omega/\pi$ for dynamical model are assumed. In order to demonstrate that the adopted control scheme can deal with both modeling inaccuracies and external disturbances, an uncertainty of $\pm 20\%$ over the values of α_2 and α_3 is considered. Moreover, the periodic excitation is treated as an unknown external disturbance. Under this assumption, $\gamma \sin(\Omega\tau)$ is not taken into account within the design of the control law. On this basis, it is assumed estimation values as

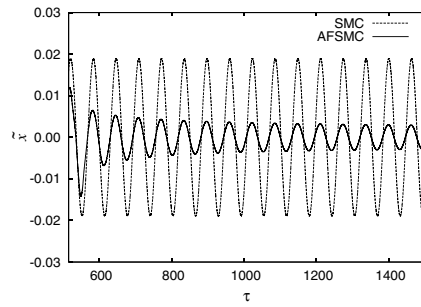


Fig. 7. Stabilization error for $\theta = 1.30$, $\Omega = 0.1$, $\xi = 0.01$ and $\gamma = 0.015$.

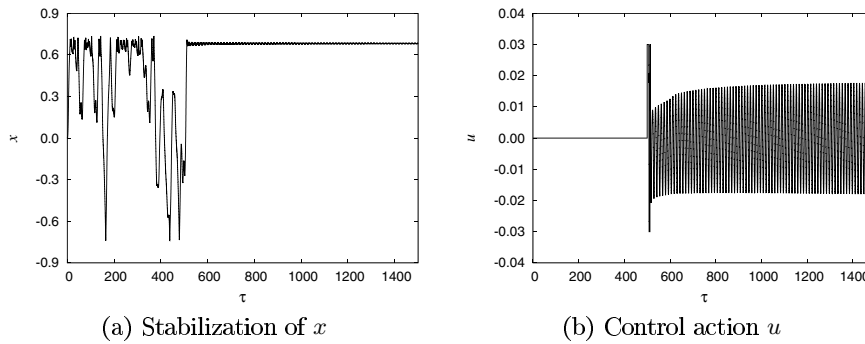


Fig. 8. Controller performance for $\theta = 0.69$, $\Omega = 0.5$, $\xi = 0.05$ and $\gamma = 0.020$.

$\hat{\alpha}_2 = 10^2$ and $\hat{\alpha}_3 = 1.15 \times 10^4$. The other estimates in \hat{f} are chosen based on the assumption that model coefficients are perfectly known. Besides, the following parameters are adopted: $\mathcal{F} = 0.05$; $\mathcal{D} = 0.05$; $\phi = 0.1$; $\lambda = 0.6$; $\eta = 0.03$ and $\vartheta = 0.1$. Concerning the fuzzy system, trapezoidal (at the borders) and triangular (in the middle) membership functions are adopted for S_r , with the central values defined respectively as $C = \{-5.0; -1.0; -0.5; 0.0; 0.5; 1.0; 5.0\} \times 10^{-2}$. It is also important to emphasize that the vector of adjustable parameters is initialized with zero values, $\hat{\mathbf{D}} = \mathbf{0}$, and updated at each iteration step according to the adaptation law (11).

The stabilization of the state vector in the neighborhood of one of the equilibrium points of the shape memory two-bar truss [21] is carried out. This approach shows that the adopted control scheme can significantly reduce the vibration level and also avoid the undesired snap-through behavior. Figures 6 and 7 show the obtained results considering $\mathbf{x}_d = [0.5, 0.0]$, $\theta = 1.30$, $\Omega = 0.1$, $\xi = 0.01$ and $\gamma = 0.015$, and Figs. 8 and 9 present the corresponding results for $\mathbf{x}_d = [0.68, 0.0]$, $\theta = 0.69$, $\Omega = 0.5$, $\xi = 0.05$ and $\gamma = 0.020$. Note that the chaotic behavior with large amplitudes of the uncontrolled response is replaced by a regular behavior with small amplitudes around the equilibrium point.

As observed in Figs. 6–9, even in the presence of modeling inaccuracies and external disturbance, the adaptive fuzzy sliding mode controller (AFSMC) is able to provide the desired stabilization with a small associated error. This is an important point to treat complex systems as SMA structures since it allows the use of simple constitutive models for control purposes.

It should also be highlighted that the proposed control law provides a smaller stabilization error when compared with the conventional sliding mode controller (SMC), Figs. 7 and 9. By considering simulation purposes, the AFSMC can be easily converted to the classical SMC by setting the adaptation rate to zero, $\vartheta = 0$.

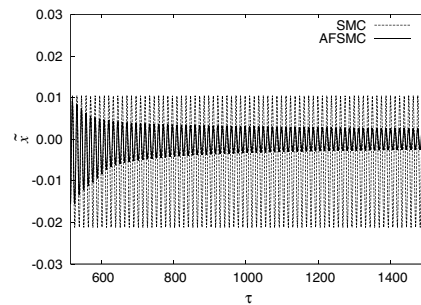


Fig. 9. Stabilization error for $\theta = 0.69$, $\Omega = 0.5$, $\xi = 0.05$ and $\gamma = 0.020$.

5 Concluding remarks

In this paper, an adaptive fuzzy sliding mode controller is considered for vibration reduction in a shape memory two-bar truss. A polynomial constitutive model is assumed to describe the constitutive behavior of the bars. Despite the deceiving simplicity, this model allows an appropriate qualitative description of system dynamics, which can exhibit chaotic behavior. Numerical simulations show the robustness of the AFSMC against modeling inaccuracies and external disturbances. The improved performance over the conventional sliding mode controller is also demonstrated. It should be highlighted that the controller robustness to modeling inaccuracies is an important issue that allows the use of a simple constitutive model for control purposes.

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